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MULTIPLE PRODUCTION OF SUPERSYMMETRIC HIGGS BOSONS IN Z^o DECAYS

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Abstract

Multi-Higgs boson production in Z^o decays is discussed in the context of low energy supergravity models. For Higgs boson masses lighter than about 20 GeV, $Z^o \to H_2^o H_2^o H_3^o$ naturally has a branching ratio in the range $10^{-4} - 10^{-6}$ and, for $m_{H_2} < 10$ GeV, $Z^o \to H_2^o H_2^o f\bar{f}$ can be within reach of LEP. Detection of these processes will give information about the structure of the Higgs sector and about the $HHZZ$ and HHH couplings.

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Future experiments in e^+e^- collisions at the Z^o peak will be able to search for the Higgs boson in a large range of masses. When LEP reaches its planned luminosity, it can probe the Standard Model (SM) Higgs boson (H^o) up to a mass of about 50 GeV, through the process [1]:

$$Z^o \to H^o \mu^+ \mu^-, \tag{1}$$

where H^o is produced via the HZZ coupling. Moreover, the radiative decay [2]:

$$Z^o \to H^o \gamma \tag{2}$$

could also be detected at LEP, although with a much smaller rate than (1)[1].

Recently, it has been pointed out [3] that the process

$$Z^o \to H^o H^o f\overline{f} \tag{3}$$

has a rate within the reach of LEP, if the Higgs boson mass is less than about 10 GeV. The detection of this decay is mainly interesting because, being sensitive to the $HHZZ$ and HHH couplings, it represents a crucial test of the Higgs sector. In view of this intriguing possibility, it is important to compare the SM results for the process (3) with the predictions coming from new theoretical models. This paper will study the processes analogous to (3) in the minimal low energy supergravity model, compare the results with the SM predictions and briefly discuss their sensitivity to the $HHZZ$ and HHH couplings.

It is worth mentioning that all processes considered in this paper can also occur in a SM scenario with two Higgs doublets. However, the strong constraints imposed by supersymmetry, which allow to relate the masses of the various Higgs bosons and the different production processes, are no longer valid.

Low energy supergravity is one of the best–motivated extensions of the SM. The introduction of supersymmetry at the Fermi scale is the only known way to describe a light elementary Higgs boson consistent with the naturalness criterion [4]. Therefore, the experimental discovery of the Higgs boson would deeply strengthen the motivation for supersymmetry. In any case, the Higgs search represents a crucial test for any low energy supergravity model.

[1] Only for a rather heavy H^o ($m_H \gtrsim 60$ GeV), the process (2) becomes dominant over (1).

The minimal $N = 1$ supergravity model requires two Higgs doublets. Then, the supersymmetric Higgs sector [5] contains two real scalars (H_1^o, H_2^o), a pseudoscalar (H_3^o) and two charged scalars ($H^{\pm}$) as physical states. Because of the constraints imposed by supersymmetry, the quartic Higgs couplings are related to the gauge couplings and the Higgs sector is fully describable in terms of only two free parameters. I choose as independent variables $m_{H_2^o}$, the mass of the lighter of the two real scalars and v_2/v_1, the ratio of the vacuum expectation values of the two Higgs neutral components. The masses of the other Higgs particles can then be written as:

$$\frac{m^2_{H_1^o}}{m^2_Z} = c^2 \left(\frac{1-r}{c^2-r}\right) \tag{4}$$

$$\frac{m^2_{H_3^o}}{m^2_Z} = r \left(\frac{1-r}{c^2-r}\right) \tag{5}$$

$$m^2_{H^{\pm}} = m^2_{H_3^o} + m^2_{W^{\pm}} \tag{6}$$

where

$$r \equiv \frac{m^2_{H_2^o}}{m^2_Z},\ c \equiv cos2\beta,\ tan\beta \equiv \frac{v_2}{v_1}, \tag{7}$$

$$r \leq c^2. \tag{8}$$

Since v_2 is the vacuum expectation value which gives mass to up-type quarks, one can choose $\frac{\pi}{4} \leq \beta \leq \frac{\pi}{2}$. From eqs. (4-8), one infers $m_{H_2^o} \leq m_{Z^o} \leq m_{H_1^o}$, $m_{H_2^o} \leq m_{H_3^o}$ and $m_{H^{\pm}} \geq m_{W^{\pm}}$. Supersymmetry implies that H_2^o should be necessarily lighter than the Z^o boson, while H_1^o and $H^{\pm}$, being heavy, do not play an interesting role at LEP.

The production of supersymmetric Higgs bosons in Z^o decays can occur via the process

$$Z^o \rightarrow H_2^o \mu^+ \mu^-, \tag{9}$$

analogous to (1)[2], or via [5-7]

$$Z^o \rightarrow H_2^o H_3^o. \tag{10}$$

The decay rate for (9) is γ^2 times the rate for the SM process (1), where γ is the $H_2^o ZZ$ coupling in units of SM Higgs coupling given by[3]:

$$\frac{H_2^o ZZ}{H^o ZZ} = \left[\frac{c^2(1-c^2)}{r^2+c^2-2rc^2}\right]^{\frac{1}{2}} \equiv \gamma. \tag{11}$$

[2]The decay $Z^o \rightarrow H_3^o \mu^+ \mu^-$ is highly suppressed, since the $H_3^o ZZ$ coupling vanishes at tree level.
[3]The Feynman rules for the supersymmetric Higgs sector can be found in ref. [5].

The rate for (10) is:

$$\frac{\Gamma(Z^o \to H_2^o H_3^o)}{\Gamma(Z^o \to \nu_e \overline{\nu}_e)} = \frac{\epsilon^2}{2}\left[\left(1 - \frac{m_{H_2}^2 + m_{H_3}^2}{m_Z^2}\right)^2 - \frac{4m_{H_2}^2 m_{H_3}^2}{m_Z^4}\right]^{\frac{3}{2}} \tag{12}$$

where ϵ is proportional to the $H_2^o H_3^o Z$ coupling:

$$\epsilon \equiv \frac{r - c^2}{(r^2 + c^2 - 2rc^2)^{\frac{1}{2}}}. \tag{13}$$

If $\frac{v_2}{v_1} \simeq 1$, the rates for the processes (9) and (1) are nearly equal, the decay (10) is suppressed, and H_2^o is undistinguishable from a SM Higgs boson. However, if $\frac{v_2}{v_1}$ is somewhat larger than 1, as suggested by supergravity models with radiative gauge symmetry breaking, H_3^o becomes almost degenerate in mass with H_2^o (see eq.(5)) and the process (10) rapidly dominates over (9), giving a different Higgs signature at LEP (see ref. [7]).

The processes analogous to (2) [8]:

$$Z^o \to H_2^o \gamma,\ Z^o \to H_3^o \gamma \tag{14}$$

could also be observed at LEP. Since these decays occur at the one-loop level, they depend on the mass spectrum of all the charged supersymmetric particles. Slight enhancements with respect to the SM are possible.

If the Higgs boson is discovered at LEP, multi-Higgs production processes can provide us with interesting information about the structure of the Higgs sector. The processes analogous to (3) are now:

$$Z^o \to H_2^o H_2^o f\overline{f},\ Z^o \to H_3^o H_3^o f\overline{f}, \tag{15}$$

which occur through the diagrams of fig. 1 and 2 respectively. These processes are sensitive to the $HHZZ$ and HHH couplings. In units of the SM Higgs couplings, one finds, in the supersymmetric case, the following Feynman rules for the relevant vertices:

$$\frac{H_2^o H_2^o H_2^o}{H^o H^o H^o} = \frac{m_{H_2^o}^2}{m_{H^o}^2}\,\delta \tag{16}$$

$$\frac{H_2^o H_3^o H_3^o}{H^o H^o H^o} = -\frac{m_{H_2^o}^2}{m_{H^o}^2}\,\frac{\gamma}{3} \tag{17}$$

$$\frac{H_2^o H_2^o ZZ}{H^o H^o ZZ} = \frac{H_3^o H_3^o ZZ}{H^o H^o ZZ} = 1, \tag{18}$$

where γ is defined in eq. (11) and

$$\delta \equiv \frac{c(r^2+c^2-2r)(1-c^2)^{\frac{1}{2}}}{(r^2+c^2-2rc^2)^{\frac{3}{2}}}. \tag{19}$$

The coefficients δ and γ appearing in eqs.(16)-(17) are, in absolute value, smaller than one and, in the light Higgs boson limit, are suppressed when $\frac{v_2}{v_1}$ is large.

Given the Higgs boson couplings eqs. (11, 16-18), one can compute the decay rates for the processes (15):

$$\Gamma(Z^o \to H_i^o H_i^o f\bar{f}) = \frac{32}{3}\frac{\pi^3\alpha^3}{sin^6\theta_W cos^6\theta_W}\frac{(C_L^2+C_R^2)}{m_Z^3}\int d\phi^{(4)} \frac{(g^{\mu\nu}m_Z^2 - p_Z^\mu p_Z^\nu)}{(2k_1k_2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}\cdot$$

$$\cdot(g^{\rho\sigma}k_1k_2 - k_1^\rho k_2^\sigma - k_2^\rho k_1^\sigma)\Lambda_{\mu\rho}^{(i)}\Lambda_{\nu\sigma}^{*(i)}\ . \tag{20}$$

where H_i^o is one of the three kinds of Higgs boson: H_2^o, H_3^o or the SM Higgs boson H^o. Also, p_Z, k_1, k_2, p_1, p_2 are respectively the Z^o, massless fermions and Higgs bosons 4-momenta and $d\phi^{(4)}$ is the phase space measure. C_L, C_R are the left- and right-handed fermion couplings to the Z^o ($C_L \equiv Q\ sin^2\theta_W - T_3$, $C_R \equiv Q sin^2\theta_W$) and

$$\Lambda_{\mu\nu}^{(i)} \equiv \frac{1}{2}g_{\mu\nu}(1+3K^{(i)}) + J^{(i)}\left[g_{\mu\nu}(D_1^{(i)}+D_2^{(i)}) + \frac{p_{1\mu}p_{2\nu}}{m_Z^2}D_1^{(i)} + \frac{p_{2\mu}p_{1\nu}}{m_Z^2}D_2^{(i)}\right], \tag{21}$$

$$D_j^{(i)} \equiv \frac{m_Z^2}{m_{H_i^o}^2 - 2p_Zp_j + i\Gamma_Z m_Z},\ j=1,2. \tag{22}$$

Finally, for the SM Higgs boson:

$$K = \frac{m_{H^o}^2}{2p_1p_2 + m_{H^o}^2},\ J=1, \tag{23}$$

for the supersymmetric Higgs boson H_2^o:

$$K = \gamma\delta\frac{m_{H_2^o}^2}{2p_1p_2 + m_{H_2^o}^2},\ J=\gamma^2, \tag{24}$$

and for the pseudoscalar H_3^o:

$$K = -\frac{\gamma^2}{3}\frac{m_{H_2^o}^2}{2p_1p_2 + 2m_{H_3^o}^2 - m_{H_2^o}^2},\ J=0. \tag{25}$$

Fig. 3 shows $\Gamma(Z^o \to H_2^o H_2^o f\bar{f})$ in units of the SM decay width $\Gamma(Z^o \to H^o H^o f\bar{f})$, for $m_{H_2^o} = m_{H^o}$[4]. The supersymmetric process is in general suppressed. Since the diagrams 2c give the leading contribution, the suppression factor with respect to the SM decay is, for small Higgs boson masses, roughly equal to $\gamma^4 \simeq \sin^4 2\beta$, where $tan\, \beta \equiv \frac{v_2}{v_1}$. Similarly to the SM case, the dependence of the total rate on the $HHZZ$ and HHH couplings is rather weak, since the diagram 2c dominates over 2a and 2b. Lacking the leading contribution, $Z^o \to H_3^o H_3^o f\bar{f}$ has a much smaller rate and is completely unobservable at LEP.

Unlike the SM, supersymmetry may lead, besides the decays (15), to triple Higgs boson production:

$$Z^o \to H_2^o H_2^o H_3^o,\ Z^o \to H_3^o H_3^o H_3^o. \tag{26}$$

The processes occur through the diagrams of figs. 4 and 5 respectively, which include the trilinear Higgs coupling, and yield the partial decay widths:

$$\Gamma(Z^o \to H_2^o H_2^o H_3^o) = \frac{\pi^2\alpha^2}{3 sin^4\theta_W cos^4\theta_W}\frac{\gamma^2\epsilon^2}{m_Z^3}\int d\phi^{(3)}\left|\sum_{i=1}^{3}\vec{p}_i A_i\right|^2, \tag{27}$$

$$\Gamma(Z^o \to H_3^o H_3^o H_3^o) = \frac{\pi^2\alpha^2}{9 sin^4\theta_W cos^4\theta_W}\frac{\gamma^2\epsilon^2}{m_Z^3}\int d\phi^{(3)}\left|\sum_{i=1}^{3}\vec{p}_i S_i\right|^2. \tag{28}$$

$d\phi^{(3)}$ is the 3-body phase space and E_i, $\vec{p}_i$ $(i = 1,2,3)$ are respectively energy and momentum of the Higgs bosons in the Z^o rest frame. In the case of eq. (27), the index $i = 3$ refers to the pseudoscalar H_3^o. Also

$$A_1 \equiv S_1' + \frac{(m_{H_3^o}^2 - m_{H_2^o}^2)}{m_Z^2}\ D_1 - D_2 \tag{29}$$

$$A_2 \equiv S_2' + \frac{(m_{H_3^o}^2 - m_{H_2^o}^2)}{m_Z^2}\ D_2 - D_1 \tag{30}$$

$$A_3 \equiv 3\frac{\delta}{\gamma}S_3 + D_1 + D_2 \tag{31}$$

$$D_i \equiv \frac{m_Z^2}{m_{H_2}^2 - 2m_Z E_i + i\Gamma_Z m_Z} \tag{32}$$

[4]In the numerical calculations throughout this paper, I have taken $m_Z = 92$ GeV, $\sin^2\theta_W = 0.23$ and $\alpha = 1/128$.

$$S_i \equiv \frac{m^2_{H_2}}{m^2_Z + m^2_{H_3} - m^2_{H_2} - 2m_Z E_i}, \; i = 1,2,3 \tag{33}$$

$$S'_i \equiv \frac{m^2_{H_2}}{m^2_Z + m^2_{H_2} - m^2_{H_3} - 2m_Z E_i}, \; i = 1,2. \tag{34}$$

The decay rate for $Z^o \to H^o_2 H^o_2 H^o_3$ as a function of m_{H_2}, for three different values of $\frac{v_2}{v_1}$, is shown in fig. 5. The mass of H^o_3 is fixed by the choice of these two parameters (see eq. (5)). For m_{H_2} smaller than about 20 GeV, the branching ratio for triple Higgs boson decay of the Z^o is naturally in the range $10^{-6} - 10^{-4}$ and a sizable number of events is expected at LEP. The leading contribution to the process comes from the diagram 4c, which is independent of the trilinear Higgs coupling. This also means that the decay rate, for light Higgs bosons, is roughly proportional to $sin^2 4\beta$, where $tan\,\beta \equiv \frac{v_2}{v_1}$. Thus the maximum signal is expected to occur for $\beta = \frac{3\pi}{8}$, i.e. $\frac{v_2}{v_1} \simeq 2.4$. The decay $Z^o \to H^o_3 H^o_3 H^o_3$, lacking the leading contribution (see fig. 5) has a much smaller rate and it is unobservable at LEP, for any value of the H^o_3 mass.

Fig. 7 compares the rates for the different Higgs boson production processes from Z^o decays, for the choice $\frac{v_2}{v_1} = 2$. The rates for $Z^o \to H^o_i H^o_i f\bar{f}$ $(i = 2,3)$ have been summed over all leptonic modes $(f = e, \mu, \tau, \nu)$, in order to allow an easier comparison with the SM predictions, given in ref. [3]. The rate for $Z^o \to H^o_i H^o_i \mu^+\mu^-$ is then about 9 times smaller. The predictions for the processes (14) are not shown, since they depend also on the unknown supersymmetric mass parameters (see ref. [8]). If kinematically allowed, $Z^o \to H^o_2 H^o_3$ dominates over $Z^o \to H^o_2 \mu^+\mu^-$, which is suppressed with respect to the analogous SM process. A sizable rate for $Z^o \to H^o_2 H^o_2 H^o_3$ is also predicted, while $Z^o \to H^o_2 H^o_2 f\bar{f}$ is suppressed with respect to the SM. The rates for $Z^o \to H^o_3 H^o_3 f\bar{f}$ and $Z^o \to H^o_3 H^o_3 H^o_3$ are unobservably small.

The effect of varying the value of $\frac{v_2}{v_1}$ is the following. Increasing $\frac{v_2}{v_1}$ will further enhance $Z^o \to H^o_2 H^o_3$ and suppress $Z^o \to H_2 \mu^+\mu^-$, $H^o_2 H^o_2 f\bar{f}$. The rate for $Z^o \to H^o_2 H^o_2 H^o_3$ has a maximum for $\frac{v_2}{v_1} \simeq 2.4$ and then decreases for larger $\frac{v_2}{v_1}$. On the contrary, if $\frac{v_2}{v_1}$ is very close to one, H^o_2 is forced to be very light, and the supersymmetric Higgs production processes will closely resemble those of the SM scenario. It is important to stress that, given the large expected value of the top quark mass, $\frac{v_2}{v_1} \simeq 1$ is a rather unnatural solution of the conditions for supergravity induced electroweak symmetry breaking. Larger values of $\frac{v_2}{v_1}$ are much more likely to occur and then, as shown in this paper, the supersymmetric Higgs sector will have very distinctive features.

In conclusion, if a light Higgs boson is discovered at LEP, detection of rare production processes can provide interesting information and constraints about the structure of non-minimal Higgs sectors. Low energy supergravity models lead to definite and distinctive predictions, which can be tested at LEP.

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Figure Captions

Fig. 1 Feynman diagrams for $Z^o \to H_2^o H_2^o f\bar{f}$.

Fig. 2 Feynman diagrams for $Z^o \to H_3^o H_3^o f\bar{f}$.

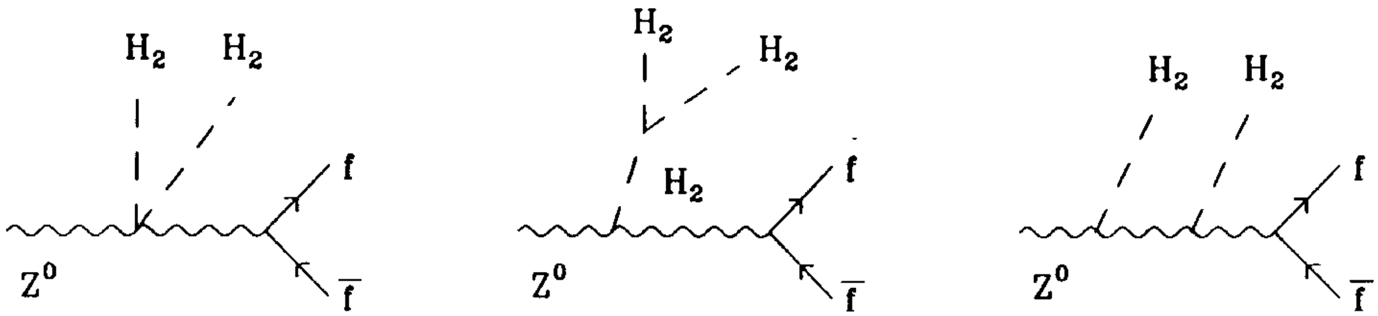
Fig. 3 $\Gamma(Z^o \to H_2^o H_2^o f\bar{f})$ in units of the SM process $\Gamma(Z^o \to H^o H^o f\bar{f})$ for $m_{H_2^o} = m_{H^o}$. In the case $\frac{v_2}{v_1} = 1.1$, consistency requires $m_{H^o} \lesssim 9$ GeV (see eq. (8)).

Fig. 4 Feynman diagrams for $Z^o \to H_2^o H_2^o H_3^o$.

Fig. 5 Feynman diagrams for $Z^o \to H_3^o H_3^o H_3^o$.

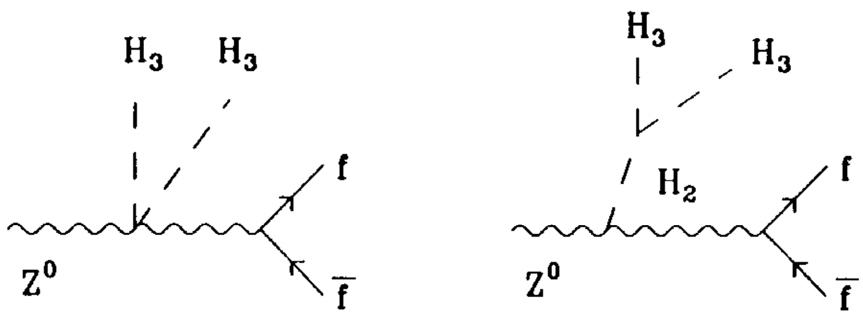
Fig. 6 $\Gamma(Z^o \to H_2^o H_2^o H_3^o)$ in units of $\Gamma(Z^o \to \mu^+\mu^-)$ as a function of $m_{H_2^o}$.

Fig. 7 The partial width of the different Z^o decays into supersymmetric Higgs bosons, for $\frac{v_2}{v_1} = 2$. The scale on the left shows the rate in units of $\Gamma(Z^o \to \mu^+\mu^-)$ and the scale on the right shows the branching ratio of the process, assuming $BR(Z^o \to \mu^+\mu^-) = 3.3\%$.



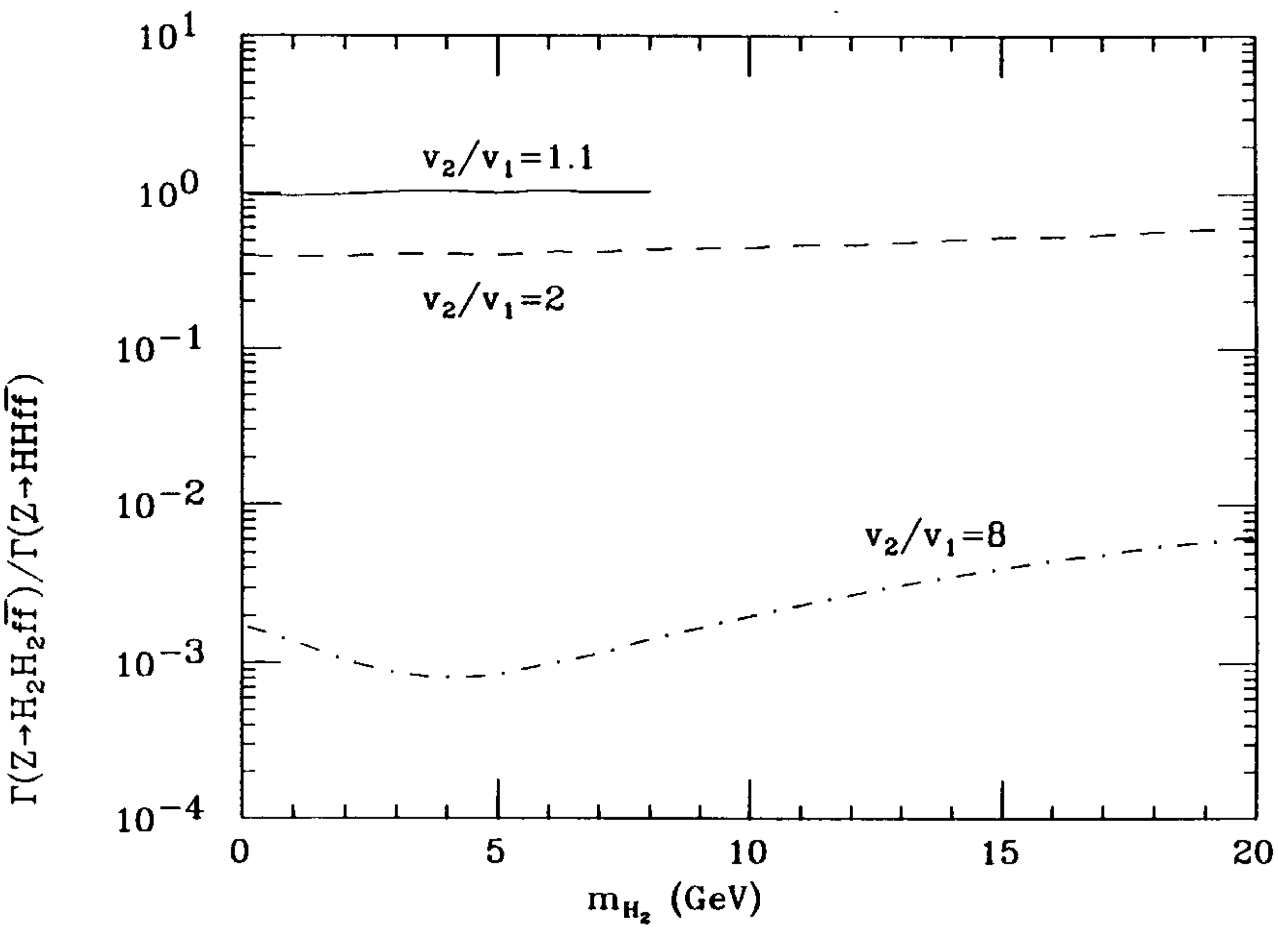
(a) (b) (c)

Fig. 1



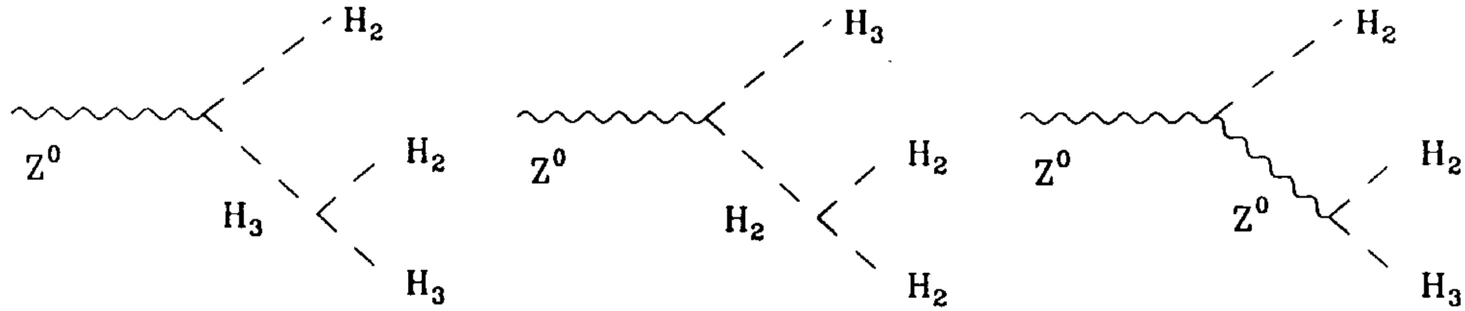
(a) (b)

Fig. 2



$v_2/v_1=1.1$
$v_2/v_1=2$
$v_2/v_1=8$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\Gamma(Z\to H_2H_2f\bar{f})/\Gamma(Z\to HHf\bar{f})$
0
5
10
15
20
m_{H_2} (GeV)

Fig. 3



(a) (b) (c)

Fig. 4

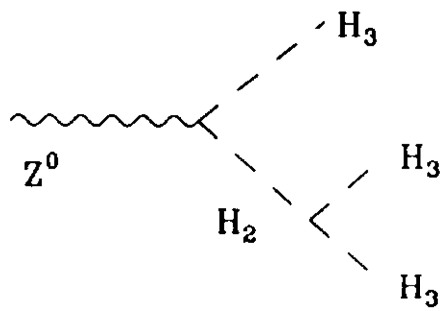
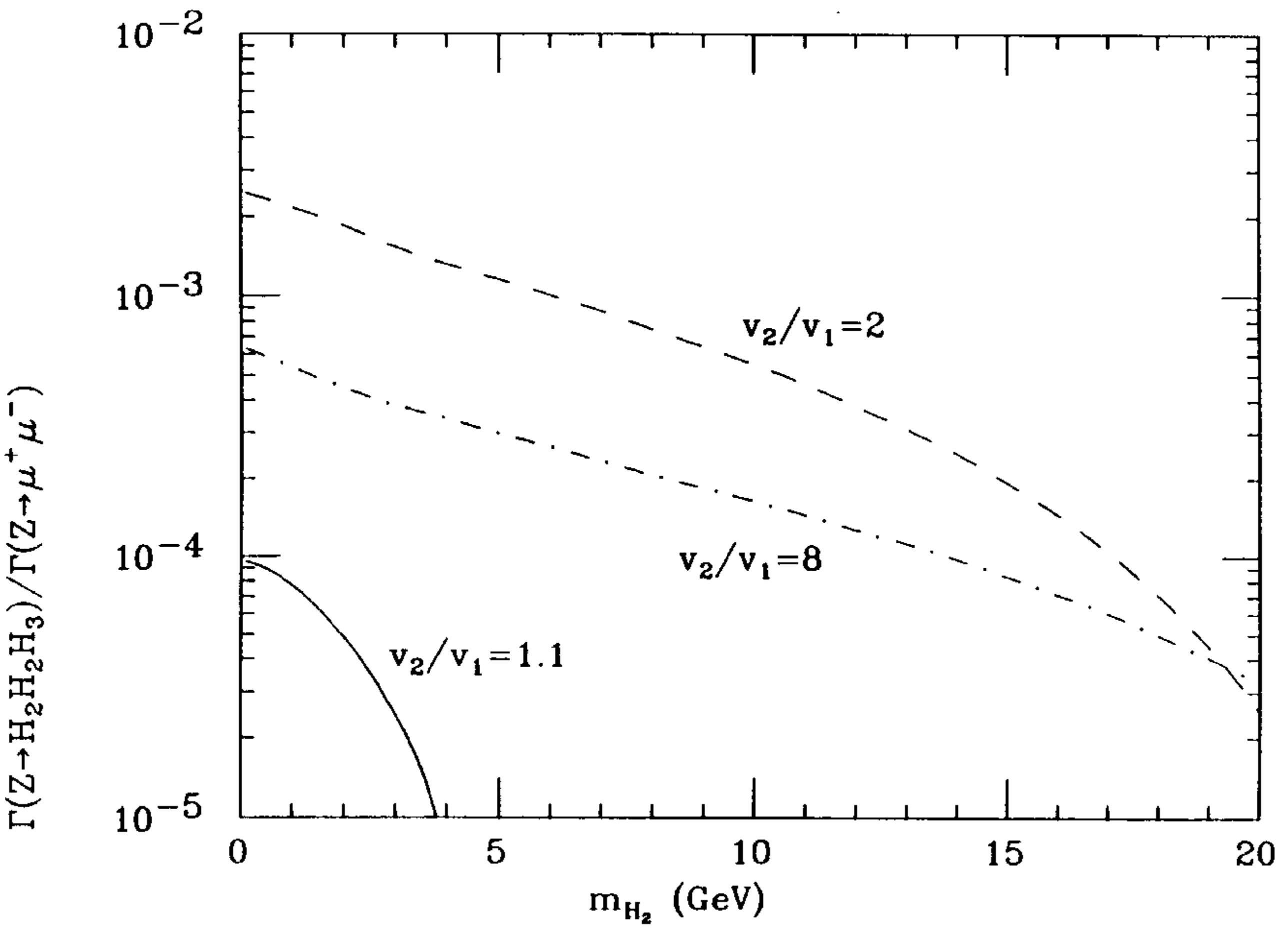


Fig. 5



10−2
10−3
10−4
10−5
0
5
10
15
20
v2/v1=2
v2/v1=8
v2/v1=1.1
m_H2 (GeV)
Γ(Z→H2H2H3)/Γ(Z→μ+μ−)

Fig. 6

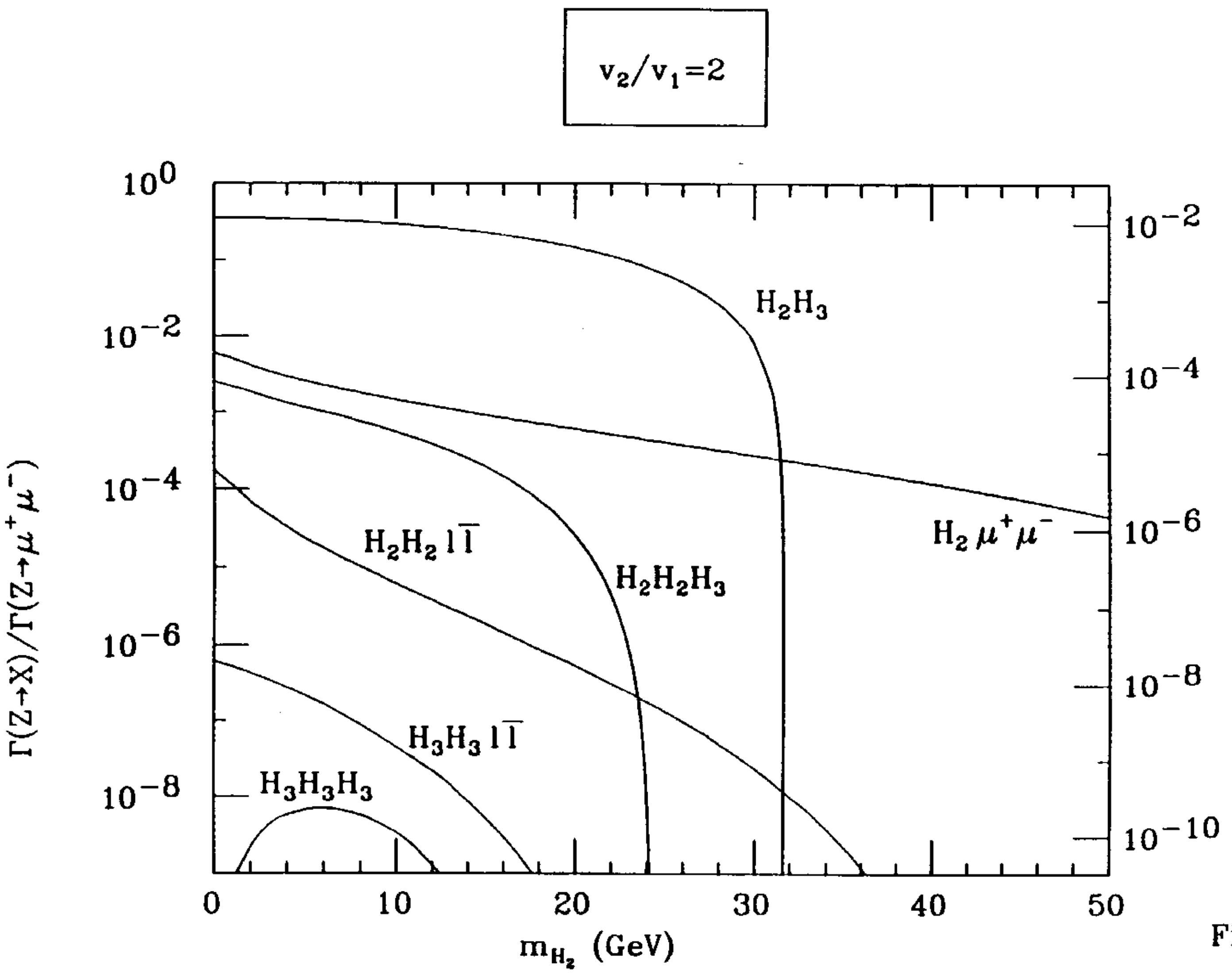

$v_2/v_1=2$
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$\Gamma(Z\to X)/\Gamma(Z\to\mu^+\mu^-)$
BR($Z\to X$)
H_2H_3
$H_2\mu^+\mu^-$
$H_2H_2H_3$
$H_2H_2 l\bar{l}$
$H_3H_3 l\bar{l}$
$H_3H_3H_3$
0
10
20
30
40
50
m_{H_2} (GeV)

Fig. 7