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On The Effects of Cosmions Upon the Structure and Evolution of Very Low Mass Stars

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ABSTRACT

A number of recent studies have suggested that cosmions, or WIMPS, may play an important role in the energetics of the solar interior; in particular, it has been argued that these hypothetical particles may transport sufficient energy within the nuclear-burning solar core so as to depress the solar core temperature to the point of resolving the solar neutrino problem [1]. Solutions to the solar neutrino problem have proven themselves to be quite nonunique [2], so that it is of some interest whether the cosmion solution can be tested in some independent manner [3]. In this Letter, we argue, first, that if cosmions solve the solar neutrino problem, then they must also play an important role in the evolution of low mass main sequence stars; and, second, that if they do so, then a simple (long mean free path) model for the interaction of cosmions with baryons leads to changes in the structure of the nuclear-burning core which may be in principal observable. Such changes include suppression of a fully-convective core in very low mass main sequence stars; and a possible thermal runaway in the core of the nuclear burning region. Some of these changes may be directly observable, and hence may provide independent constraints on the properties of the cosmions required to solve the solar neutrino problem, perhaps even ruling them out.

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Simple physical arguments suggest that if cosmions transport a significant fraction of the energy liberated in the nuclear-burning core of a very low-mass star, then it may well be possible to prevent the cores of such stars from becoming fully convective. Such changes in stellar structure might be directly observable: Suppose, for example, that the lower convection zone boundary can be pushed well outside of the core region of an ordinarily fully-convective, $0.2M_{O}$ main sequence star. The corresponding temperature and hydrostatic pressure readjustments may then lead to sufficiently large ($\gtrsim$10%) changes in the star's radius and/or luminosity that they can be confirmed or excluded by, for example, comparing such models with observations of nearby, well-measured eclipsing visual binaries of low mass [4]; alternatively, it may be possible to test for such changes in stellar structure by observing the spectrum of low-degree p-modes, as has been done for the Sun in order to measure the extent of its convection zone [5]. The detailed analysis briefly described in the following has led us not only to explore these questions, but also to obtain (unexpected) results on the possible instability of the core, which may provide yet further observational constraints on cosmions in the future.

To begin with, we note that the gravitational potential of low mass stars (we fix for convenience on a star with mass 0.2 M_{O}) is such that the rates of cosmion capture and evaporation are very similar to the corresponding rates in the Sun. So, unlike the case for giant branch stars, any cosmion relevant to the solar neutrino problem will also be relevant for these lower mass stars. Furthermore, solar evolutionary models which include cosmions show that by the time these weakly-interacting particles produce noticable effects in the solar interior, they carry an appreciable fraction of the solar luminosity in the nuclear-burning core; hence, our initial arguments suggested that cosmions may well suppress the onset of convection near the core of very low mass stars simply by flattening the actual temperature gradient below the adiabatic value within the core.

These ideas led us to investigate the problem quantitatively, based on a more accurate evaluation of the cosmion trapping efficiency by the stellar gravitational potential, and the use of a stellar evolutionary code (courtesy of I. Iben) to evolve the type of star in question on the main sequence. In the main, our procedures are very similar to those described by Gilliland *et al.* [1]; in particular, the interaction between cosmions and ordinary baryonic matter for low mass stars is best described by the small interaction cross section limit first studied by Spergel and Press [1]. We parametrize the cosmions by the same three parameters as Gilliland *et al.* [1], namely the cosmion cross section, mass, and abundance; and use the same values of these parameters which Gilliland *et al.* claim solve the solar neutrino problem. For these parameters in a $0.2M_{O}$ star, we find a capture rate [6] of 30% of the capture rate for the Sun, implying a cosmion abundance of 30% of the corresponding solar abundance at a given age. The evaporation mass [7], e.g., the mass of the lightest cosmion that will not evaporate in a solar age, is the same as for the Sun within the uncertainties of the calculations. Energy transport by cosmions is handled in exactly the same way as in Gilliland *et al.* (see their equation 3). The star is thus evolved in time, while simultaneously the number of cosmions within the potential increases because of accretion.

Figure 1 compares the interior temperature profile for two converged solutions to the stellar evolution equations which include cosmion energy transport at two successive times; the later solution is obtained near the last time step $t = t_*$ ($= \approx 2.7\times10^8$ yrs) for which we were able to obtain a converged solution for the cosmion case. At this time step, one can already see the dominance of energy transport in the core by cosmions (Figure 2); that is, cosmion energy transport so dominates the core regions of the star that thermal convection is entirely suppressed. The temperature profile is very comfortably subadiabatic, and in the very center of the core even shows evidence for an increase in temperature with radius. We can thus conclude that the inclusion of cosmions does indeed prevent the formation of a fully-convective core in very low mass main sequence stars. Can this be observed? Recall that the lowest l p-modes are sensitive to the structure of the star's interior at the depth, and on the scale, of the changes we are seeing here [5]; and that furthermore these same low l modes can be in principle detected from spatially-unresolved stellar observations [8]. For the relatively dim low-mass stars here in question, such observations cannot be sensibly carried out at present (both because of the intrinsic faintness of the stars and because the expected amplitude of these p-modes is likely to be far lower than in the solar case); but this may not be an obstacle for observations carried out with the new generation of 8 meter and larger optical telescopes.

The second point of note is the aforementioned lack of convergence at $t = t_*$. In order to convince ourselves that this failure of convergence was not simply a matter of numerical inadequacy of the code we were using, we evolved the same star using different gridding schemes and different time steps (always obtaining the same result); and evolved the same star using a totally different evolutionary code [9], which also fails to converge when the cosmion luminosity begins to dominate energy transport in the core. Close inspection of the iterates for the final time step shows that the interior solution experiences a thermal runaway in the very central core. This last result must be carefully interpreted. Strictly speaking, relaxation codes of the type we used do not follow the time evolution of the star *within* a given relaxation cycle; instead, the thermal runaway seen within the final relaxation cycle is more properly regarded as evidence that no equilibrium solution obeying the assumed equations of stellar evolution exists at that time, near to the solution obtained in the previous time step. Note that we find our result to be entirely independent of the size of the time step, suggesting that the problem is the absence of an equilibrium solution, and not the absence of a nearby equilibrium solution. Can we explain the termination of the sequence of equilibrium solutions?

In the absence of reevolving the star using a fully dynamic stellar interior code (rather than the standard hydrostatic model we used), this question can only be answered by appealing to a local stability analysis within the core. This task is simplified substantially by the fact that at the last converged time step, the nuclear-burning core has a rather flat temperature distribution, so that a local analysis should be adequate for the purpose of establishing the stability properties of this core. Our starting point is then one of the standard forms of the time-dependent energy equation [10],

$$\frac{dT}{dt} = \left(\frac{d\ \ln T}{d\ \ln \rho}\right)_{ad} \frac{T}{\rho}\frac{d\rho}{dt} + \frac{1}{c_v}\left[\epsilon_{nuc} - \epsilon_w\right], \tag{1}$$

together with the equation for hydrostatic equilibrium, $dp/dr = -\rho g$, e.g., we assume in the following that the instability occurs on time scales long when compared to typical dynamical time scales. All thermodynamic quantities have their customary meaning; and ϵ_{nuc} and ϵ_w are the specific luminosities for nuclear burning and cosmion energy transport, respectively (cf. equation 3 in Gilliland *et al.* [1]). In a one-zone approximation for the virtually isothermal core (e.g. Figure 1), and using the fact that the first term on the right-hand side of (1) remains relatively small, we obtain a model equation for the core baryon temperature T of the form

$$\frac{d\tau}{dt} = a\exp[-b/\tau^{1/3}] - c\left(\tau_w + \left[\frac{m_x}{m_b}\right]\tau\right)^{1/2}(\tau - \tau_w)\,, \tag{2}$$

where the first term on the right hand side represents nuclear burning and the second term represents cosmion transport in the large Knudsen number limit; $\tau \equiv T/T_o$, where T_o is the initial isothermal temperature, τ_w is the scaled cosmion temperature, and m_x/m_b (≈ 5) is the ratio of the cosmion to baryon mass. The coefficients a ($= 6.8$), b ($= 18.1$), and c ($= 4.3\times10^{-7}$) are all positive definite; and the coefficient c is only linearly dependent upon density. The core baryon density (ρ) evolution is again fixed by the hydrostatic equation. For simplicity, assume that the cosmion temperature remains fixed; this is not an unreasonable assumption as $\tau_w < \tau_{core}$ and increases throughout the evolution, and c varies with $exp(-\beta/\tau_w)$, so that the coefficient of the loss term in (2) is bounded from above. It is then readily shown that the core temperature evolution equation (2) has a solution which becomes unbounded in finite time. Although other physical processes not considered in this model will clearly intervene to prevent this from occuring, this result nevertheless suggests that such low-mass stars might have a drastically different evolutionary behavior than standard models predict.

The physical cause of this instability is readily found. In a normal star (or in a star in which the cosmion-baryon interaction falls in the small Knudsen number limit, so that energy transport by cosmions obeys a diffusion-like equation), the interior core temperature — and hence the nuclear burning rate — is regulated by the twin demands for energy balance and hydrostatic equilibrium. For example, if nuclear burning were to be shut off within the core, diffusive energy transport alone would tend to lower the core temperature, and hence tend to lower the core pressure; instead, the demand for hydrostatic equilibrium maintains the core pressure by leading to a contraction of the core, and hence to a corresponding heating of the core (as a result of the transformation of gravitational potential energy into the form of heat). In this way, the core temperature is maintained by virtue of the core's contraction despite the cutoff of core nuclear burning, and hence the external appearance of the star would remain the same for time scales of order the Kelvin-Helmholz contraction time for the core. More precisely, in an actual "standard" star, the consequence of this hemostat-like regulation of the core conditions is that the nuclear burning rate is fixed by a combination of the pressure stratification demanded by the equation of hydrostatic equilibrum and the temperature stratification imposed by the diffusive photon energy transport equation.

What fails in a star dominated by cosmion energy transport in the small cross section limit is the tie between pressure regulation of the core by the hydrostatic equation and regulation of the core temperature structure by the local energy balance and transport equations. In particular, energy transport in the core at the final converged time step of our stellar evolution code is beginning to be dominated by cosmions, which carry the energy liberated in the core by nuclear burning well outside the nuclear burning region. This energy transport is sufficiently effective to virtually isothermalize the core region; the resulting spatial uniformity of the temperature distribution, together with the non-local nature of cosmion energy transport in the small cosmion-baryon interaction cross section limit we are in, implies that the temperature profile in the core is no longer regulated by a local diffusion equation. As a consequence, the core baryon temperature T is fixed solely by a local balance between nuclear burning and (non-local) removal of energy by cosmions, as illustrated by equation (2) above. This balance is stable for sufficiently weak temperature dependences of the nuclear burning rate; but for a more realistic temperature-dependence of ϵ_{nuc}, this balance is unstable to a classical superheating instability familiar in gas and plasma dynamics, in which the increase of the local heating rate with temperature overwhelms the increased efficiency of energy losses with increasing temperature. In ordinary stars, such instabilities are suppressed by radiative conduction, which is here ineffective during the crucial initial phase of the instability; in fact, as shown in Figure 1, radiative transport acts to *heat* the central core at the time of the last converged solution.

The conclusions to be drawn from our results are however tempered by several limitations of our analysis. To begin with, the single-zone stability analysis cannot determine the ultimate fate of the instability; saturation of the instability is not captured by our analysis, and hence the possibility remains that there are no observable consequences of the thermal runaway in the core. This limitation can clearly be overcome with the use of more sophisticated, dynamical stellar evolution simulations. We note that the results of the evolutionary stellar structure code argue against the possibility of restabilization at drastically different, but still hydrostatic, core conditions; indeed, inspection of successive iterations suggest that the thermal runaway may lead to rapid mixing in the core on dynamical time scales, driven by Rayleigh-Taylor instabilities. However, in the absence of computations with a fully-dynamic code, this remains speculative. Stabilization by radiative transport seems unlikely in light of the rapid onset of the instability; indeed, we suspect that a significant reason for the lack of convergence of the evolution code is that — as in the case of a helium flash — standard evolutionary stellar structure code cannot follow the dynamics as the evolutionary time scale changes by so many orders of magnitude. We further note that the small interaction cross section limit considered here has the defect that the cosmion temperature is assumed to be spatially uniform; in fact, one expects the cosmion temperature to roughly follow the form of gravitational potential [11], and hence to decrease with distance from the core. Qualitatively, the principal effects of this temperature variation are to decrease the volume within which cosmions redistribute the energy liberated in the core, and to raise the cosmion temperature in the core region. The latter effect will decrease the effectiveness of cosmion energy transport within the core region; and as a result, one would expect the onset of the instability to be postponed beyond the point indicated in our simulations, but for the initial linear instability growth rate to be somewhat increased once cosmion energy transport dominates photon energy transport. Thus, it seems that the isothermal cosmion temperature assumption is very likely a satisfactory model for measuring the impact of cosmion energy transport on the evolution of low-mass stars. Finally, we note that S. Raby has recently found results rather similar to those found here, but for a evolution model for the Sun [12]; how our results and those of his relate remains to be seen.

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References

1. Spergel, D. N., and W. H. Press *Astrophys. J.*, **296**, 663 (1985); Faulkner, J., and R. L. Gilliland *Astrophys. J.*, **299**, 994 (1985); Gilliland, R., J. Faulkner, W. H. Press, and D. N. Spergel *Astrophys. J.*, **306**, 703 (1986); Spergel, D. N., and J. Faulkner *Ap. J. (Letters)*, **331**, L21 (1988).
2. Bahcall, J., and R. Ulrich *Rev. Modern Phys.*, **60**, 297 (1988).
3. Rood, R. T., and A. Renzini in *Astronomy, Cosmology and Fundamental Physics*, M. Caffo, R. Fanti, G. Giacomelli, and A. Renzini, eds. (Dordrecht: Kluwer), in press (1988); Raffelt, G. G., and D. S. P. Dearborn *Phys. Rev.*, **D37**, 549 (1988); Salati, P., A. Bouquet, and J. Kaplan, preprint (1988).
4. Popper, D. M. *Ann. Rev. Astron. Ap.*, **18**, 115 (1980).
5. See, for example, Duvall, Jr., T. L., and J. W. Harvey *Nature*, **310**, 19 (1984).
6. Gould, A. *Astrophys. J.*, **321**, 521 (1987); Gould, A., private communication.
7. Griest, K., and D. Seckel *Nucl. Phys.*, **B283**, 681 (1987); **B296**, 1034 (1988).
8. cf. review by K. G. Libbrecht, *Space Sci. Rev.*, **47**, 275 (1988).
9. Gilliland, R. L. "SOLEVL — A User's Manual" (Boulder: NCAR, 1987).
10. cf. equation (4.31) in J. P. Cox, *Theory of Stellar Pulsation* (Princeton: Princeton Univ. Press, 1980).
11. Nauenberg, M. *Phys. Rev.*, **D36**, 1080 (1987); Gould, A., and G. Raffelt, private communication.
12. Raby, S., preprint (1989).

Figure 1: Baryon temperature (in units of 10^6 K) as a function of mass (in solar mass units) for a 0.2 solar mass star at an age of 5.1×10^7 years (solid) and 2.7×10^8 years (dashed); both results are for fully-converged solutions to the stellar evolution equations including energy transport by cosmions, and in both cases, the surface temperature is 3.23×10^3 K.

Figure 2: Energy balance within a 0.2 solar mass star at an age of 2.7×10^8 years. The different contributions to the differential luminosity (dL/dM) are plotted, as well as their sum, versus the mass coordinate (in solar mass units): the nuclear contribution (solid); the wimp contribution (dash dot); the gravitational contribution (dash dot dot dot); the radiative contribution (dashed); and the sum of all contributions (dotted). ϵ_{nuc} and ϵ_{wimp} are the clearly dominant contributors, while the radiative contribution is not only small, but also positive in the core (because of a slight temperature inversion within the core).

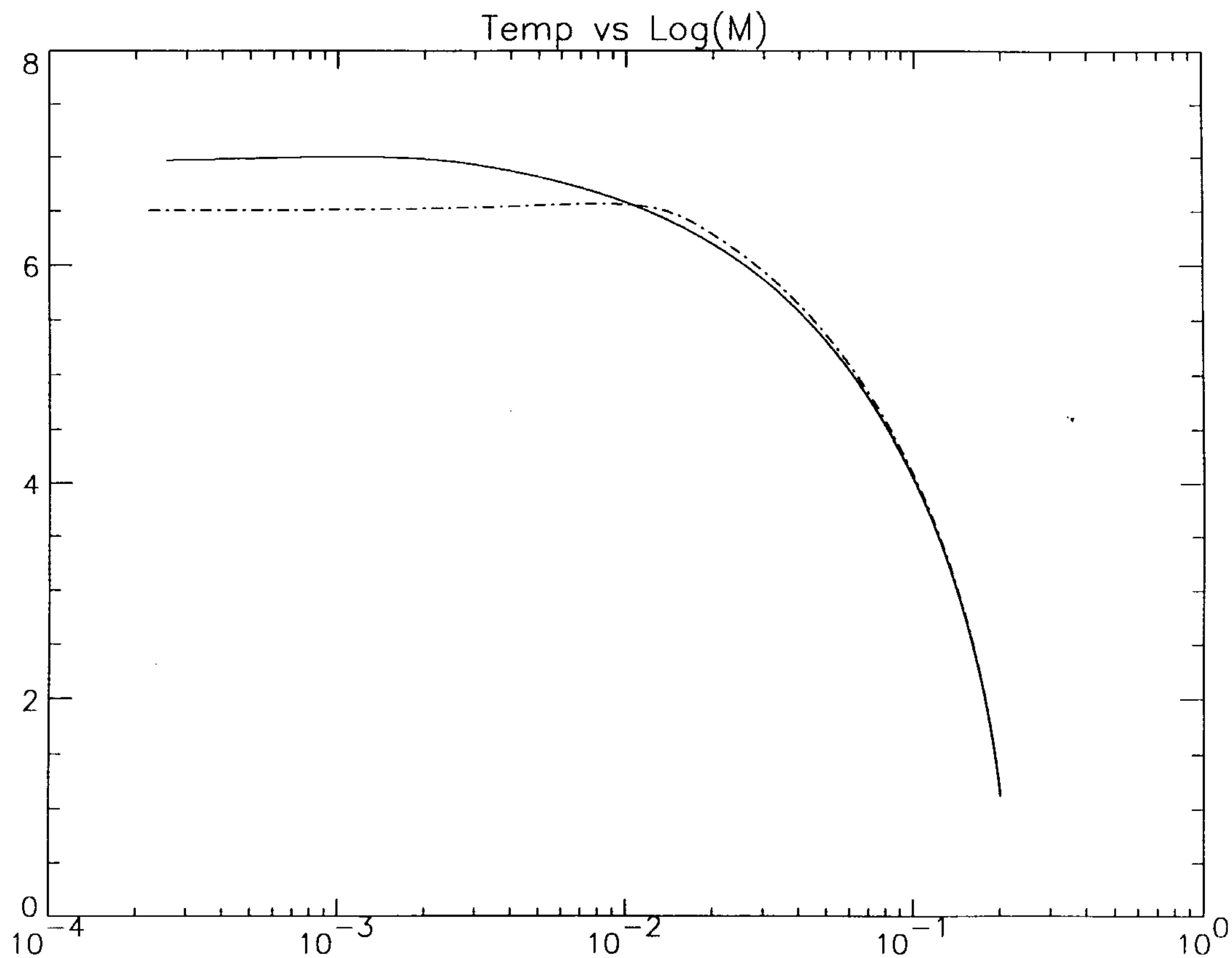

fig. 1

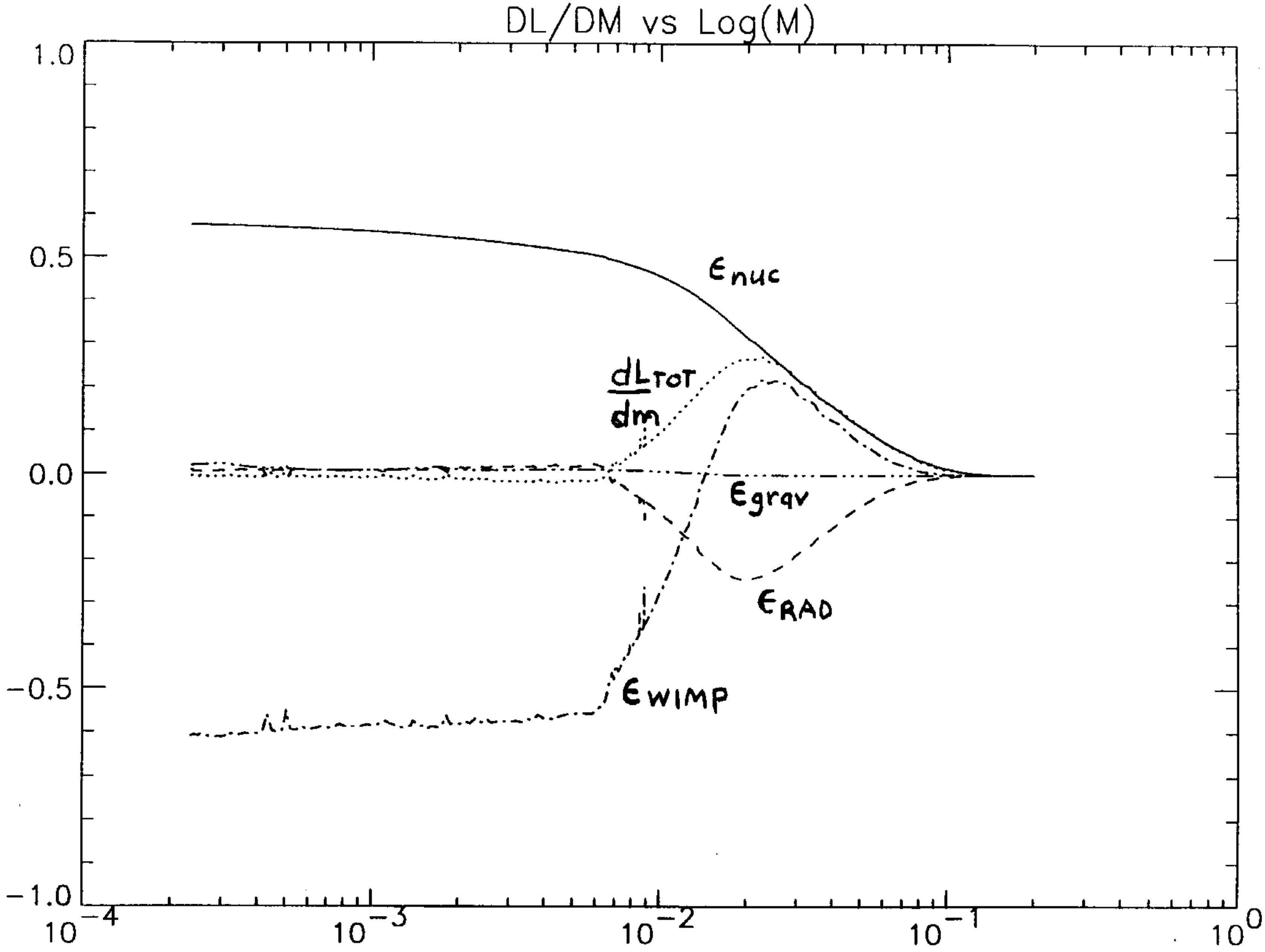
DL/DM vs Log(M)
1.0
0.5
0.0
−0.5
−1.0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
ϵ_{nuc}
$\frac{dL_{TOT}}{dm}$
ϵ_{grav}
ϵ_{RAD}
ϵ_{WIMP}

Fig. 2