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SPONTANEOUS SYMMETRY BREAKING IN 4-DIMENSIONAL HETEROTIC STRING*

Jnanadeva Maharana†

Theoretical Physics Department
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

ABSTRACT

The evolution of a 4-dimensional heterotic string is considered in the background of its massless excitations such as graviton, antisymmetric tensor, gauge fields and scalar bosons. The compactified bosonic coordinates are fermionized. The world-sheet supersymmetry requirement enforces Thirring-like four fermion coupling to the background scalar fields. The non-abelian gauge symmetry is exhibited through the Ward identities of the S-matrix elements. The spontaneous symmetry breaking mechanism is exhibited through the broken Ward identities. An effective 4-dimensional action is constructed and the consequence of spontaneous symmetry breaking is envisaged for the effective action.

The string theory[1] offers a promise of unifying all the fundamental forces of nature. One of the marvels of the string theory lies in its rich symmetry structure. It has been argued by Gross[2] that all the string states are gauge particles and most of the string symmetries are spontaneously broken leaving only the familiar local symmetries of the theory. Moreover, the high energy behavior of the scattering amplitudes at Planckian energies reveals many interesting features of the string theory[3]. However, at low energies, energies much smaller than the Planck scale, the string theory is expected to exhibit the salient features of those theories which describe the low energy phenomena adequately. Therefore, all the gauge symmetries, manifest at Planck scale, do not remain unbroken at lower energies. It is now recognized that the Higgs' mechanism plays a cardinal role in the models unifying the fundamental forces. Recently, there have been several attempts to contruct four dimensional string theories following the work of Narain.[4-9] The mechanism of symmetry breaking and the Higgs' phenomena has been envisaged for 4-dimensional heterotic string theory.[10-12]

In this talk, I shall consider the evolution of a 4-dimensional heterotic string in the background of its massless excitations and investigate the phenomena of spontaneous symmetry

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†Permanent address: Institute of Physics, Bhubaneswar - 751005, India.

breaking. The invariance of the action under world–sheet supersymmetry transformations imposes stringent constraints on the coupling of the string to the relevant background fields. We consider a 4–dimensional heterotic string model where X^μ are the four space–time coordinates and ψ^μ are their world sheet super partners. The compactified bosonic coordinates are fermionized[13,14] such that the 22 right moving compactified bosonic coordinates give 44 Majorana–Weyl fermions denoted by $\eta^A, A = 1, \cdots 44$. The left moving sector consists of 12 Majorana–Weyl fermions obtained from six compactified bosonic coordinates and six of their super partners collectively denoted by $\chi^a, a = 1, \cdots 18$. These 18 fermions must transform in the adjoint representation of a semi–simple Lie group G in order to facilitate non–linear realization of world–sheet supersymmetry[14] and the choice of G is restricted to $SU(2)^6, SU(3)\otimes SO(5)$ and $SU(4)\otimes SU(2)$ for the 4–dimensional case. In the fermionic formulation, various solutions are obtained by suitable choice of mutually commuting boundary conditions diagonalized in some general complex basis for the fermions consistent with the requirements of modular invariance(9). Let us choose a simple boundary condition where all fermions satisfy Neveau–Schwarz boundary condition. This theory has a tachyonic ground state and several massless encitations; however, the mechanism of spontaneous symmetry breaking can be demonstrated through this simple example. The ground state and the massless states of this theory are as follows:

a) The ground state is a taelyon in the vector representation of $SO(44)$

$$T_A : \eta^A_{1/2} |0> \qquad M^2 = -1/2 \tag{1}$$

b) There are massless gauge bosons in the adjoint representation of $SO(44)$ and G, respectively

$$A^{AB}_\mu : \chi^\mu_{1/2}\eta^A_{1/2}\eta^B_{1/2}|0> \tag{2}$$

$$W^a_\mu : \chi^a_{1/2}\partial X^\mu_R |0> \tag{3}$$

c) Massless scalar bosons in the adjoint representations of both $SO(44)$ and G.

$$\zeta^a_{AB} : \chi^a_{1/2}\eta^A_{1/2}\eta^B_{1/2}|0> \tag{4}$$

d) The usual massless states such as graviton, dilaton and antisymmetric tensor fields are present in addition to the massless states presented above: (2), (3) and (4). The action for the evolution of the 4–dimensional heterotic string in the background of massless fields is[15]

$$\begin{aligned} S &= \frac{1}{2}\int d^2\sigma \Big[G_{\mu\nu}(X)\partial_+X^\mu\partial_-X^\nu + \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) \\ &+ i\psi^i\partial_-\psi^i + i\psi^i\left(w^{ij}_\mu - S^{ij}_\mu\right)\psi^j\partial_-X^\mu \\ &+ i\eta^A\partial_+\eta^A + i\chi^a\partial_-\chi^a + \eta^A T^m_{AB}\eta^B A^m_\mu(X)\partial_+X^\mu \\ &- \frac{1}{2}\eta^A T^m_{AB}\eta^B F^m_{\mu\nu}(X)\psi^\mu\psi^\nu - if^{abc}\eta^A T^m_{AB}\eta^B\zeta^a_m(X)\chi^b\chi^c \\ &- i\eta^A T^m_{AB}\eta^B D_\mu\zeta^a_m(X)\psi^\mu\chi^a - \frac{i}{2}C^{mnp}\eta^A T^p_{AB}\eta^B\zeta^a_m(X)\zeta^b_n(X)\chi^a\chi^b \Big] \end{aligned} \tag{5}$$

obtained by generalizing the earlier method to construct action for heterotic string in background fields.[16] Our notations are as follows: $G_{\mu\nu}(X)$ and $B_{\mu\nu}(X)$ are the background graviton and antisymmetric fields respectively. $\psi^i = e^i_\mu(X)\psi^\mu$ being the vierbeins and w^{ij}_μ are the torsion–free connections whereas $S_{\mu\nu\gamma} = \frac{1}{2}(\partial_\mu B_{\mu\gamma} + \partial_{\gamma\mu} + \partial_\gamma B_{\mu\nu})$ is the field strength associated with $B_{\mu\nu}$; $F^m_{\mu\nu}$ are the field strength of the gauge background potential A^m_μ. The structure constants of the group G and $S0(44)$ are denoted by f^{abc} and C^{mnp} respectively; and $D_\mu \zeta^a_m(X) = \partial_\mu \zeta^a_m(X) + C^{mnp} A^n_\mu(X) \zeta^a_p(X), \zeta^a_p(X)$ being the scalar background. We have coupled only the gauge fields $A^m_\mu(X)$ to the stringth for the sake of simplicity in order to illustrate the mechanism of spontaneous symmetry breaking; however, the gauge fields $W^a_\mu(X)$ can be coupled to the string in a similar manner[16]. We set $G_{\mu\nu} = \eta_{\mu\nu}$ and $B_{\mu\nu} = 0$, in what follow, to study the mechanism of spontaneous symmetry breaking. The action is invariant under following super–symmetry transformations ($G_{\mu\nu} = \eta_{\mu\nu}$ and $B_{\mu\nu} = 0$).

$$\delta X^\mu = \epsilon \psi^\mu \tag{6}$$

$$\delta \psi^\mu = i\epsilon \partial_+ X^\mu \tag{7}$$

$$\delta \eta^A = i\epsilon \eta^B T^m_{BA} A^m_\mu(X) \psi^\mu + i\epsilon \eta^B T^m_{BA} \zeta^a_m(X) \chi^a \tag{8}$$

$$\delta \chi^a = \epsilon f^{abc} \chi^b \chi^c \tag{9}$$

Notice the Thirring–like coupling of the fermions to the scalar background and the coupling corresponds to similar ones for a constant background field considered earlier.[17] However, we note that the world–sheet supersymmetry forces us to include additional terms in the action (5). Our strategies are as follows:

Define the S–matrix generating functional

$$\sum [A, \zeta] = \int d\,[phase\ space]\, d\,[ghosts]\, exp\,(iS_H) \tag{10}$$

where $d[phase\ space]$ is the Hamiltonian phase space measure involving x^μ, ψ^μ, η^A and χ^a and the corresponding canonical momenta P_μ, π^μ, π^A and π^a respectively. The ghosts appear as a consequence of the (1,0) superconformal symmetry. S_H is the Hamiltonian action given by

$$S_H = \int d^2\sigma \left[\dot{X}^\mu P_\mu + \psi^i \dot{\psi}^i + \eta^A \dot{\eta}^A + \chi^a \dot{\chi}^a - H \right] + \int d^2\sigma L_{ghost} \tag{11}$$

and

$$\begin{aligned} H &= \frac{1}{2} \tilde{P}_\mu \tilde{P}_\nu \eta^{\mu\nu} + \frac{1}{2} X'^\mu X'^\nu \eta_{\mu\nu} + \Pi^\mu \partial_1 \psi^\nu \eta_{\mu\nu} + \Pi^a \partial_1 \chi^a - \Pi^A \partial_1 \eta^A \\ &+ \frac{1}{2} \eta^A T^m_{AB} \eta^B A^m_\mu X'^\mu + \frac{i}{4} \eta^A T^m_{AB} \eta^B F^m_{\mu\nu} \psi^\mu \psi^\nu + \frac{i}{2} f^{abc} \chi^b \chi^c \zeta^a_m \eta_A T^m_{AB} \eta^B \\ &+ \frac{i}{2} \eta^A T^m_{AB} \eta^B D_\mu \zeta^a_m \psi^\mu \chi^a + \frac{i}{4} C^{mnp} \eta^A T^m_{AB} \eta_B \zeta^a_m \zeta^b_n \chi^a \chi^b \\ \tilde{P}_\mu &= P_\mu - \frac{1}{2} \eta T \eta A_\mu \end{aligned} \tag{12}$$

L_{ghost} is the ghost Lagrangian which has the same form as the L_{ghost} for the heterotic string in the absence of background fields and we do not need its explicit form to derive WI and the effects of spontaneous symmetry breaking. Let us introduce the following infinitesimal generator of a canonical transformation of the form[18]

$$\Phi = \frac{1}{2} \int d\sigma \eta^A T^m_{AB} \eta^B \Lambda^m(X) \tag{13}$$

where $\Lambda(X)$ is an arbitrary function. The variations induced by Φ are

$$\delta_\phi \eta^A = iT^m_{AB}\eta^B \Lambda^m(X) \tag{14}$$

$$\delta_\phi P_\mu = \frac{1}{2}\eta^A T^m_{AB}\eta^B \partial_\mu \Lambda^m(X) \tag{15}$$

$$\delta_\phi X^\mu = \delta_\phi \psi^\mu = \delta_\phi \chi^a = \delta_\phi(ghosts) = 0 \tag{16}$$

The Hamiltonian action satisfies the following relation

$$\delta_\phi S_H = -\delta_G S_H \tag{17}$$

where $\delta_G S_H$ means that we perform the gauge variations of the background fields $A^m_\mu(X)$ and $\zeta^a_m(X)$ only

$$\delta_G A^m_\mu(X) = \partial_\mu \Lambda^m(X) + C^{mnp}A^n_\mu(X)\Lambda^p(X) \tag{18}$$

$$\delta_G \zeta^a_m(X) = C^{mnp}\zeta^a_n(X)\Lambda^p(X) \tag{19}$$

Now we argue that the path integral phase space measure remains invariant, at least classically, under the canonical transformations (14)–(16). Therefore, the generating functional $\sum[A,\zeta]$ exhibits the following gauge invariance property due to (15).

$$\sum[A,\zeta] = \sum[A+\delta_G A, \zeta + \delta_G \zeta] \tag{20}$$

Consequently,

$$\int dY \left(\frac{\delta \sum}{\delta A^m_\mu(Y)}\delta_G A^m_u(Y) + \frac{\delta \sum}{\delta \zeta^a_m(Y)}\delta_G \zeta^a_m(Y) \right) = 0 \tag{21}$$

Using eq. (18) and (19) in (21), we arrive at

$$< \int d^2\sigma \left[V^m_\mu(X)\left\{ \partial_\mu \Lambda^m(X) + C^{mnp}A^n_\mu(X)\Lambda^p(X) + V^a_m(X)C^{mnp}\zeta^a_n(X)\Lambda^p(X) \right\}\right] >= 0 \tag{22}$$

where $V^m_\mu(X)$ and $V^m_a(X)$ are the vertex functions given by

$$\begin{aligned} \frac{\partial L}{\partial A^m_\mu(X)} \equiv V^m_\mu(X) &= -\frac{1}{2}\eta^A T^m_{AB}\eta^B \left(\tilde{P} + X'_\mu\right) + \frac{i}{2}\eta^A T^m_{AB}\eta_B \psi^\mu \psi^\nu \partial_\nu \\ &+ \frac{i}{2}C^{mnp}\eta^A T^p_{AB}\eta^B A^n_\nu(X)\psi^\mu\psi^\nu \\ &+ \frac{i}{2}C^{mnp}\eta^A T^p_{AB}\eta_B \zeta^a_n(X)\chi^a \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{\partial L}{\partial \zeta^a_m(X)} \equiv V^m_a(X) &= \frac{i}{2}f^{abc}\chi^b\chi^c\eta^A T^m_{AB}\eta^B \\ &+ \frac{i}{2}\eta^A T^m_{AB}\eta^B \psi^\mu \chi^a \partial_\mu + \frac{i}{2}C^{pnm}\eta^A T^p_{AB}\eta^B A^n_\mu(X)\psi^\mu\chi^a \\ &+ \frac{i}{2}C^{pmn}\eta^A T^p \eta^B \zeta^b_n(X)\chi^a\chi^b \end{aligned} \tag{24}$$

$< ... >$ means expectation value with the measure $\exp(iS_H)d(phase\ space)$. Notice that $\Lambda^m(X)$ appearing in (22) is an arbitrary function, therefore, if we take functional derivative of (22) with respect to $\Lambda^q(y)$ and then set $\Lambda^q(y) = 0$ the relations still hold good.

$$
\begin{aligned}
&< \int d^2\sigma \Big(V_\mu^m(X) \left[\partial_\mu \delta\left(x(\sigma) - y\right) \delta^{mq} + C^{mnq} A_\mu^n \delta\left(X(\sigma) - y\right) \right] \\
&+ V_m^a(X) C^{mnq} \zeta_n^a(X) \delta\left(x(\sigma) - y\right) \Big) >= 0
\end{aligned} \tag{25}
$$

This is the fundamental relation that gives Ward identities for amplitudes involving gauge and scalar bosons. We take appropriate number of functional derivatives of (25) with respect to the gauge and scalar bosons and then set them to their background values.

$$
\begin{aligned}
&\prod_{i=1}^{N} \prod_{j=1}^{M} \frac{\delta}{\delta A_\mu^{m_i}(X_i)} \frac{\delta}{\delta \zeta^{a_j}(X_j)} < \int d^2\sigma \Big(V_\mu^m\left(X(\sigma)\right) \Big[\partial_\mu \left(X(\sigma) - y\right) \delta^{mq} \\
&+ C^{mnq} A_\mu^n\left(x(\sigma)\right) \delta\left(X(\sigma) - y\right) \Big] + V_m^a\left(X(\sigma)\right) C^{mnq} \zeta_n^a\left(X(\sigma)\right) \delta\left(X(\sigma) - y\right) \Big) >= 0
\end{aligned} \tag{26}
$$

Here $X_i = X_i(\sigma_i)$. The functional derivative acting on $< ... >$ brings down extra vertex functions and therefore, the $N+1$ point amplitudes are related to the lower point amplitudes. We denote the background value of gauge and scalar fields by b.y. which are required to be consistent with conformal invariance.

It is worthwhile to emphasize that the WI presented here are to be considered as the tree level result. Indeed, anomalies might creep in when we carefully compute the Jacobian associated with the fermionic measure under the transformations (14)–(16). This question has been examined by us recently and the results are reported elsewhere.[16]

Now we proceed to discuss the phenomena of spontaneous symmetry breaking in the string theory from our point of view. It is interesting to note that if the scalar background takes a constant value $\zeta_m^a(X) = \beta_m^a$ then we precisely reproduce the Thirring–like four fermion interactions considered by ABK[11] in the content of spontaneous symmetry breaking and Higgs' mechanism in the string theory. However, we encounter additional terms in the action (5) for nontrivial scalar background fields. These terms arise due to the invariance of the action under world–sheet supersymmetry transformations. We may envisage the effects of spontaneous symmetry breaking if we closely examine the two point function for the gauge fields in (26) (modulo complications due to möbius invariance). It is easy to see from (26) that the two point function exhibits a mass term for constant scalar backgrounds and we can interpret it as the analog of the Higgs' mechanism. Indeed, a more elaborate computation[19] reveals that the consistency conditions, satisfied by the gauge and scalar background fields, are obtained from the equations of motion of a four dimensional effective action

$$
\begin{aligned}
S_{eff} &= \int d^4n \Big[-\frac{1}{4} Tr \left(F_\mu^m(n) \right)^2 + \frac{i}{2} D^\mu \zeta_m^a(n) D_\mu \zeta_m^a(X) \\
&- \frac{4}{3} f^{abc} C^{mnp} \zeta_m^a(X) \zeta_n^b(X) \zeta_p^c(n) \\
&- \frac{1}{16} C^{mqp} C^{pst} \zeta_m^a(X) \zeta_s^a(X) \zeta_t^b(n) \zeta_q^b(X) \Big]
\end{aligned} \tag{27}
$$

Thus, if the scalar field acquires a non–zero vacuum expectation value, classically the system undergoes spontaneous symmetry breaking and exhibits Higgs' mechanism.

To summarize, we have considered a four dimensional heterotic string theory in the gauge and scalar backgrounds. The generaling functional is constructed in the path integral formalism and Ward identities are derived using the local symmetry properties of the generating functional. The model exhibits spontaneous symmetry breaking phenomena for constant vacuum expectation values of the scalar background fields.

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