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HIERARCHICAL CHIRAL SYMMETRY BREAKING AND QUARK MASS MATRICES REVISITED

Carl H. ALBRIGHT* and Manfred LINDNER

Fermi National Accelerator Laboratory,

P.O. Box 500, Batavia, Illinois 60510

Abstract

We analyze how the assumption of hierarchical chiral symmetry breaking can be systematically used to create phenomenologically satisfactory mass matrices. In place of postulating a particular set of mass matrices at the outset, we emphasize that once a particular basis for the first stage of chiral symmetry breaking is selected, the following steps are determined by the known information on quark masses and mixings. We illustrate this procedure for the basis originally chosen by Fritzsch and find a modified set of quark mass matrices in the minimal Higgs framework which fits the data much better provided $m_t \simeq 88 GeV$ and the $b \to u/b \to c$ mixing ratio is large.

*Permanent address: Department of Physics, Northern Illinois University, DeKalb, Illinois 60115

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Although the standard model (SM) of color and electroweak interactions based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ is highly successful, the number of generations and mass spectra of the quarks and leptons remain unexplained. Part of the problem can be traced to the fact that insight into the mass-generating mechanism taking place in the helicity-flipping Yukawa mass sector is obscured due to the freedom of choice of basis and to the question in which particular basis the mass dynamics should be studied. For example, the nonidentity of the mass and weak eigenbases leads to Cabibbo and Kobayashi-Maskawa mixing angles.[1] Clearly, all important features like quark mass hierarchies and relatively small mixing angles survive in any basis and strongly suggest a mass mechanism generated by successive stages of chiral symmetry breaking (χSB). While it may be desirable to use nearly diagonal mass matrices to pursue the consequences of the χSB chain, one can also attempt to pursue these ideas in other bases like the weak basis. Such reasoning, in fact, led Fritzsch[2] more than ten years ago to postulate a rather attractive set of mass matrices which, however, are not in good agreement with experimental information unless the Higgs structure is enlarged.[3] Here we reexamine the issue of hierarchical χSB in the light of the freedom to choose any basis for the mass matrices. In the framework of a hierarchical sequence, we shall illustrate that in comparing with experimental data it is possible to determine the form of the corrections of the second and third step of χSB once a basis is choosen for the first step. We exhibit the generality of this balance between a choice for the first stage and the necessary form of the following stages. As a concrete example we indicate how the original Fritzsch model with minimal Higgs structure can be improved systematically, before and after renormalization effects are taken into account. Comments are made for a mass matrix in the coherent basis which is in analogy to the BCS gap phenomenon.[4]

The real issue is which eigenvector basis should one adopt to shed most light on the mass-generating mechanism. This, in turn, is tied up with the existence of certain unphysical transformations which may or may not have physical significance in a proper extension beyond the SM. Within the framework of the SM, the weak eigenbasis and mass eigenbasis seem to be rather special. A typical term in the Yukawa Lagrangian, which yields mass contributions through the spontaneous symmetry-breaking mechanism, can be written in these two bases as

$$\bar{\psi}_L \mathbf{M} \psi_R = (\bar{\psi}_L \mathbf{U}_L^{\dagger}) \mathbf{U}_L \mathbf{M} \mathbf{U}_R^{\dagger} (\mathbf{U}_R \psi_R) = \bar{\Psi}_L \mathbf{D}^D \Psi_R \tag{1}$$

where $\mathbf{U}_L$ and $\mathbf{U}_R$ are SU(3) transformations in flavor space and we have allowed for the possibility that the mass (sub)matrix $\mathbf{M}$ may be non-Hermitian. In the weak basis, $\mathbf{M}_{ij} = g_{ij} < \phi >$ in terms of Yukawa couplings and the vacuum expectation value, while in the mass eigenbasis $\mathbf{D}^D = diag\ (\lambda_1,\ \lambda_2,\ \lambda_3)$. On the other hand, a typical term in the weak charged-current Lagrangian can be written as

$$\begin{aligned} \bar{\psi}_L^U \gamma_\mu \psi_L^D W_\mu^+ &= (\bar{\psi}_L^U \mathbf{U}_L^{U\dagger} \gamma_\mu \mathbf{U}_L^U \mathbf{U}_L^{D\dagger} (\mathbf{U}_L^D \psi_L^D) W_\mu^+ \\ &= \bar{\Psi}_L^U \gamma_\mu \mathbf{V}_{KM} \Psi_L^D W_\mu^+ \end{aligned} \tag{2a}$$

where the KM mixing matrix is[1]

$$\mathbf{V}_{KM} = \mathbf{U}_L^U \mathbf{U}_L^{D\dagger} \tag{2b}$$

(up to diagonal SU(3) phase transformations) and contains just four physical mixing parameters, e.g., three angles and one phase for the case of three families of quarks. Any general SU(3) transformation of type (1) simply corresponds to a different choice of basis. If one thinks of mass matrices as vectors in a suitable vector space, this means that one can rotate the vectors to any basis and only the relative angles between the up and down sectors given in (2b) have to be fixed. While these ambiguities are completely unphysical within the SM, it may very well happen that in physics beyond the SM, the spontaneous symmetry-breaking mechanism assumes more significance such that a special basis is singled out in which mass matrices are explained in a particularly simple way.

As mentioned above, the observed quark mass hierarchy strongly suggests a hierarchial χSB sequence as emphasized long ago by Fritzsch.[2] This sequence could be generated by a series of new scales associated with a series of new physics, but it seems more natural to assume that there is only one new scale and that the hierarchy is generated by small (seesaw or radiative) corrections to a rank one matrix, $\widetilde{\mathbf{M}}_1$. The fact that $\widetilde{\mathbf{M}}_1$ is rank one ensures that only one mass is generated in the first stage of χSB. In the second and third stages, the full mass matrix then receives successively smaller correction terms:

$$\widetilde{\mathbf{M}} = \widetilde{\mathbf{M}}_1 + \epsilon\widetilde{\mathbf{M}}_2 + \eta\widetilde{\mathbf{M}}_3; \quad rank\ \widetilde{\mathbf{M}}_i = i, \quad \eta \ll \epsilon \ll 1 \tag{3}$$

If the first stage of χSB selects the same basis for the up and down quark sectors, small mixings are automatically ensured, since they are attributed to the differences in the diagonalization of the full matrix coming from the second and final stage correction terms. From the above discussion, it is also clear that the choice of basis for $\widetilde{\mathbf{M}}_1$ is very important. While in a certain basis the corrections might have a very simple interpretation, it is clear that in another basis this apparent simplicity may be destroyed completely.

For example, if one boldly assumes that the basis for $\widetilde{\mathbf{M}}_1$ is already diagonal and $\widetilde{\mathbf{M}}_2$ and $\widetilde{\mathbf{M}}_3$ arise from nearest neighbor interactions in this basis, then one arrives at the well-known Fritzsch model:[2]

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & B^* & 0 \end{pmatrix} + \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad A \ll B \ll C \tag{4}$$

But the same type of correction terms in other bases lead to very different results, and one is left with a huge number of matrix types in different bases and with different corrections. Since the Fritzsch model with minimal Higgs structure fails to explain all the experimental information,[3,5] we now reexamine its basic assumptions and show

how a more general form can emerge which improves the predictions.

To construct a sequence of χSB one has to start with the limit where all quark masses vanish and the Yukawa terms of (1) are missing, so one need not adopt any particular basis. All terms in the Lagrangian are flavor-diagonal and are invariant under separate $U(3)$ transformations of the lefthanded and righthanded fields, i.e., the Lagrangian exhibits a full global $U(3)_L \times U(3)_R$ chiral symmetry. The first stage of χSB at some high scale will give a mass to the t and b quarks via two, in principle independent, rank one matrices $\widetilde{\mathbf{M}}_1^U$ and $\widetilde{\mathbf{M}}_1^D$. Although the form of these matrices is related to the unknown mass generation mechanism, it is clear that a random choice for $\widetilde{\mathbf{M}}_1^U$ and $\widetilde{\mathbf{M}}_1^D$ would typically result in some large mixings. Since the corrections of stage 2 and 3 are small, the initial basis will introduce a stiffness which will nearly preserve the mixings first introduced; therefore, it seems natural to assume[†] $\widetilde{\mathbf{M}}_1^U = (\widetilde{\mathbf{M}}_1^U)^\dagger = \mathit{const.} \times \widetilde{\mathbf{M}}_1^D$ for the basis where the first stage of χSB occurs. By a suitable SU(3) rotation $\mathbf{U}_L$, we can then always simultaneously diagonalize the up and down sector of this first stage of χSB without picking up any mixings, while the second and third step are transformed into new, probably less transparent, corrections:

$$\mathbf{M} = \mathbf{U}_L(\widetilde{\mathbf{M}}_1 + \epsilon\widetilde{\mathbf{M}}_2 + \eta\widetilde{\mathbf{M}}_3)\mathbf{U}_L^\dagger = \mathit{diag}(0,0,C) + \epsilon\mathbf{M}_2 + \eta\mathbf{M}_3 \tag{5}$$

In this basis the Lagrangian exhibits at the first stage just a chiral $U(2)_L \times U(2)_R$ symmetry in the first and second family subspace. Alternatively, one can regard the residual symmetry as a discrete chiral $Z(3)_L \times Z(3)_R$ symmetry which has certain advantages for the Goldstone sector.[7]

At the second and third stages of χSB, we shall rely heavily on the observed form

[†] An approximate relation of this type was observed some time ago for the full up and down mass matrices, ref. 6.

of the KM matrix[8]

$$\mathbf{V}_{KM} = \begin{pmatrix} 0.9754 \pm 0.0004 & 0.2206 \pm 0.0018 & 0 \pm 0.0087 \\ -0.2203 \pm 0.0019 & 0.9743 \pm 0.0005 & 0.0460 \pm 0.0060 \\ 0.0101 \pm 0.0086 & -0.0449 \pm 0.0062 & 0.9989 \pm 0.0003 \end{pmatrix} + i \begin{pmatrix} 0 & 0 & 0 \pm 0.0087 \\ 0 \pm 0.0004 & 0 \pm 0.0001 & 0 \\ 0 \pm 0.0085 & 0 \pm 0.0019 & 0 \end{pmatrix} \tag{6a}$$

which in discussions of chiral symmetry breaking is conveniently parametrized by[9]

$$\begin{aligned} \mathbf{V}_{KM} &= R_{23}(\theta_{23})R_{13}(\theta_{13},\delta)R_{12}(\theta_{12}) \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \end{aligned} \tag{6b}$$

The data suggest a rotation of $\theta_{23} \simeq 3^o$ occurs first, followed by rotations of $\theta_{13} \simeq 0.5^o$ and phase δ and finally by the Cabibbo angle rotation with $\theta_{12} \simeq 13^o$. The $\epsilon\mathbf{M}_2$ terms of (5) introduced at the second stage of χSB will bring us to rank two matrices and will turn on the c and s quark masses. If the mixing matrix element $|V_{23}|$ is not set properly at the second stage by the choice for $\epsilon\mathbf{M}_2^U$ and $\epsilon\mathbf{M}_2^D$, the even smaller $\eta\mathbf{M}_3$ final corrections have no chance to overcome the stiffness in the $|V_{23}|$ element introduced by the terms of the second stage. As we shall see, this gives generally upper and lower bounds for the top mass. Depending on the chosen form of the second step, other KM matrix elements may also get contributions at this level. If this is the case, then there is also a stiffness problem in these elements and the entries created at this level must be very close to their final values. If, however, the second step does not determine a given KM element, then it is controlled by the third step alone.

Whatever form we adopt for the mass matrices at the second stage of χSB , it must be rank two and in the basis of (5) we must therefore have

$$0 = Det\ [diag\ (0,\ 0,\ C) + \epsilon \mathbf{M}_2] \simeq \epsilon C\ Det\ \mathbf{m}_{12} \tag{7}$$

where $\mathbf{m}_{12}$ is the submatrix composed of the first two rows and columns of $\mathbf{M}_2$, and the last line follows from the fact that C is much larger than any entry in $\mathbf{M}_2$; hence we conclude that $Det\ \mathbf{m}_{12} \simeq 0$. Given the fact that the choices for $\mathbf{m}_{12}^U$ and $\mathbf{m}_{12}^D$ will, in general, introduce a Cabibbo angle already at this second stage of χSB , there are three possibilities. First, it could be that the Cabibbo angle is already determined at this second stage and that the third stage of χSB serves only to give mass to the u and d quarks; however, without a delicate cancellation it is hard to understand the smallness of V_{13} both before and after the third stage. The second possibility is that the $\mathbf{m}_{12}$ submatrices are aligned so that $C^D \epsilon^U \mathbf{m}_{12}^U \simeq C^U \epsilon^D \mathbf{m}_{12}^D$ with $Det\ (\mathbf{m}_{12}^U) = Det\ (\mathbf{m}_{12}^D) = 0$. Since the submatrices are carefully scaled, simultaneous rediagonalization is possible, and no Cabibbo angle emerges at this second stage. This could very well be the case for a dynamical mechanism involving radiative corrections; unfortunately, without a detailed model in mind, no particular form of $\mathbf{m}_{12}$ is singled out.

We now pursue the third possibility that the Cabibbo angle vanishes at this second stage without the scaling requirement above, eg., $(\mathbf{M}_2)_{11} = (\mathbf{M}_2)_{12} = (\mathbf{M}_2)_{21} = 0$, so the rank two mass matrices are of the form

$$\mathbf{M}^{(2)} = diag(0,0,C) + \epsilon \mathbf{M}_2 = \begin{pmatrix} 0 & 0 & D \\ 0 & E & B \\ D^* & B^* & C \end{pmatrix} \tag{8}$$

with D^U or E^U and D^D or E^D set equal to zero to guarantee rank 2. Note that the matrix invariants require

$$\begin{aligned} C^U + E^U &= m_t - m_c, & 2\left[|B^U|^2 + |D^U|^2\right] + (E^U)^2 + (C^U)^2 &= m_t^2 + m_c^2, \\ C^D + E^D &= m_b - m_s, & 2\left[|B^D|^2 + |D^D|^2\right] + (E^D)^2 + (C^D)^2 &= m_b^2 + m_s^2 \end{aligned} \tag{9}$$

In order to insure the upper limits $|V_{13}| \lesssim 0.0123$ and $|V_{23}| \lesssim 0.052$, we find that E^U or $|D^U| \lesssim m_c^2/m_t \lesssim 0.04$ and E^D or $|D^D| \lesssim m_s^2/m_b \simeq 0.0027$ which in turn bounds the t mass at this second stage of χSB to lie in the range

$$\begin{array}{ccccc} 33 & \lesssim & m_t(1GeV) & \lesssim & 145\ GeV \\ 25 & \lesssim & m_t^{phys} & \lesssim & 90\ GeV \end{array} \tag{10}$$

Since the magnitudes of E^U, $|D^U|$, E^D and $|D^D|$ are forced to be anomalously small, we take them identically zero at this stage. In this limit, the Lagrangian exhibits a maximal chiral $U(1)_L \times U(1)_R$ or $Z(2)_L \times Z(2)_R$ symmetry.[7]

Based on the assumption of an aligned first stage of χSB (natural smallness of all mixings), we cast the first stage into a diagonal basis and introduced a general term for the second stage. If we exclude the alternatives as indicated above, we arrive at a model which to this point is identical to that of Fritzsch, although we started with a much larger class of matrices. We chose to discuss this step in a certain basis but the results are still general in terms of symmetries and rank. The form written down is only one representation of the class of matrices which explain the data sucessfully. If we knew which basis is relevant for understanding the first stage, we could transform back and see what the second stage corrections have to be in this basis. If the initial basis were, for example,

$$\mathbf{M}_1 = \frac{1}{3}C \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{3}C\ \mathbf{E}_{111} \tag{11}$$

instead of $diag(0,0,C)$ as proposed recently,[4] then the two different rank one matrices are connected by

$$\mathbf{M}_1 = R_{12}^{-1}(\Theta_3)R_{23}^{-1}(\Theta_2)R_{12}^{-1}(\Theta_1)\ diag(0,0,C)\ R_{12}(\Theta_1)R_{23}(\Theta_2)R_{12}(\Theta_3) \tag{12}$$

where R_{ij} is a rotation in the subspace ij, Θ_1 is arbitrary, $tan(\Theta_2) = \sqrt{2}$ and $\Theta_3 = \frac{\pi}{4}$.

If we go to rank two as discussed above, we can find approximately the corresponding corrections in the new basis (11) by applying the same transformation (12), and we get ($s_1 = \sin\Theta_1,\ c_1 = \cos\Theta_1$):

$$\mathbf{M}^{(2)} = \left(\frac{C}{3} + \frac{(B+B^*)c_1}{3\sqrt{2}}\right)\mathbf{E}_{111} + \frac{1}{\sqrt{6}}\begin{pmatrix} (B+B^*)s_1 & (B-B^*)s_1 & Bs_1 - \sqrt{3}B^*c_1 \\ (B^*-B)s_1 & -(B+B^*)s_1 & -Bs_1 - \sqrt{3}B^*c_1 \\ B^*s_1 - \sqrt{3}Bc_1 & -B^*s_1 - \sqrt{3}Bc_1 & \sqrt{3}^{-1}(B+B^*)c_1 \end{pmatrix} \quad (13)$$

Depending on the arbitrary choice for Θ_1 (e.g., $\frac{\pi}{6}$ and $\frac{\pi}{3}$ seem to be special), B and B^*, we can get interesting patterns in this basis which should be ultimately explainable by the mechanism that selects this basis.

In the third and final stage of χSB, the u and d quarks obtain small masses and all mixings must take their final values consistent with (6). For given first and second stages, we can take the even smaller third stage corrections as perturbations and find which form this third stage can assume to give the right masses and mixings. We have performed this calculation in the basis where the first stage is diagonal, and with small approximations we find that the diagonal elements of $\mathbf{M}_3$ must be extremely small, even at the final stage. The element $(\mathbf{M}_3)_{12}$ is fixed very precisely by the determinant and $(\mathbf{M}_3)_{13}$ and $(\mathbf{M}_3)_{23}$ are essentially free numbers of the order of $(\mathbf{M}_3)_{12}$. Compared to these results, the original Fritzsch model with nearest neighbor interactions is a very special choice. Since $(\mathbf{M}_2)_{23}$ is anyway much larger than $(\mathbf{M}_3)_{23}$ we can safely ignore the latter correction. If we look only at the KM matrix elements and known masses, then $(\mathbf{M}_3)_{13}$ is not constrained. But in $B^0 - \bar{B}^0$ mixing experiments[10] enters a combination of the yet unknown top mass and mixings containing additional information which is quite sensitive to $(\mathbf{M}_3)_{13}$. The $(\mathbf{M}_3)_{13}$ element is then very important, and we have performed a numerical search of possible fits to the data in the presence of the minimal Higgs structure. We found that good fits are centered about

$|(\mathbf{M}_3)_{13}| = |(\mathbf{M}_3)_{12}|$ which leads us to write the full mass matrices in the form

$$\mathbf{M} = \begin{pmatrix} 0 & A & Ae^{i\phi_D} \\ A & 0 & B \\ Ae^{-i\phi_D} & B & C \end{pmatrix} \tag{14}$$

This set of matrices is then characterized by nine parameters, one less than the ten independent physical parameters for three quark families.‡ Note that our search for a solution in the SM framework suggests that we must give up the Fritzsch assumption of nearest neighbor interactions, which requires $D^U = D^D = 0$.

The conventional treatment is to assume these matrices apply at the low scale of 1 GeV, evaluate the parameter magnitudes and calculate the KM matrix elements for comparison with data. But in a previous paper,[11] we have emphasized that in general the above form of the matrices is only relevant at the χSB scale, Λ_{SB}, which we shall take to be 100 TeV for our calculations. One should then evolve the matrices downward to 1 GeV, for example, with the help of the renormalization group equations (RGE's) for the Yukawa couplings. Following the prescription discussed in detail in our previous work and with our earlier notation, we find for the more general form of (14)

$$\mathbf{M}^U(t) \simeq \begin{pmatrix} 0 & A & D \\ A + \frac{B}{C}D^*(\gamma - 1) & \frac{B^2}{C}(\gamma - 1) & B\gamma \\ D^*\gamma & B\gamma & C\gamma \end{pmatrix} \tag{15a}$$

$$\mathbf{M}^D(t) \simeq \begin{pmatrix} 0 & A' & D' \\ A'^* + \frac{B}{C}D'^*(\gamma' - 1) & \frac{B}{C}B'^*(\gamma' - 1) & B' + \frac{B}{C}C'(\gamma' - 1) \\ D'^*\gamma' & B'^*\gamma' & C'\gamma' \end{pmatrix} \tag{15b}$$

‡If one would like to minimize the number of parameters one could try the simplest case $(\mathbf{M}_3)_{12} = (\mathbf{M}_3)_{13} = (\mathbf{M}_3)_{23}$, i.e. $\phi_D = 0$, and as we will see this fits the data best. In that case we have only 8 parameters.

where

$$t = ln(\mu/1GeV), \qquad t_{SB} = ln(\Lambda_{SB}/1GeV) \tag{15c}$$

$$\gamma(t), \gamma'(t) \simeq exp\{\frac{3}{32\pi^2} b_{U,D} \int_{t_{SB}}^{t} dt' C^2(t')\} \tag{15d}$$

and the matrices are conveniently made dimensionless by dividing out by the expectation value $< \phi >\simeq 175$ GeV. The coefficients b_U and b_D are given by[11]

$$\begin{aligned} b_U &= -b_D = 1 \qquad \text{minimal Higgs model} \\ b_U &= 3b_D = 1 \qquad \text{double Higgs model} \end{aligned} \tag{15e}$$

for the two models we shall consider. Note that here both $\mathbf{M}^U$ and $\mathbf{M}^D$ evolve into non-Hermitian forms, unlike the Fritzsch mass matrices.

With the identification $D = A \exp i\phi_D$ and $D' = A' \exp i\phi_D$, the magnitudes of C, B and A can be determined from the invariant quantities $Det\ \mathbf{H}^U, Tr\ \mathbf{H}^U$ and $Tr\ (\mathbf{H}^U)^2$ evaluated in both the weak and mass eigenbases, where $\mathbf{H}^U = \mathbf{M}^U \mathbf{M}^{U\dagger}$, and similarly for $C', |B'|$ and $|A'|$. The relations obtained are quite complicated and best solved by the method of successive approximation. We use the Gasser-Leutwyler[12] quark mass determination at the 1 GeV scale and use explicitly the set

$$\begin{aligned} m_u &= 3.5\ MeV, \qquad & m_d &= 6.1\ MeV \\ m_c &= 1.35\ GeV, \qquad & m_s &= 120\ MeV \\ m_t &= ? \qquad & m_b &= 5.3\ GeV \end{aligned} \tag{16}$$

to compare with our previous results on the Fritzsch model. To determine the "physical" top quark mass, the running mass is evolved from $m_t(1\ GeV)$ to $m_t(m_t)$ by the relation

$$m_t(m_t) = \gamma_{m_t} m_t(m_t)\ |_{gauge\ evolution} \tag{17a}$$

where $\gamma_{m_t} = \gamma(t_{m_t})/\gamma(0)$, and then the first-order QCD correction is applied to obtain[11]

$$m_t^{phys} = m_t(m_t) \left[1 + \frac{4}{3\pi}\alpha_s\right] \tag{17b}$$

The projection operator technique of Jarlskog[13] can be applied to the renormalized matrices $\mathbf{H}^U$ and $\mathbf{H}^D$ to calculate the squares of the KM matrix elements at the 1 GeV scale according to

$$|V_{\alpha j}|^2 = Tr\left[P_\alpha P'_j\right] \tag{18a}$$

$$P_1 = \left(\lambda_2^2\,\mathbf{1} - \mathbf{H}^U\right)\left(\lambda_3^2\,\mathbf{1} - \mathbf{H}^U\right) / \left[(\lambda_2^2 - \lambda_1^2)(\lambda_3^2 - \lambda_1^2)\right] \ , \quad \text{etc.} \tag{18b}$$

where they can be compared directly with the data in (6). We fit all $|V_{\alpha j}|^2$ KM matrix elements squared within one standard deviation by varying $m_t(1\ GeV)$, $\phi_{A'}$ and $\phi_{B'}$ – only two of which are independent in the Fritzsch model. In the new model proposed in this paper, we also vary ϕ_D to keep the J-value[14] of CP-violation fixed near its best experimental value[15]

$$\begin{aligned} J &= Im(V_{12}V_{23}V_{13}^*V_{22}^*) \\ &= c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}sin\delta \\ &\simeq |V_{12}||V_{23}||V_{13}||V_{22}|sin\delta \\ &\simeq 0.30 \times 10^{-4} \end{aligned} \tag{19}$$

in the parametrization introduced in (6b). The physically-allowed KM regions are indicated by the annular rings in Fig. 1 for the Fritzsch and new models, both without and with evolution, where the phase angle $\phi_{B'}$ is plotted against $m_t(1\ GeV)$ and m_t^{phys} for the tightly- constrained ($\pm 2°$) phase $\phi_{A'}$. Contours of fixed ϕ_D within the rings are clearly marked which yield the correct J-value of (19) for the new model.

For each point on the $\phi_{B'}\ vs.m_t$ plots, we then calculate the lefthand side of[16]

$$m_t^2|V_{33}V_{31}^*|^2 R(z_t, z_\eta, v_2/v_1) \simeq (2.0 \pm 0.5)\frac{(0.140)^2}{B_B f_B^2} \tag{20}$$

where the righthand side is determined by the ARGUS and recent CLEO results[10] on $B_d^o - \bar{B}_d^o$ mixing. As noted in earlier publications,[3] the allowed $B - \bar{B}$ mixing band overlaps the physically-allowed KM ring only in the two-doublet Higgs version of the Fritzsch model. With the new model proposed here, the standard Higgs structure is also acceptable. The combined allowed KM and $B - \bar{B}$ mixing regions favor a top quark mass of 88 GeV in both the Fritzsch model with two-doublet Higgs and our new model with standard minimal Higgs structure. Note that the preferred value for $\phi_D \simeq 0^o$, which could be used to propose a model with fewer parameters. The top mass can range down to 70 GeV, if one insists on our new model with two Higgs doublets.

Other parameters of interest include the CP-violation phase angle δ, the $B_s^o - \bar{B}_s^o$ mixing parameter r_s, the charmless-to-charm ratio $|V_{13}|/|V_{23}|$, the bag parameter B_K in K decay and finally ϵ'/ϵ. The predictions for the "best" point in each one of the graphs are presented in Table I, along with the presently best-determined experimental or theoretical values.[10,15,17–19] A strikingly-invariant feature to be observed from the Table is that the Fritzsch model favors a relatively low $b \to u/b \to c \simeq 0.05$ ratio, while our new model favors a rather high $\simeq 0.15$ ratio. This dichotomy occurs exactly along the lines of the CLEO *vs.* ARGUS findings. It is also interesting to note that only the new model with standard Higgs structure gives a bag parameter in excellent agreement with the large N calculations of Bardeen, Buras and Gèrard.[18]

As a further comparison, we present the KM matrices for the two best fits of Fig. 1c and 1e:

$$\mathbf{V}_{KM}^{FRIT} = \begin{pmatrix} 0.9752 & 0.2209 & -0.0004 \\ -0.2206 & 0.9741 & 0.0498 \\ 0.0114 & -0.0485 & 0.9987 \end{pmatrix} \pm i \begin{pmatrix} 0 & 0 & 0.0026 \\ 0.0001 & 0 & 0 \\ 0.0026 & 0.0006 & 0 \end{pmatrix} \qquad (21a)$$

$$\mathbf{V}_{KM}^{NEW} = \begin{pmatrix} 0.9751 & 0.2211 & -0.0069 \\ -0.2205 & 0.9740 & 0.0504 \\ 0.0179 & -0.0476 & 0.9986 \end{pmatrix} \pm i \begin{pmatrix} 0 & 0 & 0.0028 \\ 0.0001 & 0 & 0 \\ 0.0027 & 0.0006 & 0 \end{pmatrix} \qquad (21b)$$

The fact that increasing V_{31} in the new model to accommodate the large $B - \bar{B}$ mixing in turn leads to a larger V_{13} and hence larger $b \to u/b \to c$ ratio can be understood from the unitarity condition

$$V_{11}V_{13}^{*} + V_{21}V_{23} + V_{31}V_{33} = 0 \tag{22a}$$

or

$$V_{13}^{*} + V_{31} \sim 0.011 \tag{22b}$$

where note must be taken that the CP-violation phase is changed considerably.

In summary, we have carefully demonstrated how the stages of chiral symmetry breaking can be phenomenologically constructed. The main point is that, without a particular dynamical theory in mind, the first stage of chiral symmetry breaking is completely arbitrary within the class of physically equivalent mass matrices and probably only in a special basis does the mass-generating mechanism become transparent. While we do not try to construct any dynamical theory, we show that the selection of a particular basis for the first stage allows one to construct the following stages by direct comparison with known masses and mixings. We have illustrated this point in the basis originally chosen by Fritzsch by showing how our procedure in the minimal Higgs framework leads to a modified set of quark mass matrices which fit the data much better provided $m_t \simeq 88GeV$. A key difference is that a large $b \rightarrow u/b \rightarrow c$ mixing ratio is expected in the new model, while the Fritzsch model gives a satisfactory fit to the data only with an expanded Higgs structure and small $b \rightarrow u/b \rightarrow c$ mixing ratio.

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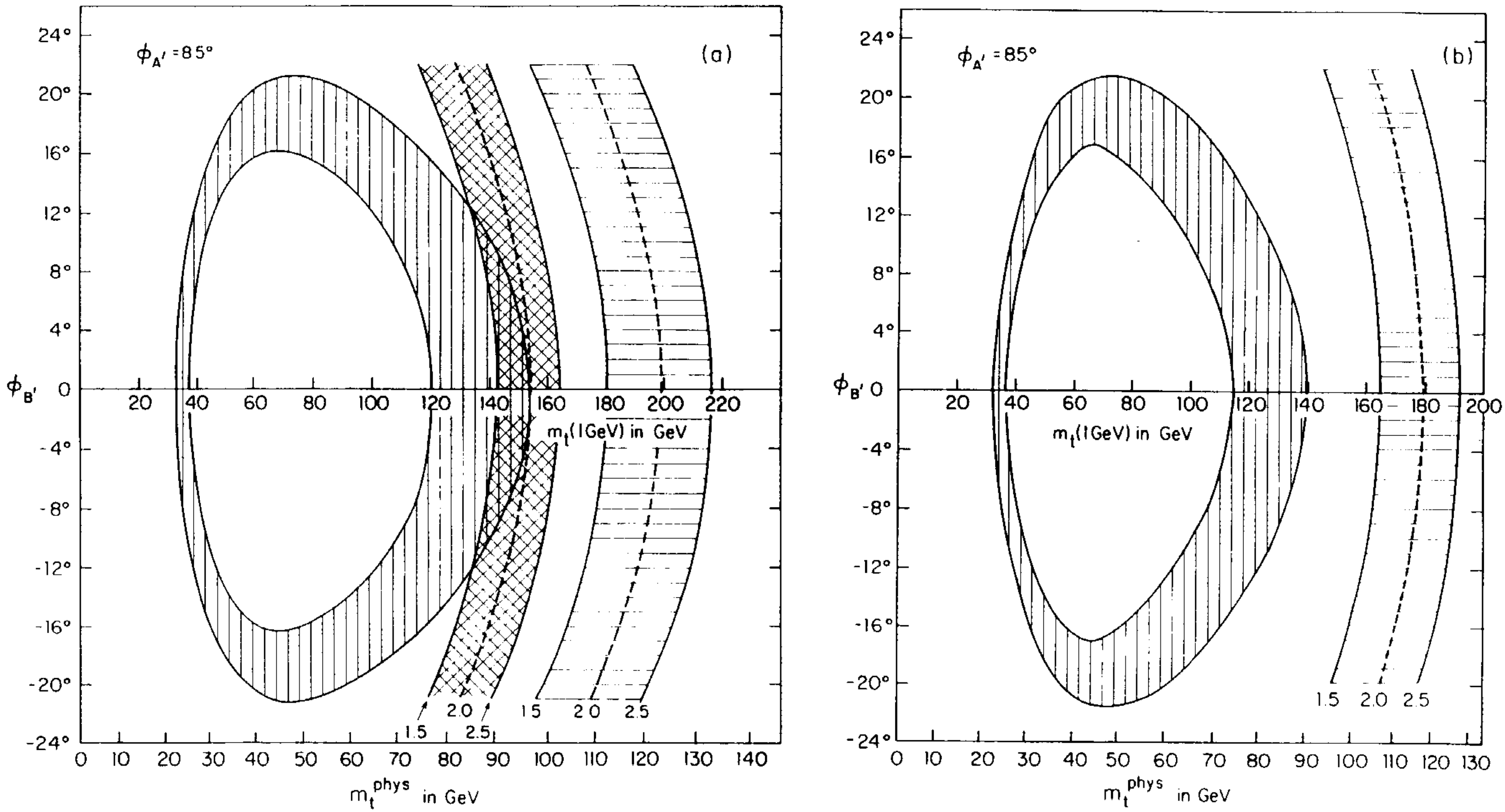
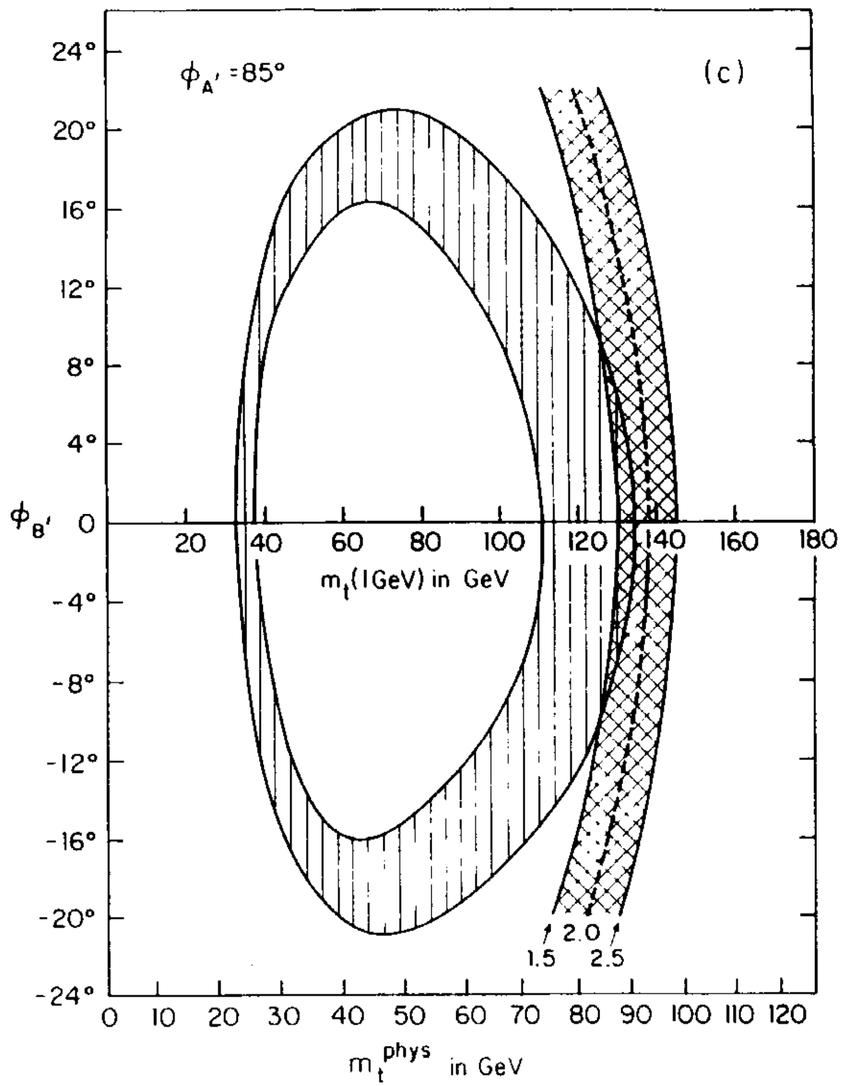
Figure Captions

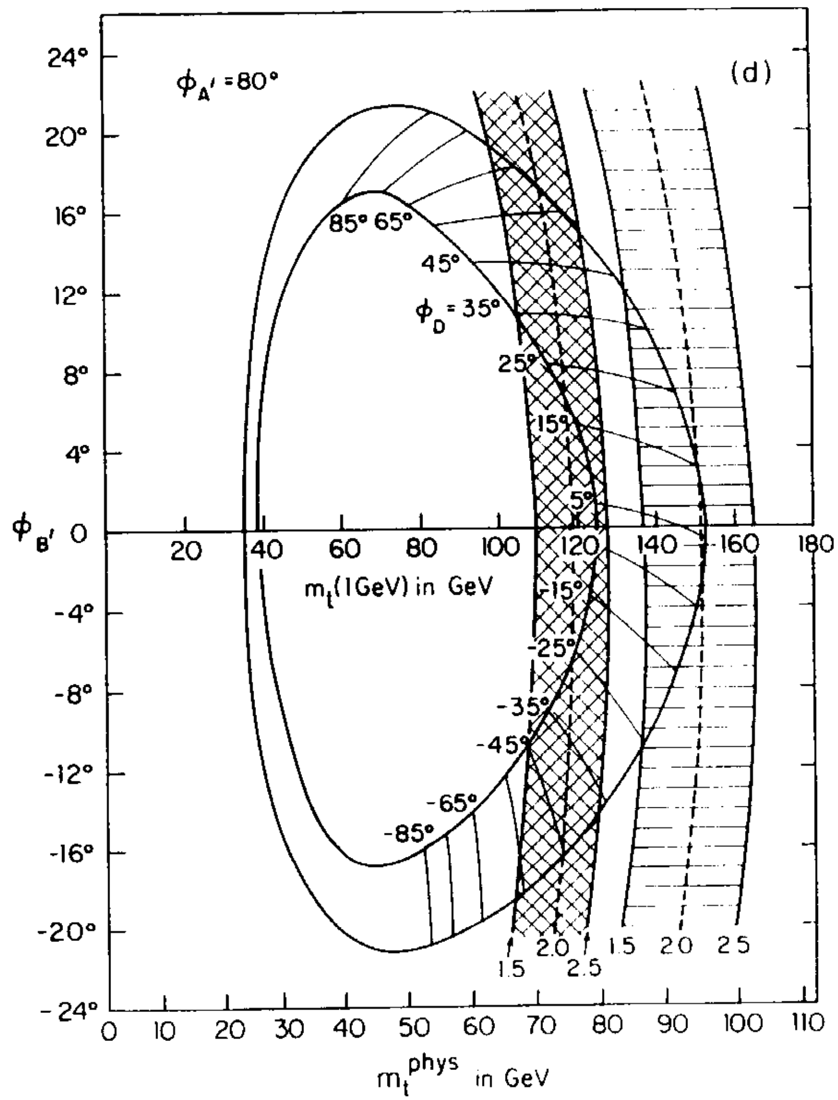
Figure 1: Phase angle $\phi_{B'}$ *vs.* $m_t(1GeV)$ and m_t^{phys} plots showing the physically-allowed annular regions for the KM matrix elements based on the one standard deviation results of Schubert given in Eq. (6a) and the $B_d^o - \bar{B}_d^o$ mixing bands single-hatched for the standard Higgs model and double-hatched for the two-doublet Higgs model. Results for the original Fritzsch model are given in (a) without evolution and in (b) and (c) with evolution; results for the new model of Eqs. (14) and (15) are presented in (d), (e) and (f), respectively. The quark masses are those of (16).



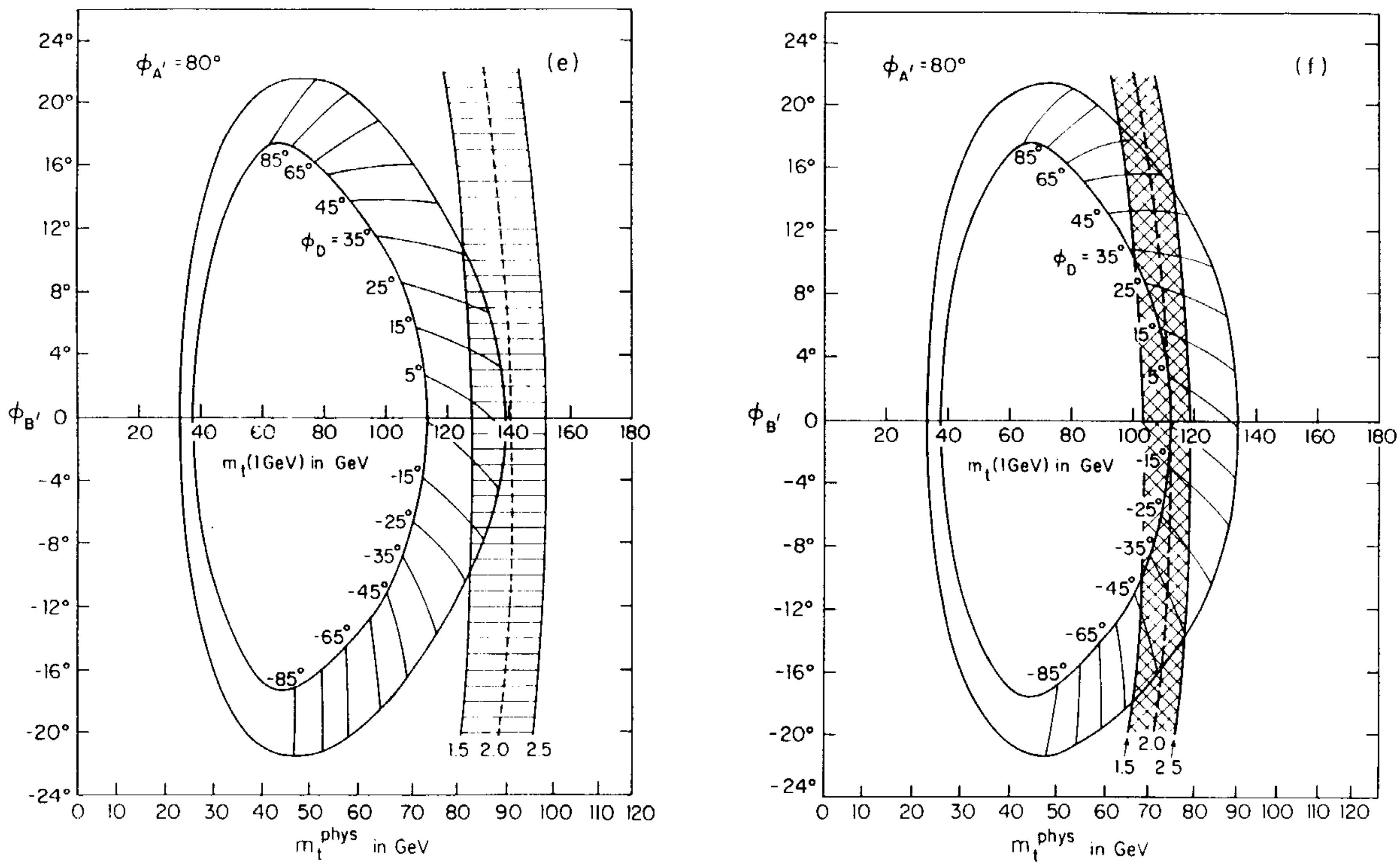
$\phi_{A'} = 85°$
(a)
$\phi_{B'}$
$m_t(1\,GeV)$ in GeV
m_t^{phys} in GeV
1.5
2.0
2.5
$\phi_{A'} = 85°$
(b)
$\phi_{B'}$
$m_t(1\,GeV)$ in GeV
m_t^{phys} in GeV
1.5
2.0
2.5



$\phi_{A'} = 85°$
(c)
$\phi_{B'}$
$m_t(1GeV)$ in GeV
m_t^{phys} in GeV
1.5
2.0
2.5



$\phi_{A'} = 80°$
(d)
$\phi_{B'}$
$m_t(1GeV)$ in GeV
m_t^{phys} in GeV
85° 65°
45°
$\phi_D = 35°$
25°
15°
5°
-15°
-25°
-35°
-45°
-65°
-85°
1.5
2.0
2.5
1.5
2.0
2.5



$\phi_{A'} = 80°$
(e)
$\phi_D = 35°$
85° 65° 45° 25° 15° 5° -15° -25° -35° -45° -65° -85°
1.5 2.0 2.5
$\phi_{B'}$
m_t(1GeV) in GeV
m_t^{phys} in GeV
$\phi_{A'} = 80°$
(f)
$\phi_D = 35°$
85° 65° 45° 25° 15° 5° -15° -25° -35° -45° -65° -85°
1.5 2.0 2.5
$\phi_{B'}$
m_t(1GeV) in GeV
m_t^{phys} in GeV

Fig.	$m_t(1GeV)$ (GeV)	m_t^{phys} (GeV)	$\phi_{A'}$	$\phi_{B'}$	ϕ_D	J $\times 10^4$	δ	$m_t^2\|V_{td}\|^2 R$	$\|V_{ts}\|^2/\|V_{td}\|^2$	r_s	$\|V_{ub}\|/\|V_{cb}\|$	B_K	ϵ'/ϵ $\times 10^4$
	Fritzsch Model with Minimal Higgs Structure												
1a	150	94	85°	0°		0.30	99°	0.85	17.2	0.94	0.053	0.94	15
1b	135	88	85°	0°		0.29	99°	0.75	17.2	0.92	0.053	1.05	17
	Fritzsch Model with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$												
1a	150	94	85°	0°		0.30	99°	1.86	17.2	0.99	0.053	0.94	15
1c	130	88	85°	0°		0.28	99°	1.53	17.2	0.98	0.053	1.12	18
	New Model with Minimal Higgs Structure												
1d	145	91	80°	2°	10°	0.30	157°	1.79	6.9	0.91	0.149	0.72	13
1e	135	88	80°	0°	5°	0.30	158°	1.74	7.0	0.91	0.147	0.74	12
	New Model with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$												
1d	110	71	80°	14°	45°	0.33	153°	2.06	6.8	0.93	0.156	1.01	22
1f	105	70	80°	12°	38°	0.32	153°	1.97	6.5	0.92	0.162	1.08	24
	Experimental Results and Theoretical Predictions												
		?				0.30	?	2.0 ± 0.5	?	?	0.07 - 0.17	0.66 ± 0.10	33 ± 11
Ref.						15		10			17	18	19

Table 1: Values of parameters associated with selected points in the plots of Figs. 1.
A mass of 120 MeV has been used throughout for the strange quark mass.