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Constraints on Muonium–Antimuonium Conversion

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ABSTRACT

We discuss a simple model in which the muonium–antimuonium conversion can occur at a level around the recent experimental bound but the process $\mu \to e\gamma$ is strictly forbidden. Measurements of the anomalous muon magnetic moment and the high energy Bhabha scattering $e^+e^- \to e^+e^-$ together provide an indirect and interesting constraint on the conversion. The model predicts anomalous events $ep \to \bar{e}\mu\mu X$ at the high energy ep collider.

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Recently the experimental limit on the muonium–antimuonium (M–$\bar{M}$) oscillation was improved by two orders of magnitudes.[1–4] However it has been commented[5] frequently in the literature that among the presently favorable models the theoretical expectation of this oscillation rate is still many orders of magnitudes smaller than the current experimental limit because of the other stronger constraints from the neutrino mass or the process $\mu \rightarrow e\gamma$ *etc.* Since it is possible to further improve this experimental bound[6] on M–$\bar{M}$, it becomes important to take a closer look at the relationship between this experimental bound on M–$\bar{M}$ and the other limits associated with the leptonic sector. In this letter we *emphasize* the essential independence between this experimental bound and most of the other stringent bounds. We work out the phenomenological consequence of a very simple extension of the Standard Model in which the lepton number is automatically conserved and $\mu \rightarrow e\gamma$ is forbidden while the M–$\bar{M}$ oscillation provides the most stringent constraint on the parameters beyond the Standard Model. The simplicity of the model means that it can be imbedded easily into a more complicated model like the left–right symmetric models or the grand unified models. Therefore our analysis also represents a whole class of model with a subsector like this one.

The experimental bounds associated with the leptonic sector can be classified into the following types:

Type (a) in which the lepton number is broken. The majorana neutrino mass and the neutrinoless double beta decay provide the possible signature.

Type (b) in which the lepton flavor is not conserved by an odd unit. These include experimental limits on $K \rightarrow \mu\bar{e}$, $\mu \rightarrow e\gamma$, $\mu \rightarrow ee\bar{e}$.

Type (c) in which the lepton flavor is not conserved by an even unit. These include the M–$\bar{M}$ oscillation, $\pi^- \rightarrow \bar{\mu}ee\bar{\nu}$.

From this it is easy to see how to forbid the experimental signatures of Type (a) and Type (b) while allowing Type (c) signature to occur at the rate of the present upper bound.

The simplest model is to merely add a doubly charged singlet scalar boson k^{++} to the Standard Model. The lepton number is automatically conserved just as the Standard Model with $L(k^{++}) = -2$. In order to forbid Type (b) signature, we have to impose a discrete symmetry, P_e, which changes the signs of the left–handed $SU(2)$ fermion doublet associated with the electron and the right–handed electron e_R,

$$P_e : \begin{cases} \psi_{eL} \rightarrow -\psi_{eL} \; ; \\ e_R \rightarrow -e_R \; . \end{cases}$$

Note that without loss of generality, we assume that the usual Yukawa coupling of the lepton is diagonal. Of course, we could have imposed the symmetry on the muon instead of the electron. Such change only affects the physics associated with the τ lepton which will not concern us in this letter. We also could have used a triplet scalar boson instead of the singlet. However in that case we have to impose the lepton number in addition to P_e which we considered it as a complication. Now we have the lepton number to forbid the Type (a) signature and P_e to forbid the Type (b) signature. The Type (c) signature becomes the most significant phenomenon beyond the model.

The relevant couplings of the k^{++} boson to leptons are given by

$$\mathcal{L}_Y = g_l l_R^T C l_R k^{++} + H.c.$$

As illustrated in Fig. 1, the k^{++} exchange diagram is the source of the M–$\bar{M}$ oscillation, which is described by the effective Hamiltonian[7]

$$\mathcal{H}_{eff} = (G_{M\bar{M}}/\sqrt{2})[\bar{\mu}\gamma^\lambda(1+\gamma_5)e]^2 + H.c.$$

This form is obtained with the help of the Fierz transformation and the definitions,

$$G_{ll'} = g_l g_{l'}/M_k{}^2 \ ,$$

$$G_{M\bar{M}} = G_{e\mu}/(4\sqrt{2}) \ .$$

The integrated probability that the muonium $M(\mu^+e^-)$ decay as μ^- rather than μ^+ is

$$\mathcal{P}(\bar{M}) = 64^3\Big(\frac{3\pi^2\alpha^3}{G_F m_\mu^2}\Big)^2\Big(\frac{m_e}{m_\mu}\Big)^6\Big(\frac{G_{M\bar{M}}}{G_F}\Big)^2 = 2.5\times 10^{-5}\Big(\frac{G_{M\bar{M}}}{G_F}\Big)^2 \ .$$

The experimental bound translates into

$$G_{e\mu}/G_F < \begin{cases} 5.1 & \text{(Ref. 1)}, \\ 2.8 & \text{(Ref. 2)}, \\ 1.7 & \text{(Ref. 3)}. \end{cases}$$

If one takes the coupling g_l optimistically to be the size of the gauge coupling, the doubly charged scalar boson k^{++} can still be lighter than the W. However one has to be careful about some other indirect constraints on the model. One of the significant constraints comes from the measurement of the muon anomalous magnetic moment. The experimental bound on the contribution to $a_\mu = \frac{1}{2}(g-2)$ beyond the Standard Model is[8]

$$\delta a_\mu = (27 \pm 69)\times 10^{-10} \ .$$

The extra contribution to δa_μ in this model comes from the diagrams in Fig. 2. Direct calculations[9] give

$$\delta a_\mu = -G_{\mu\mu} m_\mu^2/6\pi^2 \ ,$$

which implies that

$$G_{\mu\mu}/G_F < 1.9 \ .$$

As $G_{e\mu}^2 = G_{ee}G_{\mu\mu}$, we need additional experimental information about G_{ee} to establish the relationship. Bhabha scattering provides the link. It has been studied in the context of probing the compositeness of the electron[10] with the effective interaction:

$$\mathcal{L}_{ee} = (2\pi/\Lambda^2)(\bar{e}_R\gamma_\mu e_R)^2 \ .$$

The PETRA measurements[11] imply the bound[10]

$$\Lambda > 750\text{GeV} \ .$$

In the present model, the exchange of the doubly charged boson k^{++} in the t-channel (see Fig. 3) gives rise to the effective interaction. By identifying $G_{ee} = 16\pi\Lambda^{-2}$, we obtain

$$G_{ee}/G_F < 7.7 \ .$$

Combining the bounds from δa_μ and $e^+e^- \to e^+e^-$, we derive an indirect bound

$$G_{e\mu}/G_F < 3.8 \ .$$

Therefore, the recent experimental results on M–$\bar{M}$ oscillation have just caught up to the constraints imposed by $g-2$ and $e^+e^- \to e^+e^-$ together. Since $g-2$ constraint is plagued with theoretical uncertainty from the hadronic polarization, it is difficult to extract new physics even with further improvement on the $g-2$ experiment.

The same setups for the M–$\bar{M}$ experiments can be used to search for the rare decay $\pi^- \to \bar{\mu}ee\bar{\nu}$ at a branching fraction as low as 10^{-12}. Our model allows this process through the diagram of the k^{++} exchange. However, its rate is highly suppressed by,

$$\frac{\Gamma(\pi^- \to \bar{\mu}ee\bar{\nu})}{\Gamma(\pi^- \to \mu\bar{\nu})} \sim \Big(\frac{G_{e\mu}m_\pi^2}{16\pi^2}\Big)^2 \sim 10^{-18} \ ,$$

which is far below the present attainable level.

Another interesting test for models of Type (c) is to search for events like $ep \to \bar{e}\mu\mu X$ at the ep collider such as HERA. In comparison with the ordinary QED process $ep \to e\mu\bar{\mu}X$, the production ratio at large transverse momenta p_t can be estimated to be of the order

$$\frac{d\sigma(ep \to \bar{e}\mu\mu X)}{d\sigma(ep \to e\bar{\mu}\mu X)} \sim \Big(\frac{G_{e\mu}p_t^2}{\alpha}\Big)^2 \ .$$

The event rate can be substantial for the transverse momenta p_t of leptons about 10 GeV if $G_{e\mu} \sim G_F$.

Note that the discrete symmetry P_e can be elevated to an anomaly free continuous symmetry if two or more k^{--} are used. Also a more complicated model of Type (c) can be found in Ref. 12 where many doublets are used instead of a singlet k. However, for future experimental study the simplest model is most useful.

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Figure Captions

Fig. 1. The k^{++} exchange diagram for the muonium–antimuonium conversion.

Fig. 2. Contributions to the muon anomalous magnetic moment.

Fig. 3. The t-channel k^{--} exchange diagram for the Bhabha scattering $e^+e^- \to e^+e^-$.

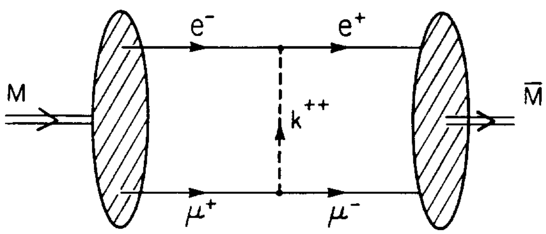

Fig. 1

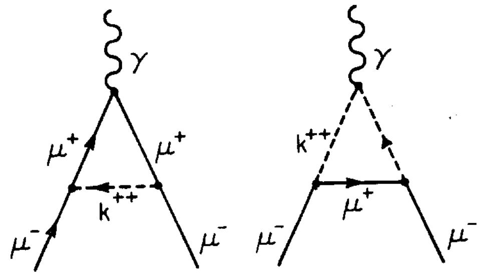

Fig. 2

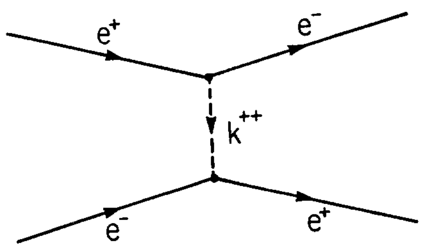

Fig. 3