



**Fermi National Accelerator Laboratory**

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**A New Method of Determining  $\text{SIN}^2 \theta_w$  in  
Deep-Inelastic  $\nu_\mu\text{-N}$  Scattering \***

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# A NEW METHOD OF DETERMINING $\sin^2 \theta_W$ IN DEEP-INELASTIC $\nu_\mu N$ SCATTERING

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## Abstract

The value of  $\sin^2 \theta_W$  can be determined to  $\pm 0.002 - 0.004$  by using the semileptonic decays of the  $K_L$  to provide a beam of  $\nu_\mu$  and  $\bar{\nu}_\mu$  and measuring the ratio  $R' = \sigma(\bar{\nu}_\mu, NC)/\sigma(\nu_\mu, NC)$ . Systematic errors which have limited the world-average of previous  $\nu_\mu N$  determinations of  $\sin^2 \theta_W$  to  $\pm 0.008$  are largely eliminated. This experiment will determine the radiative corrections  $\Delta r$  in  $\nu_\mu N$  scattering to  $\pm 0.007$  and in combination with  $W, Z$  mass measurements will provide precise tests of the Standard Model at the tree and one-loop level.

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<sup>†</sup> Based on talk presented at XII International Workshop on Weak Interactions and Neutrinos, 9 - 14 April 1989, Ginosar, Israel.

## 1. Introduction

Improved measurements of  $\sin^2 \theta_W$  are of central importance to an improved understanding of the Standard Model. We can write the  $W$  and  $Z$  masses as:

$$M_W = \frac{37.281 \text{ GeV}/c^2}{\sin^2 \theta_W (1 - \Delta r)^{1/2}}$$

and

$$M_Z = \frac{M_W}{\cos \theta_W}$$

where  $\Delta r$  describes the radiative corrections. Within the Standard Model the value of  $\Delta r$  is predicted to be  $0.07 \pm 0.013$  for  $m_t = 45 \text{ GeV}/c^2, m_H = 100 \text{ GeV}/c^2$  and is currently determined to only  $\pm 0.037$ . The  $W, Z$  mass determinations from the colliders will provide one measurement of  $\Delta r$  with errors of  $\pm 0.006$  but a measurement in a single process is not sufficient. Each determination of  $\sin^2 \theta_W$ , from the  $W, Z$  masses,  $\nu e$  scattering, or deep-inelastic  $\nu_\mu N$  scattering, depends on new physics in different ways and so an ensemble of experiments is necessary to fully exploit the data. A comparison of  $\sin^2 \theta_W$  measured in  $\nu_\mu N$  measured with precision similar to the collider determinations will provide tight constraints on physics beyond the Standard Model and possibly point to new phenomena.<sup>[1]</sup>

The experiment will measure the ratio  $R' = \sigma(\bar{\nu}, NC)/\sigma(\nu, NC)$ . As a function of  $y$ ,

$$R'(y) = \frac{\frac{d\sigma_{\bar{\nu}NC}}{dy}}{\frac{d\sigma_{\nu NC}}{dy}} = \frac{[g_R^2 + g_L^2(1-y)^2] + [g_L^2 + g_R^2(1-y)^2] \left(\frac{\bar{Q}}{Q}\right)}{[g_L^2 + g_R^2(1-y)^2] + [g_R^2 + g_L^2(1-y)^2] \left(\frac{Q}{\bar{Q}}\right)} \quad (1)$$

where

$$g_L^2 = \epsilon_L(u)^2 + \epsilon_L(d)^2 = \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W \quad (2)$$

$$g_R^2 = \epsilon_R(u)^2 + \epsilon_R(d)^2 = \frac{5}{9} \sin^4 \theta_W \quad (3)$$

$$\epsilon_L = T_i^3 - \sin^2 \theta_W Q_i \quad (4)$$

$$\epsilon_R = -\sin^2 \theta_W Q_i \quad (5)$$

and we have substituted the values of  $T_i^3$  ( $3^{rd}$  component of weak isospin) and the quark charge  $Q_i$  for the individual species.  $Q$  and  $\bar{Q}$  are the quark and antiquark structure functions evaluated at a specific  $y$ ; for any species  $Q, \bar{Q} = \iint x Q(x, q^2) dx dq^2$ .

A determination of  $\sin^2 \theta_W$  from this neutral-current ratio has significant systematic advantages over previous methods which have used  $R_\nu = \sigma(\nu, NC)/\sigma(\nu, CC)$ . The first Section describes the principles of the tagged-line and the second will describe the measurement. We conclude with a comparison of the expected errors to those of the current data.

## 2. The Tagged-Neutrino Line

The tagged line uses a beam of  $K_L$  and the semileptonic decays  $K_L \rightarrow \pi\mu\nu_\mu$  and  $K_L \rightarrow \pi e\nu_e$  to produce a beam of  $\nu_\mu, \nu_e$  and their antiparticles. A tagging spectrometer reconstructs the charges, momenta and species of the hadron and lepton from the  $K_L$  decay. We then know whether the neutrino is a  $\nu_e$  or  $\nu_\mu$  and can distinguish neutrino from antineutrino. We can also use the momenta as measured in the tagging spectrometer to calculate the momentum vector of the neutrino. This provides an energy determination with  $\sigma/E \approx 7\%$  and a prediction of the impact point in the neutrino detector of about 10 cm. This paper concentrates on the *use* of the tagging scheme in a determination of  $\sin^2 \theta_W$ ; details of a particular neutrino-tagging scheme have been presented in Fermilab Proposal P-788.<sup>[2]</sup> Fig. 1 is a schematic of the tagging spectrometer. We show a  $\pi$  and  $\mu$  passing through the spectrometer. It contains drift chambers for tracking the hadron and lepton, and a large aperture (2.5 m) dipole with a  $p_T$  kick of 0.5 GeV/c for momentum analysis. Particle identification is accomplished in two stages: a TRD separates electrons from  $\pi$ 's and  $\mu$ 's and a Fe filter followed by scintillator is used to identify muons. The systematic errors from particle misidentification, both  $\nu_e$  with  $\nu_\mu$  and  $\nu_\mu$  with  $\bar{\nu}_\mu$ , have been studied in P-788 and determined to be negligible; the contaminations will be small and can be measured from the data.

The neutrinos will be detected in a 3500-ton magnetized iron target instrumented with drift chambers and scintillators; the design will be similar to that of the CDHS detector.<sup>[3]</sup>

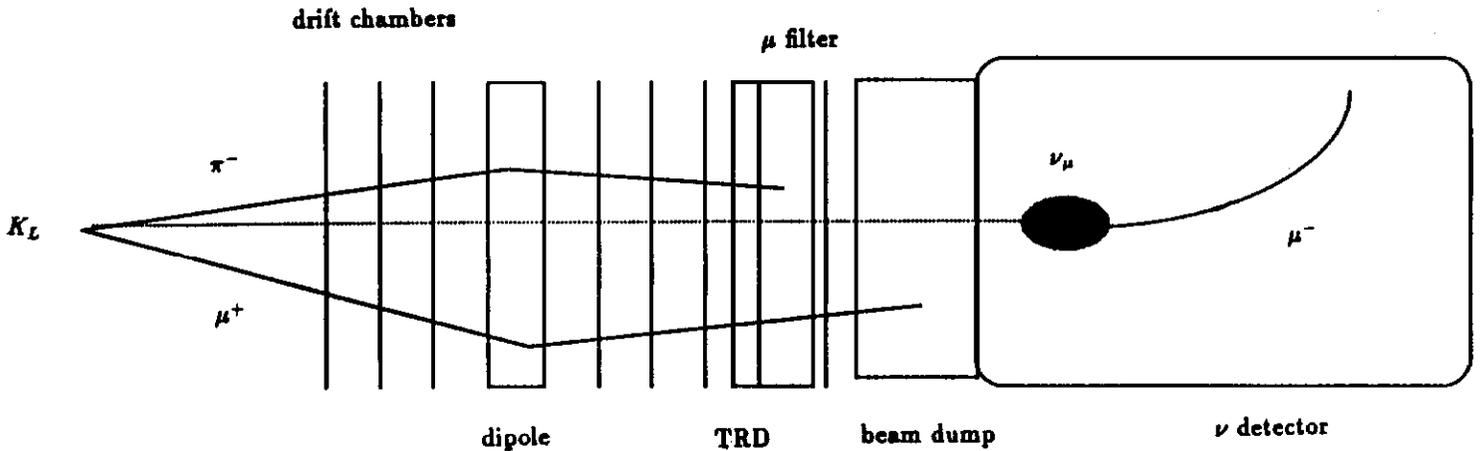


Fig. 1. A schematic of the Tagged-Neutrino Spectrometer and Neutrino Detector (not drawn to scale). A  $K_L \rightarrow \pi^- \mu^+ \nu_\mu$  decay is pictured. The  $\pi$  and  $\mu$  pass through drift chambers and a dipole which measure their momenta and determine the  $K_L$  decay point. They next pass through a TRD (useful for  $K e_3$  decays) and then into a muon filter. The  $\pi$  is absorbed and the  $\mu$  continues downstream, first firing a bank of scintillators and then passing into a beam-dump. The  $\nu_\mu$  (dotted line) interacts in a neutrino detector downstream and a  $\mu^-$  is observed in a charged-current interaction.

There are two errors which have limited previous DIS measurements of  $\sin^2 \theta_W$  through  $R_\nu$  : charged-to-neutral current cross-talk from unreconstructed muons, and the effect of charm-production in the charged-current cross-section, modeled by slow-rescaling.<sup>[4]</sup> Both errors arise because  $R_\nu$  compares the neutral-current and charged-current cross-sections: the first is an “experimental” error from misclassification of events, and the second results from the breakdown of the simple quark-parton model in the presence of the heavy charmed quark.

The tagging scheme greatly reduces both sets of errors. Since  $R'$  is a neutral-current ratio, charged-current muons feed into both numerator and denominator and tend to cancel in the ratio; a  $y$ -cut, using  $E_\nu$  from the tagger, will reduce the error still further. The error in the past on  $\sin^2 \theta_W$  in the Fe experiments has been  $\approx 0.005$  but in this experiment we expect an order-of-magnitude reduction. The second systematic error in  $R_\nu$  arises from charm-production:  $s$  and  $d$  can produce charm through the charged-current but not the neutral; the difference is then a direct correction to  $R_\nu$  and the error in the calculation is the single largest theoretical error in extracting  $\sin^2 \theta_W$ . The size has been debated and estimates range from 0.004 to 0.007.<sup>[1,4]</sup>  $R'$  itself is free of this correction but the extraction of  $\sin^2 \theta_W$  from  $R'$  will reintroduce the error since we must make assumptions about  $Q$  and  $\bar{Q}$ . We see from Eq.(1) that  $R'$  is sensitive to  $\bar{Q}/Q$ , the antiquark-to-quark ratio, since the numerator and denominator of  $R'$  reverse the roles of quark and antiquark.  $\bar{Q}/Q$  will be determined through a measurement of  $\sigma(\bar{\nu}, CC)/\sigma(\nu, CC)$  from the charged-current sample: a fit to the ratio of the  $y$ -distributions of the  $\nu_\mu$  and  $\bar{\nu}_\mu$  cross-sections will measure the quantities  $(\bar{D} + \bar{S})/U$  and  $\bar{U}/(D + S)$ , from which we can then extract  $\bar{Q}/Q = (\bar{U} + \bar{D} + \bar{S})/(U + D + S)$ .<sup>1</sup> The resultant statistical error on  $\sin^2 \theta_W$  from  $\bar{Q}/Q$  will be  $\pm 0.002$ . Slow-rescaling errors will re-enter here, but at a level down by approximately an order-of-magnitude from an  $R_\nu$  determination: the error from the standard variation of  $m_c = 1.5 \pm 0.3 \text{ GeV}/c^2$  is only  $\pm 0.0005$ . It is important to note that the determination of  $\sigma(\bar{\nu}, CC)/\sigma(\nu, CC)$  will be free of flux normalization errors since  $\nu_\mu$  and  $\bar{\nu}_\mu$  are made in equal numbers (up to the small  $CP$ -violation correction), leaving only the  $\nu/\bar{\nu}$  difference in resolution smearing in  $y$  as a bias; however, in charged-current events we have two determinations of  $y$  with which to study resolutions: one from the tagger  $E_\nu$  and the muon energy, and another from the neutrino event itself. Systematic errors from non-isoscalarity are reduced because the charged-current data will be applied to the neutral-current sample within the same target, measuring an effective  $\bar{Q}/Q$  for the target. Finally, we have made the same cuts on  $y$  and  $E_\nu$  in both samples, and hence the charged- and neutral-current samples will have the same  $x$ ,  $y$ , and  $q^2$  distributions. A preliminary study places the errors (excluding slow-rescaling) at  $\pm 0.0005$ .

Although most of the attention on the theoretical errors has been devoted to charm-production, a variety of other effects exist in an  $R_\nu$  analysis and contribute an amount almost equal to the slow-rescaling errors: in the analysis of Amaldi *et al.* <sup>[1]</sup> the error

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<sup>1</sup>The cross-sections will be fit to  $A + B(1 - y)^2 + C(1 - y)$ , where  $C$  represents the  $R = \sigma_L/\sigma_T$  and higher-twist contributions; all three terms will then be applied to the neutral-current analysis.

almost equal to the slow-rescaling errors: in the analysis of Amaldi *et al.* [1] the error from slow-rescaling is  $\pm 0.0041$  and the remaining errors add up to  $\pm 0.0035$ . A substantial fraction of these errors come from the determination of  $(\bar{U} + \bar{D})/(U + D)$  and  $\bar{S}/D$ ; the determination of  $\bar{Q}/Q$  within the experiment makes these errors much smaller and leads to an error of  $\pm 0.0005$  as discussed above. Errors from  $W\gamma$  box diagrams which contribute an error of  $\pm 0.003$  to  $R_\nu$  do not appear in the neutral current  $R'$ .

The experiment will provide a number of other measurements. The prediction of the neutrino momentum and direction will allow us to determine the neutrino cross-sections for  $\nu_\mu$  and  $\nu_e$  to  $\approx 1\%$ , limited by systematic errors in the prediction. The clean identification of neutrino species will open new regions for neutrino oscillation searches (discussed in P-788). We expect a factor of  $10^3$  improvement in the  $\nu_e \rightarrow \nu_\tau$  oscillation limits at small  $\sin^2 2\theta_{e\tau}$ , probing to  $\Delta m^2 \approx 100 eV^2$  for  $\sin^2 2\theta_{e\tau} \approx 10^{-3}$ . We also expect an order-of-magnitude improvement in the limits on wrong-sign muon production to  $5 \times 10^{-5}$ .

### 3. Apparatus and Statistical Errors

For our estimates we have used a 900 GeV primary proton beam with  $3.0 \times 10^{18}$  *pot* at the Tevatron. A fixed-target energy of 1.2 TeV instead of 900 GeV would provide an additional doubling of statistics, equally divided between an improved acceptance and the increased neutrino cross-section. It would also make charged-current muons more energetic and less likely to be missed by a detector. Hence the planned fixed-target luminosity upgrades from the Fermilab Main Injector<sup>[5]</sup> and proposed energy upgrades would only improve the measurement. Table 1 shows the event sample ( $E_\nu > 30$  GeV/c) in each category after analysis cuts of  $\approx 25\%$  (primarily on agreement between the predicted and measured  $\nu$  impact point).

Table 1. Numbers of Expected Events

	$\nu_\mu$	$\bar{\nu}_\mu$
CC	218K	92K
NC	70K	31K

Previous detectors, such as CCFR (690 tons of Fe) or CDHS (1250 tons) would only provide a  $\approx 3\%$  measurement. In order to further reduce the statistical errors, we are proposing a detector with three times the mass of CDHS, or 3500 tons.

### 4. Conclusions

The Amaldi *et al.* world average for  $\sin^2 \theta_W$  from  $R_\nu$  with  $\rho^2 = 1$  fixed is (experimental error followed by theoretical):

$$\sin^2 \theta_W = 0.233 \pm 0.0033 \pm 0.0054.$$

Given that a precision measurement of  $\sin^2 \theta_W$  is largely motivated by searching for deviations from the Standard Model, it is perhaps more appropriate to quote errors based on allowing  $\rho^2$  to float. In that case, Amaldi *et al.* find a best fit of

$$\sin^2 \theta_W = 0.232 \pm 0.014 \pm 0.008.$$

Since  $R'$  is a purely neutral-current ratio it is independent of  $\rho^2$ . The planned experiment at 900 GeV would provide a statistical error of 1.6% and would find (statistical errors on  $R'$  and  $\bar{Q}/Q$  added in quadrature; it is followed by the systematic error for  $m_c = 1.5 \pm 0.3$ , and our estimated upper limits on the systematic error on  $\sin^2 \theta_W$  from  $\bar{Q}/Q$  and the muon subtraction):

$$\delta(\sin^2 \theta_W) = \pm 0.0040 \pm 0.0005 \pm 0.0005 \pm 0.0005.$$

With fixed-target upgrades to higher energy we may hope for an improved statistical error and lowered systematic error; an eventual error of  $\pm 0.002 - 0.003$  should be achievable, since increased energy and increased statistical power will enable us to make cuts to reduce the size of the muon subtraction and residual slow-rescaling corrections.

These precise results for  $\sin^2 \theta_W$  will then be applied to constrain the value of  $\rho^2$ , from either this experiment (and the Paschos-Wolfenstein<sup>[6]</sup>  $R^-$ ) or the sum of previous DIS experiments. For the two-parameter fit, the  $R'$  errors provide a factor of five improvement in the  $\sin^2 \theta_W$  error. In either case, the method provides a unique, precision DIS measurement of  $\sin^2 \theta_W$  largely free of slow-rescaling and other QCD corrections. Such a measurement is a critical part of a continuing program of detailed exploration of the Standard Model and is a logical continuation of neutrino physics at Fermilab into the 1990's.

## References

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