Fermi National Accelerator Laboratory

FERMILAB-PUB-89/141-T

CERN-TH.5433/89

Quenching The Cosmological Constant

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Abstract

We propose and try to justify a model – inspired by known features of one-loop quantum gravity corrections – in which the effective cosmological constant is consistent with inflationary needs at early times and sufficiently suppressed at later epochs.

Ref.CERN-TH.5433/89

May 1989

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

1. Introduction

In a fundamental theory of quantum gravity, such as Superstring Theory, the physical cosmological constant Λ is best defined as the coefficient of $g_{\mu\nu}$ on the right hand side of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} + \text{sources} + \text{higher deriv. terms}, \tag{1}$$

seen as the Euler-Lagrange equations of the low-energy effective action of the theory.

The great mystery about the smallness of Λ is reinforced by the need to have a large Λ [say $\mathcal{O}(1)$ in Planck units] in the early universe in order to sustain inflation. As the universe expands and cools down, and after many changes due to all sorts of phase transitions, symmetry breakings *etc.*, Λ should eventually relax to an infinitesimally small value [say $\mathcal{O}(10^{-120})$ in Planck units] in the present epoch.

A new line of approach to this long-standing problem was started a few years ago by Baum [1] and Hawking [2], and was later developed by Coleman in a very influential and stimulating paper [3]. Related ideas have been also put forward by Banks [4]. The general framework invoked in these papers is that of Euclidean Quantum Gravity (EQG) and the specific configurations employed in the functional integral are the so-called wormholes [5].

After a few months of great excitement, the wormhole scenario appears now to suffer from many problems, both at the "conceptual" and at the "phenomenological" levels. In our opinion, the most serious objections in the first category are related to the fact [6,7] that standard wormholes *lower* the Euclidean action in such a way as to make it *unbounded* from below at large Euclidean volumes V. It is precisely this unboundedness that gives Coleman's result $\Lambda \to 0 \Leftrightarrow V \to \infty$, and that casts, at the same time, great doubts on the legitimacy of the conclusion. In particular, $\Lambda = 0$ does *not* correspond to any stationary point of the non-local, wormhole-induced effective action.

In spite of this and other criticisms, we believe that it would be too hasty to abandon the hope that EQG contains the solution of the Cosmological Constant Problem. The general idea that some quantum gravity corrections could become sizeable at large distances and thus affect the present value of Λ (without changing it in the early universe) does seem very appealing.

A wormhole-like mechanism using just saddle points and doing precisely this job was put forward in ref.[6]. It would leave a positive Λ_0 unaffected, thus saving inflation in the early universe. However, as the universe cools and Λ_0 rolls down towards its would-be final, negative value, the non-local wormhole term effectively renormalizes it to a very tiny positive quantity. The problems with that proposal are basically two:

i) wormholes should contribute to the action with a sign opposite to Coleman's;

ii) the mechanism becomes ineffective if one assumes that the contribution of other universes leads to Coleman's double exponential.

In this short note, we present an attempt which does not involve exotic and poorly understood effects of the wormhole type: it is supposed to generate a similar mechanism by invoking nothing other than "standard" quantum gravity radiative corrections. We should point out that this idea is not entirely new. Other authors [8] in the past have analyzed EQG corrections to Einstein's action with a cosmological term, with the goal of finding an instability of the classical (DeSitter) solution in favour of flat space-time. The previous proposals, however, suffered from being either limited to "on-shell" calculations or from being affected by the notorious gauge-dependence problem of quantum corrections to the off-shell effective action. Our approach is free of these problems. However, as we shall see later on, it is not yet complete enough for deciding whether quantum effects lead to the desired rearrangement of the gravitational vacuum. We hope to convince the reader that every effort should be made to improve our understanding of radiative corrections at large distances in EQG in order to find out if our proposal – or a suitable modification thereof – will work.

2. A toy model that works

We shall start by producing a sort of toy model which works and which possesses many of the features that radiative corrections are supposed to exhibit. We shall then try to find out how close we can get to what we need by some bona-fide calculations.

The toy model assumes that the tree-level action:

$$S = \Gamma_0 = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{g}\,(2\Lambda_0 - R) \tag{2}$$

is modified by quantum effects at large $V = \int \mathrm{d}^4x\sqrt{g}$ by a term:

$$\Delta S = \Gamma_1 = \beta\,\Lambda_0^2\,V\log(V/\lambda^4)\,, \tag{3}$$

where β is a pure number and λ is the ultra-violet cut-off of the theory in position space. We do not know of any other way of making sense of EQG other than assuming the existence of a finite UV cut-off somewhat larger than Planck's length [9]. A candidate theory providing just such a cut-off is, of course, String Theory, where:

$$\lambda = \lambda_s = \sqrt{2\hbar\alpha'}\,, \qquad \alpha' = \text{ inverse string tension} \tag{4}$$

and

$$\lambda_s\sqrt{\alpha_G} \ = \ \lambda_P \ = \ \sqrt{\hbar G}\,, \qquad \alpha_G = \ \text{Grand Unified coupling constant.} \tag{5}$$

Although we shall have in mind string theory as the way to regulate EQG, the reader can replace it with her/his favourite way provided it does not spoil general covariance.

A few remarks are in order before we proceed to using eq.(3). The fact that, at one loop, the logarithm of the UV cut-off multiplies a term proportional to $\Lambda_0^2 V$ is well known [10,11]. In any given theory the coefficient β can be reliably computed too (see below). What is much less obvious – and which we shall discuss later – is that the infra-red (IR) scale of the logarithm is just the volume itself. Adding now eqs.(2) and (3) into the effective action $S + \Delta S$, we find the modified "classical" equations:

$$R_{\mu\nu} \ - \ \tfrac{1}{2}\, g_{\mu\nu} R \ = \ -\{\Lambda_0 \ + \ 8\pi\beta G\,\Lambda_0^2\,[\log(V/\lambda^4) + 1]\}\; g_{\mu\nu}\,, \tag{6}$$

whose solution is again a DeSitter universe, $R_{\mu\nu} = \Lambda g_{\mu\nu}$, $V = 24\pi^2/\Lambda^2$, where the renormalized, or effective Λ is given by the "bootstrap" condition:

$$\Lambda \ = \ \Lambda_0 \ - \ 16\pi\beta G\,\Lambda_0^2\,[\log(\Lambda\lambda^2) \ + \ \text{const}\,] \tag{7}$$

If $\beta > 0$, Λ is found graphically as shown in fig.(1). Its dependence on Λ_0 is shown in fig.(2). Evidently, as long as Λ_0 is positive, $\Lambda_0 \approx \Lambda$, while, when Λ_0 becomes negative and not too large in Planck units, Λ becomes very tiny, approaching the value:

$$\Lambda \ = \ \lambda^{-2}\,\exp(-16\pi\beta G|\Lambda_0|\,)^{-1}. \tag{8}$$

If the tree-level, zero temperature value of Λ_0 satisfies:

$$0 \ < \ -G\Lambda_0 \ < \ \mathcal{O}(10^{-2})\,, \tag{9}$$

which should be easy to achieve with some supersymmetry protection, the resulting value of Λ will be certainly compatible with experimental bounds!

In case the reader might be worried about our derivation of eq.(8), we mention that exactly the same result follows from replacing the non-local action by a local action through the introduction of a wormhole-like auxiliary variable α_0 [3]. One finds that the integral over α_0 is dominated by a saddle point at which the effective cosmological constant is precisely given by eq.(8).

The most appealing feature of the model discussed here is the existence of a phase transition at $\Lambda_0 = 0$. The magnitude of $|\Lambda_0|$ does not play any rôle in the determination of the gravitational vacuum, as long as $G|\Lambda_0|$ is small enough, see eq.(9). One can readily envisage an inflationary scenario, in which the transition from the expanding universe to flat space-time is triggered by the change of the sign of Λ_0 due to the rearrangement of the vacuum state in the matter sector.

3. Real life is harder

We shall now proceed with a bona-fide attempt at computing Γ_1. We shall first review existing results and then try to go beyond them. However, even before presenting any calculations, we stress three basic requirements to be fulfilled in order for the result to be both reliable and useful. It is important:

i) to have a *finite* theory of quantum gravity which preserves *general covariance*;

ii) to work with an *off-shell* background field effective action;

iii) to maintain *general covariance* even *off-shell*.

String theory is an "existence proof" for point i), but, as we stressed, other regulators, if they exist, can be used. As for point ii), it comes from the very definition of the cosmological constant. It is well known that errors can be made if classical equations are inserted too early into the calculation of the effective action. An example was recently provided by Duff [12]. Another one, more relevant for our present problem, concerns terms in Γ_1 in which the logarithm multiplies the curvature scalar or its square. If such terms are rewritten using the tree-level equation $R_{\mu\nu} = \Lambda_0\, g_{\mu\nu}$, one ends up with the wrong one-loop-corrected equations, as it is easy to check. Finally requirement iii) is almost obvious if one wants to get unambiguous answers; it has been widely discussed in the quantum gravity literature [11,13,14], but is apparently not so well known by the particle physics community. As stressed by Vilkovisky [13], the crucial point is that, if one does not correct the usual background field effective action, different gauges provide answers that differ by equations of motion (*i.e.* by first variations of the classical action). In order to determine the new, quantum corrected, stationary points, however, one has to "vary" the off-shell action making results obtained in different gauges differ by "second" variations which do not vanish even on-shell.

The requirements ii) and iii) can be satisfied provided that one computes the so-called Vilkovisky – DeWitt (VD) background field effective action [13,14] (or unique effective action) Γ_{VD}. We shall consider spherical backgrounds of arbitrary radii r. As we shall discuss later on, this will be sufficient to argue about the volume dependence of quantum corrections in the general case. The one-loop correction to Γ_{VD} has the formal expression:

$$\Gamma_1 \;=\; \tfrac{1}{2}\log\det\{\, \delta^2(S + S_{GF} + S_{VD})\,/\,(\delta h)^2\,\} \;-\; \log\det\{\, \delta^2 S_G \,/\, \delta v \delta v^* \,\}\,, \qquad (10)$$

where S is the classical action and S_G the ghost action. The functional derivatives are taken with respect to the metric fluctuations $h_{\mu\nu}$ and the ghost fields v_μ. In eq.(10),

S_{GF} is the gauge fixing term taken to be of the form:

$$S_{GF} = \frac{1}{32\pi G\alpha}\int d^4x\sqrt{g}\,\nabla^\rho \bar{h}_{\rho\mu}\nabla^\sigma \bar{h}_\sigma^{\ \mu}\,, \tag{11}$$

The corresponding ghost action is:

$$S_G = \frac{1}{32\pi G\alpha}\int d^4x\sqrt{g}\,v_\mu^*(-\Box - 3/r^2)v^\mu\,. \tag{12}$$

Finally, S_{VD} is the additional term prescribed by VD. Fortunately, for the case of spherical backgrounds under consideration, the one-loop VD action can be obtained simply by omitting the VD term in eq.(10) and by taking the limit $\alpha \to 0$ at the end [11]. The computation of Γ_1 is presented in detail in ref.[15]. Here, we restrict ourselves to the discussion of a couple of important technical points.

It is well known that some of the metric fluctuations contribute with the "wrong" sign [16] to the quadratic part of the action; in these cases, the appropriate Wick rotation is performed in order to give meaning to the functional integral or, if we wish, in order to keep the eigenvalues of the kinetic energy operators positive. Special care has to be taken for the zero eigenvalues. There are two types of zero modes: those which arise from a symmetry of the problem and that, as such, do not depend on the values of r or Λ_0, and those which occur when these two quantities are in a certain relationship. It is now widely accepted that the first type of zero modes have just to be taken out since they correspond to residual gauge transformations, *i.e.* to incomplete gauge fixing. On the other hand, it is clear that the second type of zero modes cannot (and should not) be subtracted.

The determinants involved in eq.(10) contain ultra-violet (UV) divergences coming from arbitrarily large eigenvalues of the kinetic energy operators. This problem can be dealt with in several ways and, certainly, String Theory must have its own. We shall assume that the way String Theory works is to introduce a lower cut-off λ^2 on the proper time t integration encountered in the heat kernel method [17] for evaluating the determinant. There are strong indications that the physical results discussed here do not depend on this choice of regularization. Most of the existing results on heat kernels concern their small t behaviour, which is sufficient to determine the UV cut-off dependence of Γ_1.

This is not sufficient, however, for our purposes since we are interested in the actual dependence of the effective action on the radius r. In order to have some handle on that, we need to study also the large t, or infra-red (IR), behaviour of heat kernels. The IR behaviour is controlled by the lowest eigenvalues, which have been the subject of many investigations by mathematicians [18]. Their results lend support to the idea that the IR behaviour of heat kernels is controlled by global, non-local properties of the manifold, such as the diameter of the largest ball that can be inscribed in the manifold. This non-locality of the effective action was pointed out long ago by DeWitt [19] and, later on, discussed to some extent by Vilkovisky [13]. This feature of the effective action is precisely what we want in order to play the game described earlier for the $V \log V$ toy model.

The final result [15] for the one-loop Vilkovisky – DeWitt effective action is:

$$\begin{aligned} \Gamma_1 \;&=\; r^4 \left[-\tfrac{1}{12}\,\lambda^{-4} - \Lambda_0 \lambda^{-2} + 2\Lambda_0^2 \log(\lambda M) \right] \\ &\quad + r^2 \left[\tfrac{17}{3}\,\lambda^{-2} - 16\Lambda_0 \log(\lambda M) \right] \;+\; \mathcal{O}(\log r), \end{aligned} \tag{13}$$

where:

$$M \;\approx\; \max\{\, |\Lambda_0|^{1/2}, 1/r \,\}\,. \tag{14}$$

The following remarks are in order:

i) One has generated, as expected, a "large" cosmological constant proportional to the cut-off. This contradicts the assumption that Λ_0 was the "tree-level" cosmological constant. In other words, we have implicitly assumed that some (supersymmetric, for instance) protection is at work. The same protection must necessarily wash out the term proportional to λ^{-4} in eq.(13).

ii) There is a finite renormalization of G: a small one if $\lambda \gg \lambda_P$, and one proportional to Λ_0 times the logarithm. The latter could be relevant if large logarithms are present.

iii) There is a pure $\log r$ term corresponding to a renormalization of R^2-type terms, which will play no rôle in the search for large volume solutions.

iv) Finally, there are logarithms (with just a constant in front) that can become infinite at special values of r and Λ_0 (i.e. when a zero mode occurs), which we neglect since we will be looking for solutions far from these singularities.

In accordance with the general discussion we made earlier, we shall now interpret the factors r occurring under the logarithms as $V^{1/4}$, since it is the global size of the manifold which controls the small eigenvalues. For $\lambda \gg \lambda_P$, the term dominant at large V in the effective action of eq.(13) is:

$$\Gamma_1 \;\approx\; \frac{3}{4\pi^2}\,\Lambda_0^2\,V\log(\lambda M)\,. \tag{15}$$

We notice that Γ_1 falls just short of giving our toy model's action of eq.(3). The two main differences are:

a) The argument of the logarithm which multiplies $\Lambda_0^2\,V$ is not a power of the volume, but $M \approx \max\{\,|\Lambda_0|^{1/2}, V^{-1/4}\,\}$, see eq.(14). This, unfortunately, prevents the logarithm from becoming very large. On the other hand, the scale of a logarithm is often difficult to determine by a one-loop calculation. Higher loops might change the scale and replace it, for instance, by $\max\{\Lambda^{1/2}, V^{-1/4}\}$, where the final $\Lambda \propto V^{-1/2}$ appears. This would be sufficient for our purposes.

b) The sign of the logarithm under discussion is the opposite of the one of the toy model. As such, it would rather affect a positive Λ_0 than a negative one. Actually, after an easy graphical check [similar to the one of fig.(1)], one finds that, for positive Λ_0, there are two solutions and that the one with $\Lambda \approx \Lambda_0$ has lower action. This conclusion depends on the integration contours over the conformal factor of the metric and could be reversed if some reason forces one to change it.

We thus conclude that our toy model almost comes out of a bona-fide one-loop calculation. It is not at all excluded that higher loop effects or the use of a more complicated and realistic theory (with dynamically broken supersymmetry, for instance) could just lead to our toy model. Unfortunately we do not know, at present, how to approach either case.

One can also take a different attitude towards our toy model and regard $S + \Delta S$ of eqs.(2) and (3) as the effective action of a more fundamental theory. In superstring scenarios, for instance, the appearance of a cosmological constant term comes together with (local) supersymmetry breaking. Both in the gluino condensation [20] and in the gravitino condensation [21] scenarios the effective actions below the supersymmetry breaking scale resemble those of supersymmetric gauge theories [22] and contain logarithms of fields including those related to the size of various spaces [9,23]. In principle, such a mechanism could yield a correction proportional to ΔS.

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Figures

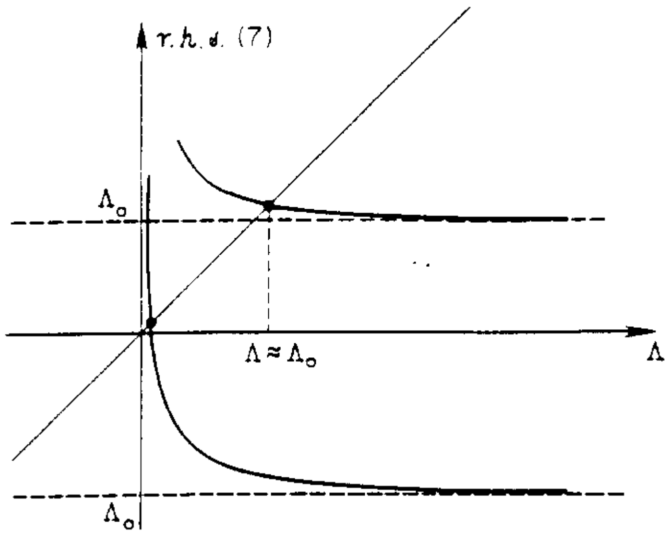

Fig.1) Graphical solutions of the "bootstrap" eq.(7); the upper and lower curves correspond to the *r.h.s.* of eq.(7) as the functions of Λ in the cases of positive and negative Λ_0, respectively.

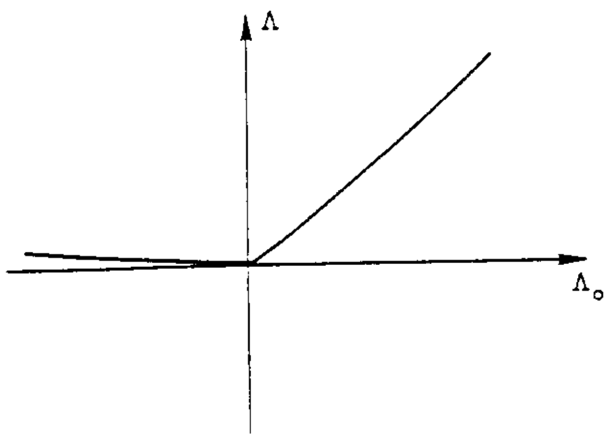

Fig.2) The effective cosmological constant Λ as the function of the tree-level cosmological constant Λ_0.

It is straightforward to repeat the self-consistent determination of the effective cosmological constant in our toy model, with the coefficient of $V \log V$ kept positive and unrelated to Λ_0^2. One finds that Λ remains positive and large as far as Λ_0 stays positive, and becomes positive and exponentially small if Λ_0 becomes negative and large compared to the coefficient of $V \log V$ times G. It remains to be checked whether such a string-based model is feasible. A more radical alternative, which we are reluctant to take into serious consideration at this point, is to abandon the locality principle, and postulate $S + \Delta S$ as the fundamental *classical* action of EQG.

In conclusion, we have given arguments to support the idea that quantum corrections to the classical action might be accompanied by logarithms which blow up at large Euclidean volume and/or small cosmological constant. It is not excluded that, just as in our toy model's effective action, these logarithms drive, at late epochs, the ground state of the theory away from its naïve, tree-level configuration, into the Big Universe of today.

We are grateful to B. Allen, W.A. Bardeen, I. Batalin, E. Fradkin, R. Kallosh, A. Linde and T.N. Tomaras for interesting discussions.