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Chern-Simons Terms in Four Dimensional Heterotic String Theory

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Abstract

We consider a four-dimensional heterotic string theory whose massless states are the N=4 supergravity multiplet and the gauge multiplet of SO(44). The string is coupled to the gauge background maintaining the world-sheet super-symmetry. It is found that the non-Abelian gauge current conservation becomes anomalous at the quantum level. The anomalies, however, can be eliminated by requiring the antisymmetric tensor field to transform nontrivially under the gauge transformation. This, in turn, introduces a Chern-Simons term in the definition of the corresponding field strength for it to be gauge invariant.

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The heterotic string is an attractive candidate for the unification of all fundamental forces of nature. There has been a lot of work in examining various aspects of the model following the original construction[1] where the right moving sector is the ten dimensional closed superstring and the left moving sector is the 26 dimensional closed bosonic string 16 of whose coordinates are compactified on tori. Narain[2] envisaged the construction of four dimensional string theories and obtained a large class of heterotic like string theories[3]. Subsequently, there has been several proposals to construct four dimensional string theories providing us with a large number of models[4]. It is argued that there is only one underlying string theory, namely, the heterotic string; however, the abundance of models with different gauge groups is the manifestation of different vacua of a single fundamental theory[5]. Indeed, there have been attempts to understand the mechanisms for symmetry breaking in order to construct realistic 4-dimensional string theories[6].

It is well known from the seminal work of Green and Schwarz[7] that the Yang-Mills and the Lorentz Chern-Simons (C-S) terms are necessary for the cancellation of various anomalies in the superstring theories. This also requires the antisymmetric tensor field to transform nontrivially under the non-Abelian gauge transformation. The existence of C-S terms for the ten dimensional heterotic string and their implications have been investigated by Sen[8] in the recent past. The purpose of this letter is to demonstrate the existence of Yang-Mills C-S terms in four-dimensional heterotic string theories. The antisymmetric tensor field, in four-dimensional string theories, is identified with the axion. The phenomenology of the axion has attracted considerable attention in the recent past and it would be interesting to investigate the phenomenology in the presence of the Chern-

Simons terms.

We consider a d=4 heterotic string which admits the N=4 supergravity multiplet coupled to the SO(44) Yang-Mills multiplet and present the explicit computations for the appearance of the Yang-Mills C-S term. The path integral approach, employed earlier for the compactified chiral bosonic string(9), provides an elegant method to obtain our results. Let us recall some of the salient features of the 4-dimensional heterotic string in the fermionized version for the compactified bosonic coordinates. In what follows we work in the light-cone gauge for the sake of simplicity. The right moving sector consists of bosonic coordinates x^μ and ψ^μ ($\mu = 3,4$) which are the world-sheet supersymmetry partners. The six compactified bosonic coordinates are fermionized to give 12 right moving Majorana-Weyl fermions. Thus there are 18 fermions appearing from the compactified coordinates which are required to be in the adjoint representation of a semi-simple group G.

The left moving sector consists of the bosonic coordinates x^μ, $\mu = 3,4$ and 44 left moving Majorana-Weyl fermions which are obtained from the 22 compactified left moving bosonic coordinates. It is possible to construct a large class of consistent 4-dimensional string theories by suitable choice of boundary conditions for the fermionic coordinates. Thus we adopt the standard set theoretic(10) notations for the fermions in the theory and recapitulate essential features of d=4, N=4 heterotic string theory coupled to SO(44) Yang-Mills gauge group. The 18 right moving internal fermions are denoted by (x^I, y^I, z^I), $I = 1,\ldots,6$.

We choose $\alpha = \{\psi^3, \psi^4; z^1, \ldots, z^6\}$ and take the simplest possible group structure

$$\Xi = \{F, F\bullet\alpha, \alpha, \phi\} \tag{1}$$

where F is the set of all fermions and $F \bullet \alpha = F \cup \alpha - F \cap \alpha$ and ϕ is the null set. The massless spectrum generated by the α sector gives the N=4 supergravity multiplet which consists of the graviton, the dilaton, the antisymmetric tensor, six graviphotons, 4 gravitinos and 4 Majorana fermions. Furthermore, we also have a gauge boson, 6 scalars and 4 gauginos in the adjoint representation of SO(44).

Our strategy for deriving the C-S term is as follows. The string is coupled to the massless gauge bosons and the antisymmetric tensor background so that these couplings satisfy world sheet (1,0) supersymmetry. Although, it is possible to write a manifestly supersymmetric action in the super-space, we take the Lagrangian in component fields since it is more suitable for the path integral derivation of the C-S term.

$$
\begin{aligned}
S = \frac{1}{2} \int d^2\sigma \Big[& \eta^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu \eta_{\mu\nu} + \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu} \\
& + \frac{i}{2}\psi^\mu \partial_- \psi_\mu + \psi^\mu \rho^\alpha \rho_5 \psi^\nu H_{\mu\nu\lambda}\partial_\alpha x^\lambda \\
& + \frac{i}{2} z^I \partial_- z^I + \frac{i}{2} x^I \partial_- x^I + \frac{i}{2} y^I \partial_- y^I \\
& + \frac{i}{2}\eta^A \partial_+ \eta^A - \frac{i}{2} f^{IJK} z^I z^J W^K_\mu \partial_- x^\mu \\
& + \frac{i}{2}\eta^A T^m_{AB}\, \eta^B A^m_\mu \partial_+ x^\mu - \frac{i}{4}\psi^\mu \psi^\nu F^m_{\mu\nu} \eta^A T^m_{AB} \eta^B \Big]
\end{aligned}
\tag{2}
$$

Our two dimensional Dirac matrices are

$$\rho^0 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \rho^{0\dagger}$$

$$\rho^1 = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\rho^{1\dagger}$$

and $$\rho_5 = \rho^0 \rho^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \rho_5^\dagger$$

Furthermore, $\partial_\pm = \partial_\tau \pm \partial_\sigma$ and $H_{\mu\nu\lambda}$ is the field strength associated with $B_{\mu\nu}$.

$$H_{\mu\nu\lambda} = \frac{1}{2}\left(\frac{\partial}{\partial x^\lambda} B_{\mu\nu} + \frac{\partial}{\partial x^\mu} B_{\nu\lambda} + \frac{\partial}{\partial x^\nu} B_{\lambda\mu}\right)$$

$W^K_\mu(x)$ are the gauge-bosons of the N=4 supergravity multiplet and f^{IJK} denote the structure constants of the group SO(4). $A^m_\mu(x)$ are the gauge-bosons in the adjoint representation of SO(44) and $F^m_{\mu\nu}$ is the corresponding field strength. Notice that the fermions z^I are in the adjoint representation of SO(4) whereas η^A, A = 1,...,44 belong to the fundamental representation of SO(44). Consequently, T^m_{AB} represent generators of SO(44) in the fundamental representation. The action is invariant under the following non-Abelian gauge transformations.

$$\delta z^I = f^{IJK} z^J \theta^K_R(x)$$

$$\delta W^I_\mu = \partial_\mu \theta^I_R(x) + f^{IJK} W^J_\mu(x)\theta^K_R(x) \tag{3}$$

$$\delta\eta^A = i\left(\theta^m_L(x) T^m \eta\right)^A$$

$$\delta A^m_\mu = \partial_\mu \theta^m_L(x) + iC^{mnp} A^n_\mu(x)\theta^p_L(x) \tag{4}$$

where $\theta^I_R(x)$ and $\theta^m_L(x)$ are the two gauge parameters and C^{mnp} are the structure constants of SO(44). Since the gauge bosons are coupled to the world-sheet Majorana-Weyl fermions we have to examine carefully if the gauge transformations (3) and (4) are anomalous in the quantum theory[11]. We rewrite the action in a modified form which is more suitable to compute anomalies.

$$S = \frac{1}{2}\int d^2\sigma\left[\eta^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu \eta_{\mu\nu} + \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}\right.$$

$$+ \frac{i}{2} \psi^\mu \partial_- \psi^\mu + \psi^\mu \rho^\alpha \rho_5 \psi^\nu H_{\mu\nu\lambda} \partial_\alpha x^\lambda + \frac{i}{2} x^I \partial_- x^I + \frac{i}{2} y^I \partial_- y^I$$

$$+ \frac{i}{2} \bar{z}^I \rho^\alpha \left(\partial_\alpha \delta^{IJ} - f^{IJK} W^K_\mu(x) \partial_\alpha x^\mu\right) \left(\frac{1-\rho_5}{2}\right) z^J$$

$$+ \frac{i}{2} \bar{\eta}^A \rho^\alpha \left(\partial_\alpha \delta^{AB} + T^m_{AB} A^m_\mu \partial_\alpha x^\mu\right) \left(\frac{1+\rho_5}{2}\right) \eta^B$$

$$- \frac{i}{4} \psi^\mu \psi^\nu F^m_{\mu\nu} \eta^A T^m_{AB} \eta^B] \qquad (5)$$

Notice that in the left moving sector the gauge background couples to the Majorana-Weyl fermions precisely as in the compactified bosonic string except for the coupling of the world-sheet fermions ψ^μ to the field strength(8,12).

It is well-known that the fermionic measure, in the path integral, is not always invariant under gauge transformations. First, we will compute the anomalies associated with the gauge transformations (3) on the right moving sector. The anomalies associated with the SO(44) gauge transformations can be computed similarly in a straight forward manner(13).

Let us define the variable

$$a^I_\alpha = \partial_\alpha x^\mu W^I_\mu \qquad (6)$$

The relevant fermionic Lagrangian, with this notation, is

$$L_f = \frac{i}{2} \bar{z}^I \not{D}^{IJ} z^J \qquad (7)$$

where the Dirac operator $\not{D}^{IJ}$ in the Euclidean space is

$$\not{D}^{IJ} = \rho_\alpha \left(\partial_\alpha \delta^{IJ} - f^{IJK} a^K_\alpha\right) \left(\frac{1-\rho_5}{2}\right) \qquad (8)$$

Note that because of the appearance of ρ_5 terms in the Lagrangian (7), we have to be careful in computing the consistent anomalies from the change in measure[14]. We rewrite the Dirac operator

$$\not{D}^{IJ} = \rho_\alpha\left(\partial_\alpha\delta^{IJ} - \frac{1}{2} f^{IJK}v^K_\alpha + \frac{1}{2} f^{IJK}\rho_5 a^K_\alpha\right) \tag{9}$$

where v^I_α are introduced with the understanding that we set $v^I_\alpha = a^I_\alpha$ at the end of the computation; furthermore we analytically continue $a^I_\alpha \to ia^I_\alpha$ so that the Dirac operator is hermitian in the Euclidean space. (For details see ref. 9.)

$$\not{D}^{IJ} = \rho_\alpha\left(\partial_\alpha\delta^{IJ} - \frac{1}{2} f^{IJK}v^K_\alpha + \frac{i}{2} f^{IJK}\rho_5 a^K_\alpha\right) \tag{10}$$

and we shall analytically continue a^I_α back again at the end of our calculations. The eigenstates ϕ_n of the hermitian operator $\not{D}^{IJ}$ are used to expand the fermions

$$z = \sum_n f_n\phi_n$$

$$\bar{z} = \sum_n g_n\phi^\dagger_n \tag{11}$$

since they form a complete basis. The fermionic path integral measure can be expressed in terms of the expansion coefficients f_n and g_n as

$$D\bar{z}\ Dz = \prod_n df_n\ dg_n \tag{12}$$

Under the gauge transformation (3)

$$D\bar{z}'\ Dz' = (\det C_{nm} \det \tilde{C}_{nm})^{-1/2}\ D\bar{z}\ Dz \tag{13}$$

where

$$C_{nm} = \delta_{nm} + \int d^2\sigma\ \phi^\dagger_n \frac{i}{2}\ (1-\rho_5)\theta_R\cdot\tau\ \phi_m \tag{14}$$

$$\tilde{C}_{nm} = \delta_{nm} - \int d^2\sigma\ \phi^\dagger_n \frac{i}{2}\ (1+\rho_5)\theta_R\cdot\tau\ \phi_m \tag{15}$$

where $(\tau^I)^{JK} = -if^{IJK}$ are the generators in the adjoint representation of SO(4) and $\theta_R \bullet \tau = \theta_R^I \tau^I$.

Using the standard technique of Fujikawa[11] we compute C_{nm} and $\tilde{C}_{nm}$ for infinitesimal gauge transformations given in (3) (for explicit calculation in the case of the string see ref. 9 and ref. 13).

$$\det C_{nm} = \exp\left[\mathrm{Tr} \sum_n \int d^2\sigma\, \phi_n^\dagger \frac{i}{2} (1-\rho_5)\, \theta_R \bullet \tau\, \phi_n\right] \tag{16}$$

After proper regularization, continuing a_α back analytically to $-ia_\alpha$ and setting $v_\alpha = a_\alpha$ we obtain

$$\det C_{nm} = \exp\left[\frac{iC_2}{4\pi} \int d^2\sigma \theta_R^I\left(-i\partial_\alpha a_\alpha^I - \epsilon_{\alpha\beta}\partial_\alpha a_\beta^I\right)\right] \tag{17}$$

where we have used

$$\mathrm{Tr}(\tau^I \tau^J) = C_2 \delta^{IJ} \tag{18}$$

with C_2 representing the Casimir invariant of SO(4). A similar calculation gives

$$\det \tilde{C}_{nm} = 1 \tag{19}$$

Therefore, the Jacobian factor multiplying the fermionic measure is given by

$$\left(\det C_{nm} \det \tilde{C}_{nm}\right)^{-1/2}$$

$$= \exp\left[\frac{iC_2}{8\pi} \int d^2\sigma\, \theta_R^I\left(i\partial_\alpha a_\alpha^I + \epsilon_{\alpha\beta}\partial_\alpha a_\beta^I\right)\right] \tag{20}$$

Rotating back to Minkowksi space the change in measure can be written as an anomalous action

$$S^{(W)}_{\mathrm{anomalous}} = -\frac{C_2}{8\pi} \int d^2\sigma\, \theta_R^I \left(\partial^\alpha a_\alpha^I - \epsilon^{\alpha\beta}\partial_\alpha a_\beta^I\right) \tag{21}$$

The first term can be removed by adding a local counterterm of the form

$$S_{CT}^{(W)} = -\frac{C_2}{16\pi}\int d^2\sigma\ a_\alpha^I\ a^{\alpha I} \tag{22}$$

to the action, whose gauge variation will cancel the first term in (21). Therefore, the minimal anomalous action is

$$\begin{aligned} S_{\text{anomalous}}^{(W)} &= \frac{C_2}{8\pi}\int d^2\sigma\ \theta_R^I\ \epsilon^{\alpha\beta}\ \partial_\alpha\ a_\beta^I \\ &= \frac{C_2}{8\pi}\int d^2\sigma\ \theta_R^I\ \epsilon^{\alpha\beta}\ \partial_\alpha\left(\partial_\beta x^\mu W_\mu^I\right) \\ &= -\frac{C_2}{8\pi}\int d^2\sigma\ \epsilon^{\alpha\beta}\ \partial_\alpha\ x^\nu\ \partial_\beta\ x^\mu\ W_\mu^I\ \partial_\nu\ \theta_R^I \end{aligned} \tag{23}$$

We note, however, that if $B_{\mu\nu}$ transforms under the gauge transformation as

$$\delta_W B_{\mu\nu} = \frac{C_2}{2}\ \partial_{[\mu}\theta_R^I(x)W_{\nu]}^I(x) \tag{24}$$

then the noninvariance of the action under this transformation would precisely cancel the term in (23) and gauge invariance will be restored.

Now let us turn our attention to the SO(44) gauge background coupled to the string coordinates. As before, we can calculate and show that the minimal anomalous action has the form(9,13)

$$S_{\text{anomalous}}^{(A)} = -\frac{1}{16\pi}\int d^2\sigma\ \epsilon^{\alpha\beta}\ \partial_\alpha\ x^\mu\ \partial_\beta\ x^\nu\ \partial_\mu\ \theta_L^m\ A_\nu^m \tag{25}$$

which will be cancelled if the antisymmetric tensor field transforms as

$$\delta_A\ B_{\mu\nu} = \frac{1}{4}\ \partial_{[\mu}\theta_L^m(x)A_{\nu]}^m(x) \tag{26}$$

Note that the fermions ψ^μ, $\mu = 3,4$ couple to the field strength $H_{\mu\nu\lambda}$ of $B_{\mu\nu}$. But $H_{\mu\nu\lambda}$ is not gauge invariant under the gauge transformations (24) and (26). We can, however, define a new field strength

$$S_{\mu\nu\lambda} = H_{\mu\nu\lambda} + (C\text{-}S)_W + (C\text{-}S)_A \tag{27}$$

which is invariant both under the Abelian gauge transformation $\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu(x) - \partial_\nu \Lambda_\mu(x)$ as well as the non-Abelian gauge transformations (24) and (26).

This demonstrates that the quantum consistency of the four-dimensional N=4 heterotic string requires the Yang-Mills C-S term in the field strength of the antisymmetric tensor $B_{\mu\nu}$. As we have noted earlier, the antisymmetric tensor in four dimensional string theories is identified with the axion and several authors have constructed low energy effective Lagrangians[17] inspired by string theories to study the phenomenology of axions. The low energy effective actions contain interactions of the type $H_{\mu\nu\lambda}H^{\mu\nu\lambda}$. Our analysis suggests that these would modify to $S_{\mu\nu\lambda}S^{\mu\nu\lambda}$. It would be interesting to explore the phenomenological consequences of such a modification.

We would like to emphasize that the existence of a C-S term is not special to the particular d=4, N=4 heterotic string model that we analyzed. In fact, such a term would necessarily arise in any other solution of d=4 heterotic string primarily from the requirement of cancellation of the non-Abelian gauge anomaly. Therefore, the results presented in this paper are quite general and the particular d=4, N=4 model is chosen as an illustrative example.

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