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Statistical Fluctuations as the Origin of Nontopological Solitons

Kim Griest, Edward W. Kolb, and Alessandro Massarotti

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Il. 60510

Astronomy and Astrophysics Center
The University of Chicago
Chicago, Il. 60637

Abstract

Nontopological solitons can be formed during a phase transition in the early universe as long as some net charge can be trapped in regions of false vacuum. It has been previously suggested that a particle-antiparticle asymmetry would provide a source for such trapped charge. We point out that, for the model and parameters considered, statistical fluctuations provide a much larger concentration of charge, and are therefore, the dominant source of charge fluctuations in solitogenesis.

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to the mass of Q free ϕ's in the true vacuum, $M_{\rm free} = Qm_\phi = Qh^{1/2}|\sigma_- - \sigma_0|$. The NTS configuration will have a lower mass, and hence be stable, for charge Q greater than some minimum charge, given by

$$Q_{\rm MIN} = \frac{1231}{h^2}\frac{\Lambda}{(\sigma_- - \sigma_0)^4}. \tag{3}$$

In this paper we will study in detail the case $\lambda_2/\lambda_1 = 0.15$. For this choice of λ_2/λ_1, $\Lambda = 0.6\lambda_1\sigma_0^4$, $Q_{\rm MIN} = 18\lambda_1/h^2$, and $M_{\rm MIN} = 46(\lambda_1/h^{3/2})\sigma_0$.

A scenario for the cosmological origin of NTS was proposed by Freeman, Gelmini, Gleiser, and Kolb[8] (hereafter, FGGK). In the FGGK scenario, there is a critical temperature, $T_C \simeq 2\sigma_0$, below which the Universe divides into domains of true ($\sigma = \sigma_-$) and false ($\sigma = \sigma_0$) vacuum. The characteristic size of these domains is determined by the correlation length, ξ, of the σ field at the transition. At high temperatures thermal fluctuations can cause a correlation volume to make the transition between the two minima. These fluctuations freeze out at the "Ginzburg" temperature, T_G. FGGK estimate T_G by the criterion that T_G is equal the maximum free energy of the correlation volume in the transition $F_M = U_M V_\xi$ (U_M is the maximum value of the potential in the region $\sigma_- \leq \sigma \leq \sigma_0$). For $\lambda_2/\lambda_1 = 0.15$, $T_G = 1.3\sigma_0/\lambda_1^{1/2}$. Of course T_G can never be larger than $T_C \simeq 2\sigma_0$.

At T_G, the probabilities of being in the false vacuum, $p(\sigma_0)$, and true vacuum, $p(\sigma_-)$, are Boltzmann distributed according to the difference in free energies of a correlation volume in the different minima

$$\frac{p(\sigma_0)}{p(\sigma_-)} = \exp[-\Delta F/T_G] = \exp[-\Lambda V_\xi/T_G] \tag{4}$$

(recall that $U(\sigma_-) \equiv 0$ by the addition of Λ). If $p(\sigma_0)/p(\sigma_-) \leq 0.3$ then only finite regions of "false" vacuum will be populated. If the regions of false vacuum contain a will also be Gaussian distributed; with means $\overline{N} = \overline{N}_\phi + \overline{N}_{\overline{\phi}}$, $\overline{Q} = |\overline{N}_\phi - \overline{N}_{\overline{\phi}}| = \eta\overline{N}$; and variance $\sigma^2 = \overline{N}$. Therefore, the probability of finding a charge Q in a volume containing a mean number $\overline{N}$ of $(\phi + \overline{\phi})$'s is[9]

$$P(Q, \overline{N}) = \frac{1}{\sqrt{2\pi\overline{N}}} \exp\left[-(Q - \eta\overline{N})^2/2\overline{N}\right]. \tag{8}$$

As described by FGGK, below T_G the Universe divides into cells of correlation volume $V_\xi \simeq (4\pi/3)\xi^3$. Adjacent cells of false vacuum form "clusters" with density per unit cluster of

$$f(r) = br^{-1.5}e^{-cr} \tag{9}$$

for volume $V = rV_\xi$. The constants b and c are unknown. Scaling arguments imply that $c \to 0$ as $p(\sigma_0) \to p_c$ (where p_c is the critical probability for percolation, $p_c \sim 1/3$) and $b \to 0$ as $p(\sigma_0) \to 0$. It is expected that b and c are of order unity otherwise. The number density of r-clusters produced in the transition is simply $n(r) = f(r)V_\xi^{-1}$. In a volume $V = rV_\xi$, the mean number of $(\phi + \overline{\phi})$'s is $\overline{N} = r\overline{N}_\xi$, where $\overline{N}_\xi$ is the mean number of $(\phi + \overline{\phi})$'s in a correlation volume

The number density of false-vacuum domains with charge Q is simply given by $n_Q = \Sigma_{r=1}^{\infty} n(r)P(Q; \overline{N} = r\overline{N}_\xi)$, where $n(r) = f(r)V_\xi^{-1}$ as before, with $f(r)$ given by Eq.(9). Approximating the sum over r by an integral,[10] n_Q becomes

$$\begin{aligned} V_\xi n_Q &= \frac{b\exp(Q\eta)}{\sqrt{2\pi\overline{N}_\xi}} \int_0^\infty dr\, r^{-2} \exp\left[-(\eta^2\overline{N}_\xi/2 + c)r - Q^2/2\overline{N}_\xi r\right] \\ &= \frac{2b}{\sqrt{\pi}} \frac{\exp(Q\eta)}{Q} (\eta^2\overline{N}_\xi/2 + c)^{1/2} K_1\left[(\sqrt{2}Q/\overline{N}_\xi^{1/2})(\eta^2\overline{N}_\xi/2 + c)^{1/2}\right], \end{aligned} \tag{10}$$

where $K_1(z)$ is a modified Bessel function of the second kind of order one. For large argument, the expansion $K_1(z) \to e^{-z}\sqrt{\pi/2z}$ gives[10]

$$V_\xi n_Q = b\frac{\overline{N}_\xi^{1/2}}{Q^{3/2}}(\eta^2 + 2c/\overline{N}_\xi)^{1/4} \exp\left[Q\eta - Q(\eta^2 + 2c/\overline{N}_\xi)^{1/2}\right]. \tag{11}$$

the large-z expansion of the Bessel function $K_1(z)$ was used. In this conclusion section we present some numerical result and discuss the range of validity of the above approximations.

Clearly for "large" Q, $Q \geq 10$–20, Gaussian statistics will be a good approximation. In Fig. 1 we compare an integration over r of Gaussian statistics, Eq.(10), to the more accurate sum over r of Poisson statistics. The Gaussian results are presented for η =0, 0.25 and 0.5, while the Poisson results are given for $\eta = 0$ only. It is clear that the Gaussian approximation is an adequate one. Integration over r rather than summing also introduces only a small error.

In Fig. 2 we present the large-z expansion of the Bessel function in Eq.(11). Comparison of Fig. 1 and Fig. 2 shows that for $\sqrt{2}Q\overline{N}_\xi^{-1/2}(\eta^2\overline{N}_\xi/2 + c)^{1/2} \geq 2$, the expansion is accurate. In Fig. 2 we also show for comparison the results of FGGK for $n_Q V_\xi$. Clearly it is a serious underestimate for n_Q unless $\eta^2 \gg 1.96\lambda_1^3 c$.

We conclude by illustrating the importance of the calculation of Y_Q. We use the example discussed in the introduction, $\lambda_2 = 0.15\lambda_1$, which gives $Q_{\rm MIN} = 18\lambda_1/h^2$, and $M(Q_{\rm MIN}) = 46\lambda_1\sigma_0/h^{3/2} = 2.5Q_{\rm MIN}h^{1/2}\sigma_0$. Assuming that the contribution to Ω from NTSs is dominated by those with $Q = Q_{\rm MIN}$, $Y_{NTS} \simeq 10^{-3}Q_{\rm MIN}^{-3/2}e^{-Q_{\rm MIN}}$, the present NTS energy density is $\rho_{NTS} = Y_{NTS}M_{NTS}s_0$, where s_0 is the present entropy density, s_0 =2800 cm^{-3}. Comparison of ρ_{NTS} to the critical density, ρ_C=$1.88\times10^{-29}h_0^2$g cm^{-3}, where h_0 is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}, gives

$$\Omega_{NTS}h_0^2 = 10^6 h^{1/2}(\frac{\sigma_0}{\rm GeV})\frac{e^{-Q_{\rm MIN}}}{Q_{\rm MIN}^{1/2}}. \qquad (15)$$

For NTSs to be dynamically relevant today, $\Omega_{NTS}h_0^2$ should be in the range $10^{-2} \leq \Omega_{NTS}h_0^2 \leq 1$. Relevant values of $Q_{\rm MIN}$, or equivalently λ_1/h^2, are shown in Table I.

The conclusion of this paper is that statistical fluctuations are the dominant source of charge fluctuations in solitogenesis, not a cosmic asymmetry as assumed by FGGK.

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7. In general, one should include a term in the potential proportional to $|\phi|^4$. Inclusion of this term will not substantially alter our conclusions.

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9. When $\eta = 0$ the corresponding formula using Poisson statistics is $P(Q,\bar{N}) = e^{-\bar{N}} I_Q(\bar{N})$, where I_Q is a modified Bessel function of order Q. Turning the sum over r into an integral, $V_\xi n_Q \approx \bar{N}_\xi^{1/2}(z^2-1)^{1/4}\Gamma(Q-\frac{1}{2})P_{1/2}^{-Q}\left(z(z^2-1)^{-1/2}\right)$ where $P_{1/2}^{-Q}$ is an associated Legendre function of order 1/2, Γ is the gamma function, and $z = c + \bar{N}_\xi \approx c + \lambda_1^{-3}$.

10. The validity of this approximation will be discussed in the final section.

FIGURE CAPTIONS

Figure 1: A comparison of Poisson (Footnote 9) and Gaussian probabilities (Eq. 10) as a function of Q. $\overline{N}_\xi = b = c = 1$ was assumed.

Figure 2: The result of the large-z expansion of the Bessel function in Eq. 11 is shown by the points marked *This Work.* Comparison of these points with the corresponding points in Fig. 1 shows that the large-z expansion is a good approximation. Also indicated by the points marked *FGGK* are the results of FGGK[8] which ignored statistical fluctuations.

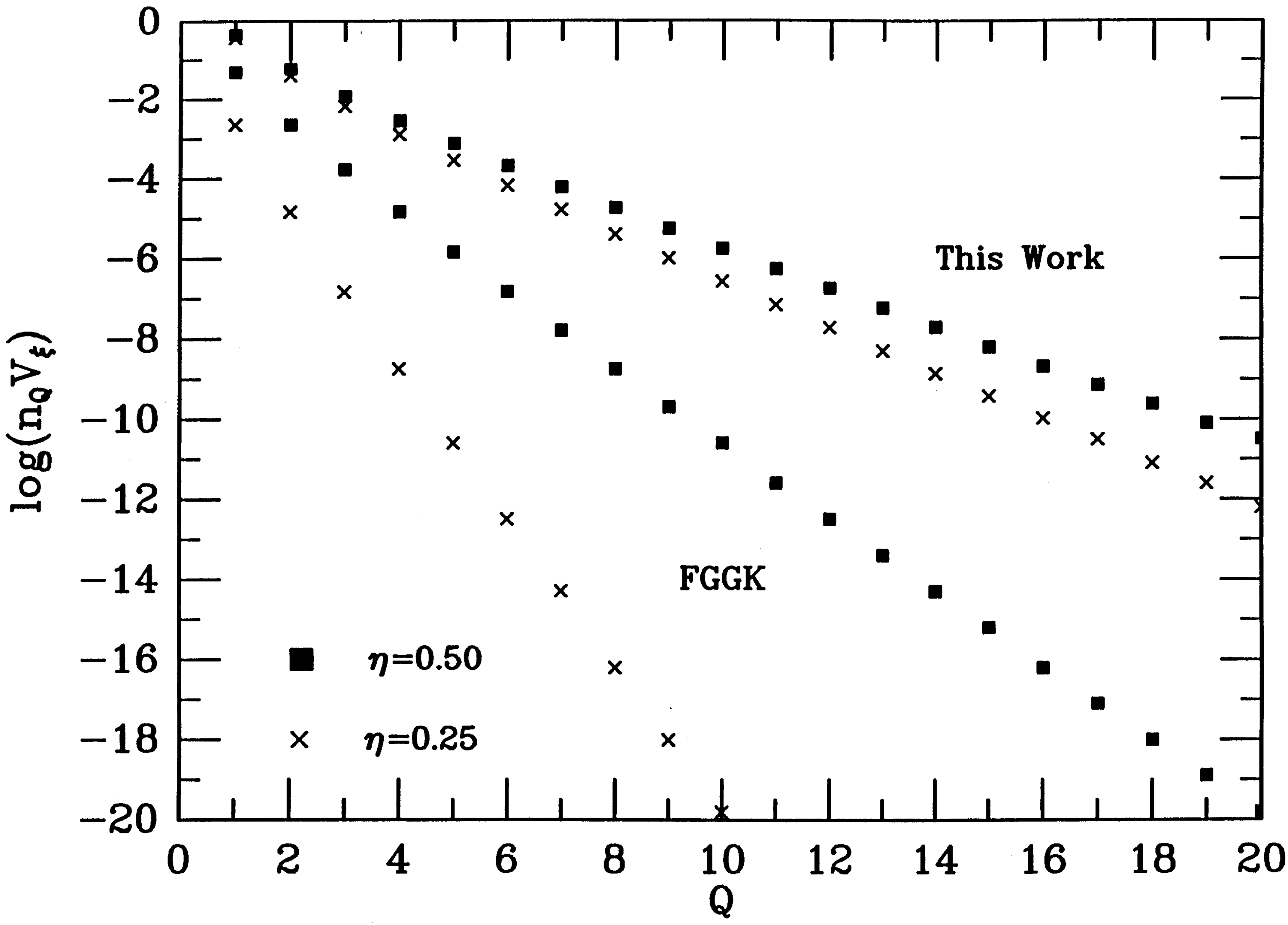
log($n_Q V_\xi$)
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
−20
0
2
4
6
8
10
12
14
16
18
20
Q
This Work
FGGK
η=0.50
η=0.25

FIG. 2