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Numerical Rates for Nucleon-Nucleon, Axion Bremsstrahlung

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Abstract. We numerically evaluate the axion emission rate from nucleon-nucleon, axion bremsstrahlung for arbitrary nucleon degeneracy(s). Our numerical rates agree with analytical results previously derived in the degenerate and non-degenerate limits. While the conditions in the newly-born, hot neutron star associated with SN 1987A are semi-degenerate, the non-degenerate, analytical rate is found to be a very good approximation (accurate to better than a factor of 2), with the degenerate, analytical rate *overestimating* axion emission by a factor of $\sim 20 - 100$.

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Introduction

To date Peccei-Quinn symmetry provides the most attractive solution to the strong CP problem (for discussion of the strong CP problem and Peccei-Quinn symmetry see Refs. 1). The axion is the pseudo-Nambu-Goldstone boson associated with the spontaneous breakdown of Peccei-Quinn symmetry. Its mass and couplings are related to the Peccei-Quinn symmetry breaking scale f_a:

$$m_a \simeq 0.62 \text{ eV}[10^7 \text{ GeV}/(f_a/N)],$$

where N is the color anomaly of the Peccei-Quinn symmetry. [Here we have followed the normalization conventions of Refs. 2; for a complete discussion of the axion and its couplings see Refs. 2, 3.]

Astrophysics and cosmology have placed very stringent limits to the allowed mass of this hypothetical, light pseudoscalar boson (for a review see Refs. 4). Requiring that the cosmological population of coherently-produced axions does not contribute too much mass density today leads to the bound[5]:

$$m_a \gtrsim 3.6 \times 10^{-6} \text{ eV}\gamma^{-0.85}(\Lambda_{\text{QCD}}/200 \text{ MeV})^{-0.6}$$

where Λ_{QCD} is the QCD scale parameter, and $\gamma \gtrsim 1$ accounts for any entropy produced in the Universe after axion production: $\gamma =$ (entropy per comoving volume after/entropy per comoving volume before).

Light axions (if they exist) should be emitted from stars of all varieties (main sequence, red giants, white dwarfs, neutron stars), and should thereby affect stellar evolution. The most stringent stellar emission bound is the recently derived bound based upon axion emission from the newly-born, hot neutron star associated with SN 1987A.[6,7,8] For the conditions that pertain in the core of the hot neutron star just after its formation:[9] $T \sim 30-80$ MeV, $\rho \simeq (6-10)\times 10^{14}$ g cm^{-3}, the dominant emission process is nucleon-nucleon, axion bremsstrahlung (NNAB): $N + N \rightarrow N + N + a$ ($N =$ neutron or proton).

The matrix element for this process, as well as the emission rate in the degenerate limit, have been calculated by Iwamoto.[10] Using the matrix element computed by Iwamoto, the author of Ref. 6 has calculated the emission rate in the non-degenerate limit. Those two rates for the process $n + n \rightarrow n + n + a$ ($n =$ neutron) are:

$$\dot{\epsilon}_a(\text{D}) = 5.3 \times 10^{44} \text{ erg cm}^{-3} \text{ s}^{-1} f^4 g_{an}^2 (X_n\rho_{14})^{1/3} T_{\text{MeV}}^6 \tag{1a}$$

$$\dot{\epsilon}_a(\text{ND}) = 1.1 \times 10^{47} \text{ erg cm}^{-3} \text{ s}^{-1} f^4 g_{an}^2 (X_n\rho_{14})^2 T_{\text{MeV}}^{3.5} \tag{1b}$$

where $f \sim 1$ is the pion-nucleon coupling, $g_{an} \sim m/(f_a/N)$ is the axion-neutron coupling, $m \simeq 0.94$ GeV is the nucleon mass, X_n is the mass fraction of neutrons, $\rho_{14} = \rho/10^{14}$ g cm^{-3}, and $T_{\rm MeV} = T/1$ MeV. [For a detailed discussion of the axion-nucleon coupling g_{an} for various axion models, see Refs. 2,3,6,7. For the moment we will focus on the process $n+n \to n+n+a$; later we will extend our discussions to all the NNAB processes.] The degenerate (D) and non-degenerate (ND) axion emission rates for $n+n \to n+n+a$ (or $p+p \to p+p+a$) are shown in Fig. 1 for $X_n\rho_{14} \simeq 4$, as a function of temperature.

The neutron Fermi momentum $p_F = 0.237$ GeV$(X_n\rho_{14})^{1/3}$, so that $p_F^2/2mT \simeq 30(X_n\rho_{14})^{2/3}/T_{\rm MeV} \simeq 75/T_{\rm MeV}$ (for $X_n\rho_{14} \simeq 4$). That is, one would expect the ND rate to be valid for $T >> 75$ MeV and the D rate to be valid for $T << 75$ MeV; the temperatures that pertain just after collapse are $\sim 30-80$ MeV, corresponding to neither strongly ND or D conditions. The two rates $\dot\varepsilon_a$(D) and $\dot\varepsilon_a$(ND) are equal for $T \simeq 20$ MeV: in the D limit ($T >> 75$ MeV) the ND rate overestimates axion emission - as one would expect since blocking factors are ignored, and in the ND limit ($T << 75$ MeV) the D rate overestimates axion emission - as one would also expect since the D rate is more temperature dependent (see Fig. 1).

Which rate is appropriate for SN 1987A? Since the two analytic rates cross each other for $T \simeq 20$ MeV (where $p_F^2/2mT \simeq 3.5$) one might naively expect that the ND rate is the better approximation (as we will show, that is in fact the case). The authors of Refs. 7, 8 use the D rate to compute axion emission from SN 1987A, while the author of Ref. 6 uses the ND rate: for $T \simeq 75$ MeV they differ by a factor of $\dot\varepsilon_a(\mathrm{D})/\dot\varepsilon_a(\mathrm{ND}) \simeq 20$. Since any limit to the axion mass is $\propto \dot\varepsilon_a^{-1/2}$, this corresponds to a discrepancy of a factor of ~ 5—a significant difference. For the same form of the axion-nucleon coupling, the authors of Ref. 7 derive the bound, $m_a \lesssim 0.9 \times 10^{-4}$, and the author of Ref. 6, $m_a \lesssim 0.75 \times 10^{-3}$ - a factor of ~ 8 difference, much of which apparently traces to the different axion emission rate used. Since the axion mass bound based upon SN 1987A is the most stringent upper bound to the axion mass, we feel it is important to resolve the discrepancy due solely to the axion emission rates.[12] This is the motivation for the present work.

In this brief paper we numerically integrate the axion emission rate from the processes $N+N \to N+N+a$ ($N =$ neutron or proton) for arbitrary degeneracy(s). Our numerical results are shown in Figs. 1-3 and are compiled in Tables I, II. The numerical results smoothly connect the ND and D limits, and indicate that for the conditions that pertain in the newly-born, hot neutron star associated with SN 1987A the ND axion emission rate is the better approximation (accurate to better than a factor of 2), with the D rate *overestimating* axion emission by a factor of $\sim 20-100$. This is our main result.

Axion Emission, One Chemical Potential

To begin we will focus on the process $n_1+n_2 \to n_3+n_4+a$ (or equivalently, $p+p \to p+p+a$, or $n+p \to n+p+a$ with $X_n = X_p$) as there is but one chemical potential; in the next section we extend our discussion to $n+p \to n+p+a$ and two unequal chemical potentials. The axion emission rate is given by a 15-dimensional phase space integral:

$$\dot{\varepsilon}_a = \int d\Pi_1\, d\Pi_2\, d\Pi_3\, d\Pi_4\, d\Pi_a\, (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_1+p_2-p_3-p_4-p_a)$$
$$\times \; E_a f_1 f_2 (1-f_3)(1-f_4) \tag{2}$$

where $d\Pi_i = d^3p_i/(2\pi)^3 2E_i$, the labels $i = 1-4$ denote the incoming (1, 2) and outgoing (3, 4) nucleons (at present, neutrons), the label a denotes the axion, the matrix element squared $|\mathcal{M}|^2$ is summed over initial and final spins and includes the usual symmetry factor of $1/4 = 1/2 \times 1/2$ for identical particles in the initial and final states, and the neutron phase space distribution functions $f_i = [\exp(E_i/T - \mu/T)+1]^{-1}$. [For the axion masses of interest, $m_a \sim 10^{-3}$ eV, axions simply 'free stream' out, and there is no need to take into account reabsorption[6], or include the $1+f_a$ factor for stimulated emission.] In keeping with the assumptions of previous authors (and for simplicity), we will assume the nucleons are NR, i.e., $T/m << 1$, and take the matrix element squared to be constant:[13]

$$|\mathcal{M}|^2 = \frac{1}{4} \times 256 f^2 g_{an}^2 m^2/m_\pi^4, \tag{3}$$

where $m_\pi \simeq 135$ MeV is the pion mass and the factor of 1/4 is the symmetry factor for identical particles in the initial and final states. [The matrix element squared is discussed in more detail in Ref. 10 and our *Appendix*.] In the NR limit, $E_i \cong m + p_i^2/2m$, and we define the NR chemical potential $\hat{\mu} \equiv \mu - m$. Further, we define the dimensionless quantities

$$y \equiv \hat{\mu}/T,$$
$$u_i \equiv p_i^2/2mT.$$

With these definitions the phase space occupancy factors f_i are: $f_i = 1/(e^{u_i - y}+1)$. The number density (per cm^3) of neutrons (or protons) is then

$$n_n = 2\int_0^\infty \frac{d^3p}{(2\pi)^3} f = (\sqrt{2}/\pi^2)(mT)^{3/2} \int_0^\infty \frac{u^{1/2}du}{e^{u-y}+1}$$
$$= 4.1 \times 10^{-6}\ \mathrm{GeV}^3 T_{\mathrm{MeV}}^{3/2} g(y)$$

where $g(y) \equiv \int_0^\infty u^{1/2}du/(e^{u-y}+1)$. Throughout we use units where $\hbar = c = k_B = 1$, so that: 1 GeV$^3 = 1.3 \times 10^{41}$ cm^{-3}, and GeV$^5 = 3.2 \times 10^{62}$ erg cm^{-3} s^{-1}. We also note that $X_n \rho_{14} \cong 9.0 \times 10^{-3} g(y) T_{\mathrm{MeV}}^{3/2}$. The function $g(y)$ has the following familiar limiting forms:

$$g(y) = \int_0^\infty \frac{u^{1/2}du}{e^{u-y}+1} \cong \begin{cases} (\pi^{1/2}/2 \simeq 0.886)e^y & y << -1 \\ \frac{2}{3}y^{3/2} & y >> 1 \end{cases}$$

For intermediate values $g(y)$ is well approximated by its Taylor expansion (to better than 1% for $-1 \lesssim y \lesssim 5$):

$$g(y) = 0.678 + 0.536y + 0.1685y^2 + 0.0175y^3 - 3.24 \times 10^{-3}y^4$$

The neutron Fermi momentum is: $p_F \equiv (3\pi^2 n_n)^{1/3}$; in the limit of $y >> 1$, $p_F^2/2mT \cong y$; while in the limit of $y << -1$, $(p_F^2/2mT) \cong 1.2\ \exp(2y/3)$.

It is convenient to define the center-of-mass (CM) and relative momenta: $\vec{p}_+ \equiv (\vec{p}_1 + \vec{p}_2)/2$ and $\vec{p}_- \equiv (\vec{p}_1 - \vec{p}_2)/2$, and the momenta of n_3 and n_4 in the CM frame: $\vec{p}_{3c} = \vec{p}_3 - \vec{p}_+$ and $\vec{p}_{4c} = \vec{p}_4 - \vec{p}_+$. In the NR limit ($T/m << 1$), the outgoing neutrons (n_3 and n_4) carry essentially all of the momentum and the axion momentum can be neglected. Momentum conservation then implies: $\vec{p}_{4c} = -\vec{p}_{3c}$, while energy conservation: $E_a = p_-^2/m - p_{3c}^2/m$. With these definitions and the aid of the delta function 10 of the 15 integrals can be immediately performed, yielding:

$$\dot{\varepsilon}_a = \frac{|\mathcal{M}|^2 T^{6.5} m^{0.5}}{2^{3.5}\pi^7} \int_0^\infty du_+ \int_0^\infty du_- \int_{-1}^{1} d\gamma_1 \int_0^{u_-} du_{3c} \int_{-1}^{1} d\gamma_c$$

$$(u_+ u_- u_{3c})^{1/2} (u_- - u_{3c})^2 f_1(u_1, y) f_2(u_2, y)(1 - f_3(u_3, y))(1 - f_4(u_4, y)) \tag{4}$$

where $u_{1,2} = u_+ + u_- \pm 2(u_+ u_-)^{1/2}\gamma_1$, $u_{3,4} = u_{3c} + u_+ \pm 2(u_{3c}u_+)^{1/2}\gamma_c$, $\gamma_1 = \vec{p}_+ \cdot \vec{p}_- / |\vec{p}_+||\vec{p}_-|$, $\gamma_c = \vec{p}_+ \cdot \vec{p}_{3c}/|\vec{p}_+||\vec{p}_{3c}|$, and the constant matrix element squared has been taken out of the integral. From this expression for $\dot{\varepsilon}_a$ it is immediately clear that the axion emission rate is proportional to $T^{6.5}$ times a function of $y \equiv \hat{\mu}/T$ only. The limiting D and ND rates discussed in the *Introduction*, cf., Eqs. (1a, b), are of this form - which is reassuring!

At this point it is straightforward to compute the axion emission rate in the ND limit ($y << -1$) by neglecting the $1 - f_3$, $1 - f_4$ 'blocking' factors and setting $f_i = e^{y - u_i}$. The integrand becomes independent of γ_c and γ_1, and the γ_c, γ_1 integrals can be done trivially. The other integrations can also be done, giving:

$$\dot{\varepsilon}_a(\mathrm{ND}) = \frac{1}{4 \cdot 35 \cdot \pi^{6.5}} |\mathcal{M}|^2 m^{0.5} T^{6.5} e^{2y}, \tag{5a}$$

$$= 2.68 \times 10^{-4} e^{2y} m^{2.5} T^{6.5} m_\pi^{-4} g_{an}^2 f^4, \tag{5b}$$

$$= 1.1 \times 10^{47}\ \mathrm{erg\ cm^{-3}\ s^{-1}} f^4 g_{an}^2 (X_n \rho_{14})^2 T_{\mathrm{MeV}}^{3.5} \tag{5c}$$

where Eq. (5b) follows from (5a) by substituting $|\mathcal{M}|^2 = 64 g_{an}^2 f^4 m^2/m_\pi^4$, and Eq. (5c) from (5b) by substituting $e^y \simeq 125(X_n\rho_{14})T_{\mathrm{MeV}}^{-3/2}$ (valid for $y << -1$). This agrees with the result previously derived in Ref. 6.[11,14]

The D limit ($y >> 1$) has been derived by Iwamoto;[10] he obtains:

$$\dot{\varepsilon}_a(\mathrm{D}) = \frac{31 \cdot \sqrt{2}}{3780\pi} m^{2.5} T^{6.5} m_\pi^{-4} g_{an}^2 f^4 y^{1/2}, \tag{6a}$$

$$= 3.69 \times 10^{-3} y^{1/2} m^{2.5} T^{6.5} m_\pi^{-4} g_{an}^2 f^4, \tag{6b}$$

$$= 5.3 \times 10^{44} \text{ erg cm}^{-3} \text{ s}^{-1} f^4 g_{an}^2 (X_n \rho_{14})^{1/3} T_{\text{MeV}}^6 \tag{6c}$$

By first performing the γ_1 and γ_c integrations (see below), and then expanding the rapidly varying parts of the integrand in a series of step functions, delta functions, and their derivatives, with some effort we have verified Iwamoto's result for the degenerate limit. In addition, we have determined that the next term in the expansion is of order $O(y^{-1})y^{1/2}$. The ND and D limit axion emission rates are shown in both Fig. 1 (as a function of T) and in Fig. 2 (as a function of $y \equiv \hat{\mu}/T$).

Returning to the general case (arbitrary y), both the γ_1 and γ_c integrations can be performed, and with the further substitutions: $v \equiv u_{3c}/u_-$ and $q_\pm = e^{-u_\pm}$, $\dot{\varepsilon}_a$ can be expressed as a 3-dimensional integral:

$$\dot{\varepsilon}_a = \frac{|\mathcal{M}|^2 m^{0.5} T^{6.5}}{2^{5.5}\pi^7} \int_0^1 dq_+ \int_0^1 dq_- \int_0^1 dv\ u_+^{-1/2} u_-^3 (1-v)^2 q_+ q_- [e^{-2y} - q_+^2 q_-^2]^{-1}$$

$$\times [1 - \exp(2y - 2vu_- - 2u_+)]^{-1} \ln\left[\frac{\cosh^2[(u_+^{1/2} + u_-^{1/2})^2/2 - y/2]}{\cosh^2[(u_+^{1/2} - u_-^{1/2})^2/2 - y/2]}\right]$$

$$\times \ln\left[\frac{\cosh^2[((vu_-)^{1/2} + u_+^{1/2})^2/2 - y/2]}{\cosh^2[((vu_-)^{1/2} - u_+^{1/2})^2/2 - y/2]}\right] \tag{7a}$$

$$\equiv |\mathcal{M}|^2 m^{0.5} T^{6.5} I(y). \tag{7b}$$

This 3-dimensional integral must be evaluated numerically. We have used two different numerical techniques to evaluate this integral: Monte Carlo integration and direct integration. For the Monte Carlo technique, the integrand was evaluated at 10^6 randomly-chosen points in the domain of integration: q_1, q_2, $v \in [0,1]$, and the integral was taken to be the average value of the integrand times the volume of the domain of integration ($= 1$). To estimate the error, we grouped the 10^6 points into 10 subsamples of 10^5 points each, and computed the individual means of the integrand, and then took the variance of the 10 means. The estimated errors for the Monte Carlo method were typically $\lesssim 10\%$. Because of the severe effects of degeneracy for $y \gtrsim 5$, the integrand becomes strongly peaked, and the Monte Carlo technique becomes unreliable. And for this reason we also used a direct gaussian technique to numerically integrate Eq. (7). By a judicious series of transformations the integrand can be made very smooth, making direct numerical integration both accurate and fast. The estimated accuracy for all our direct integrations is better than 1%. Our numerical results are shown in Figs. 1 and 2 and compiled in Table I. The following expression is a closed form fit to $I(y)$ which for all values of y is accurate to better than 10%,

$$I_{\rm fit}(y) = \Big[1.79\times10^5 e^{-y} + 2.39\times10^5 e^{-2y} + 1.73\times10^4(1+|y|)^{-1/2}$$
$$+6.92\times10^4(1+|y|)^{-3/2} + 1.73\times10^4(1+|y|)^{-5/2}\Big]^{-1}.$$

Our chosen range of y spans $y = -10 \to 50$. From Fig. 2 and Table I it is clear that the numerical results smoothly join on to the asymptotic limits (D, ND). The approach to the ND limit is much more rapid than the approach to the D limit, which is easy to understand. In the ND limit the expansion parameter is e^y, while in the D limit the expansion is in powers of y^{-1}. From Fig. 1 it is also clear that the ND, analytic rate provides a very good approximation to the actual rate for $p_F^2/2mT \lesssim 3$, or $T \gtrsim 30$MeV for $X_n\rho_{14} = 4$.

Two Chemical Potentials

To this point we have assumed that the chemical potentials for all 4 nucleons are equal. For the processes $n_1 + n_2 \to n_3 + n_4 + a$ and $p_1 + p_2 \to p_3 + p_4 + a$ this is of course true. However, if one wishes to consider the process $n_1 + p_2 \to n_3 + p_4 + a$ this assumption is only valid if $X_n = X_p$. It is straightforward to relax the assumption of equal chemical potentials by defining separate neutron ($y_1 \equiv \hat{\mu}_n/T$) and proton ($y_2 \equiv \hat{\mu}_p/T$) chemical potentials. In this case the analogue of Eq.(7) is

$$\dot{\varepsilon}_a = \frac{|\mathcal{M}|^2 m^{0.5}T^{6.5}}{2^{5.5}\pi^7}\int_0^1 dq_+ \int_0^1 dq_- \int_0^1 dv\, u_+^{-1/2}u_-^3(1-v)^2 q_+q_-[e^{-y_1}e^{-y_2} - q_+^2q_-^2]^{-1}\times$$

$$[1-\exp(y_1+y_2-2vu_- - 2u_+)]^{-1}\ln\left[\frac{[1+\exp[(u_-^{1/2}+u_+^{1/2})^2 - y_1]][1+\exp[y_2-(u_-^{1/2}+u_+^{1/2})^2]]}{[1+\exp[(u_-^{1/2}-u_+^{1/2})^2 - y_1]][1+\exp[y_2-(u_-^{1/2}-u_+^{1/2})^2]]}\right]$$

$$\times\ln\left[\frac{[1+\exp[((vu_-)^{1/2}+u_+^{1/2})^2 - y_1]][1+\exp[y_2-((vu_-)^{1/2}+u_+^{1/2})^2]]}{[1+\exp[((vu_-)^{1/2}-u_+^{1/2})^2 - y_1]][1+\exp[y_2-((vu_-)^{1/2}-u_+^{1/2})^2]]}\right] \tag{8a}$$

$$\equiv |\mathcal{M}|^2 m^{0.5}T^{6.5} I(y_1, y_2). \tag{8b}$$

In the limit that $y \equiv y_1 = y_2$ this expression reduces to Eq.(7), and $I(y,y) = I(y)$. Also note that $\dot{\varepsilon}_a$ is again proportional to $T^{6.5}$ times a function of y_1 and y_2 alone. As before, the ND limit (y_1, $y_2 \ll -1$) is straightforward to obtain,

$$\dot{\varepsilon}_a(ND) = \frac{1}{4\cdot 35\cdot \pi^{6.5}}|\mathcal{M}|^2 m^{0.5}T^{6.5}e^{y_1+y_2} \tag{9a}$$

$$= 4.4\times10^{47}\mathrm{erg\ cm^{-3}\ s^{-1}} f^4 g_{aN}^2 X_n X_p \rho_{14}^2 T_{MeV}^{3.5} \tag{9b}$$

where Eq. (9b) follows by substituting $|\mathcal{M}|^2 = 256m^2g_{aN}^2f^4/m_\pi^4$ (no symmetry factor), and g_{aN} is the effective axion nucleon coupling for $n+p \to n+p+a$ (see *Appendix*). With some effort, by expanding the integrand as before one obtains the following expression in the D limit (y_1, $y_2 \gg 1$)

$$\dot{\varepsilon}_a(D) = \frac{31\cdot\sqrt{2}}{945\pi}m^{2.5}T^{6.5}m_\pi^{-4}g_{aN}^2f^4\bar{y}^{1/2}(1-\Delta y/2\bar{y}) \tag{10a}$$

$$= 2.1\times 10^{45}\mathrm{erg\ cm^{-3}s^{-1}}g_{aN}^2f^4\rho_{14}^{1/3}T_{\mathrm{MeV}}^6(\frac{X_n^{2/3}+X_p^{2/3}}{2})^{1/2}(1-\Delta y/2\bar{y}) \tag{10b}$$

where $\bar{y}=(y_1+y_2)/2$ and $\Delta y = |y_1-y_2|/2$.

Finally, in the limit that $y_1 \ll -1$ and $y_2 \gg 1$ (one degenerate and one non-degenerate species) with a similar amount of effort we find that

$$\dot{\varepsilon}_a(D,ND) = \frac{1}{2^{1.5}\pi^{6.5}}m^{2.5}T^{6.5}m_\pi^{-4}g_{aN}^2f^4e^{y_1}$$

$$\times\int_0^\infty dw/w^2\int_0^\infty z^2dz\ln(1+w)\ln(1+we^{-z}) \tag{11a}$$

$$= 5.43\times10^{-3}m^{2.5}T^{6.5}m_\pi^{-4}g_{aN}^2f^4e^{y_1} \tag{11b}$$

$$= 1.8\times10^{46}\mathrm{ergcm^{-3}s^{-1}}g_{aN}^2f^4X_1\rho_{14}T_{MeV}^5 \tag{11c}$$

We have numerically evaluated $I(y_1,y_2)$ for y_1, $y_2 \in [-10,10]$ using the same direct integration technique as before. Our results, all accurate to better than 1% , are compiled in Table II. These results agree with the analytic expressions derived above in the appropriate limits. The grid of 231 values in Table II can be used to interpolate for all intermediate values. For a point $\bar{y}$, Δy in between grid points $A = (\bar{y}_i,\ \Delta y_i)$, $B = (\bar{y}_i,\ \Delta y_{ii})$, $C = (\bar{y}_{ii},\ \Delta y_i)$, $D = (\bar{y}_{ii},\ \Delta y_{ii})$, the double, logarithmic linear interpolation

$$\ln I(\bar{y},\Delta y) \cong \left(\frac{\bar{y}-\bar{y}_i}{\bar{y}_{ii}-\bar{y}_i}\right)\left[\left(\frac{\Delta y-\Delta y_i}{\Delta y_{ii}-\Delta y_i}\right)\ln I(\bar{y}_i,\Delta y_i)+\left(\frac{\Delta y_{ii}-\Delta y}{\Delta y_{ii}-\Delta y_i}\right)\ln I(\bar{y}_i,\Delta y_{ii})\right]$$

$$+\left(\frac{\bar{y}_{ii}-\bar{y}}{\bar{y}_{ii}-\bar{y}_i}\right)\left[\left(\frac{\Delta y-\Delta y_i}{\Delta y_{ii}-\Delta y_i}\right)\ln I(\bar{y}_{ii},\Delta y_i)+\left(\frac{\Delta y_{ii}-\Delta y}{\Delta y_{ii}-\Delta y_i}\right)\ln I(\bar{y}_{ii},\Delta y_{ii})\right]$$

provides a value for $I(\bar{y},\Delta y)$ which is accurate to better than 5%.

In addition, the following is a closed form fit to $I(y_1,y_2)$ which is accurate to better than 25% for all values of y_1, y_2 (and typically, better than 10%),

$$I_{\mathrm{fit}}(y_1,y_2) = \Big[2.39\times10^5(e^{-y_1-y_2}+0.25e^{-y_1}+0.25e^{-y_2})+1.73\times10^4(1+|\bar{y}|)^{-1/2}$$

$$+6.92\times10^4(1+|\bar{y}|)^{-3/2}+1.73\times10^4(1+|\bar{y}|)^{-5/2}\Big]^{-1}.$$

To illustrate our results we have computed $\dot{\varepsilon}_a$ for $\rho_{14} = 8$ as a function of temperature for two sets of abundances: (i) $X_n = 0.9$, $X_p = 0.1$; and (ii) $X_n = 0.7$, $X_p = 0.3$. The case $X_n = X_p = 0.5$ has already been done, since in this instance $y_1 = y_2$ (see Fig. 1). The results for these two cases are shown in Fig. 3, along with the analytic rates for the degenerate and non-degenerate limits (for simplicity, the $(1 - \Delta y/2\bar{y})$ factor has not been included for the degenerate limit). Again, it is clear that for the conditions that pertain in the post-collapse core the non-degenerate rate is a very good approximation, overestimating $\dot{\varepsilon}_a$ by at most a factor of 2, while the degenerate rate can overestimate $\dot{\varepsilon}_a$ by as much as a factor of 100.

Discussion and Concluding Remarks

We have numerically calculated the axion emission rate from nucleon-nucleon, axion bremsstrahlung for arbitrary neutron and proton degeneracy using both Monte Carlo and direct numerical integration techniques. Our numerical results agree with the analytical results previously obtained in the D ($y >> 1$) and ND ($y << -1$) limits,[6,10] and with the analytic expressions we have derived in the various limiting regimes with two chemical potentials.

Somewhat surprisingly, the transition from the D regime to the ND regime occurs for $p_F^2/2mT \sim 3.5$ (rather than ~ 1). For $p_F^2/2mT \lesssim 3$, the non-degenerate rate provides a reasonable approximation to the actual axion emission rate. In the non-degenerate regime convergence to the ND rate is rapid. The degenerate regime is rather more complicated. The leading order term in the degenerate regime expansion varies as the square root of the average chemical potential, and if the two chemical potentials are quite different (as in the case for $X_n = 0.9$ and $X_p = 0.1$) this can be quite a poor approximation. The convergence to the D limit quite slow because the expansion is in powers of y^{-1}, and for two chemical potentials because of the additional exacerbating effect of unequal chemical potentials. The slow approach to the degenerate limit is clearly illustrated in Figs. 1-3.

Our motivation for this work was to accurately calculate axion emission from the newly-born, hot neutron star associated with SN 1987A, where the conditions are that of intermediate degeneracy. From Figs. 1, 3 it is clear that in the pertainent regime ($\rho_{14} \simeq 8$, $T \sim 70$ MeV), the ND rate is a good approximation (overestimating the true emission rate by at most a factor of 2), and that the D rate is a poor approximation (overestimating the true emission rate by a factor of $\sim 20 - 100$). Since any axion mass limits which are derived scale as the axion emission rate to the $-1/2$ power, mass limits derived using the D axion emission rate should be scaled *upward* by a factor of $\sim 5 - 10$. Applying such a factor to the limit derived in Ref. 7 (where the D rate was used), brings this limit into better accord with the limit derived in Ref. 6 (where the ND rate was used). Given the overall uncertainties in deriving these axion mass limits (see Refs. 6-8 for discussion of the uncertainties), there now seems to be reasonable agreement that the axion mass limit based upon SN 1987A is: $m_a \lesssim 10^{-3}$ eV.

We gratefully acknowledge the suggestion of E. W. Kolb which stimulated this work, and useful conversations with E. W. Kolb, G. Steigman, and J. N. Fry. This work was supported in part by the DoE (at Chicago and Fermilab), the NASA (at Fermilab), The German National Scholarship Foundation through a scholarship for RPB, and The Alfred P. Sloan Foundation through a fellowship for MST.

Appendix

Here we briefly discuss the matrix element squared $|\mathcal{M}|^2$; for a more complete discussion we refer the reader to Ref. 10. In the 1-pion exchange approximation there are 4 direct and 4 exchange diagrams, corresponding to the axion being emitted by any one of the 4 nucleons (see Fig. 4). In the limit that $3mT \gg m_\pi^2$, $|\mathcal{M}|^2$ is constant; Iwamoto[10] has computed the first correction to this approximation in the degenerate limit. In the relevant temperature-density regime, this correction is of order $0.5 - 1$.

Isospin invariance relates the 4 pion-nucleon-nucleon vertex couplings needed to compute $|\mathcal{M}|^2$: $\pi p\bar{n}$: $\pi n\bar{p}$: $\pi n\bar{n}$: $\pi p\bar{p}$ :: 1:1:$\sqrt{2}$/2:$\sqrt{2}$/2. Following Iwamoto[10] and denoting the coupling $\pi n\bar{n}$ by f, and the axion nucleon couplings by g_{an} and g_{ap} (for the neutron and proton respectively), it follows that $|\mathcal{M}|^2 = 64m^2m_\pi^{-4}f^4g_{an}^2$ $(n+n \to n+n+a)$; $64m^2m_\pi^{-4}f^4g_{ap}^2$ $(p+p \to p+p+a)$; and $5\cdot 128m^2m_\pi^{-4}f^4[(g_{an}^2+g_{ap}^2)/2]$ $(n+p \to n+p+a)$. The matrix element squared for the pn process is larger for two reasons: (i) because there are no identical particles in the initial or final states there is no factor of $\frac{1}{4}$; (ii) the charged pion coupling to nucleons is $\sqrt{2}$ times that of the neutral pion, which enhances the exchange diagram by a factor of 2. [Put in the language of isotopic spin, the nn or pp initial state is pure $I=1$, while the pn initial state is equal parts $I=1$ and $I=0$, and so there are two isospin channels for the pn process and $\mathcal{M} = \mathcal{M}(I=0)/\sqrt{2} + \mathcal{M}(I=1)/\sqrt{2}$. Because there are two isospin channels for the pn process one might expect that irrespective of the 1-pion exchange approximation used here that the pn process would have a larger value for $\mathcal{M}^2$. Supporting this notion is the fact that the cross section for pn elastic scattering is larger than that for nn or pp elastic scattering.] We also mention that the effective axion-nucleon coupling for the pn process defined just below Eq. (9) is just: $g_{aN}^2 = 2.5(g_{an}^2+g_{ap}^2)/2$.

Bringing this all together it follows that in the 1-pion exchange approximation the total axion emission rate from all NNAB processes is

$$\dot{\varepsilon}_a = 64(m^{2.5}T^{6.5}/m_\pi^4)f^4\left[g_{an}^2 I(y_1,y_1) + g_{ap}^2 I(y_2,y_2) + 10(\frac{g_{an}^2+g_{ap}^2}{2})I(y_1,y_2)\right] \qquad (A1)$$

where the first term accounts for $n+n \to n+n+a$, the second for $p+p \to p+p+a$, and the third for $n+p \to n+p+a$. $I(y_1,y_2)$ is as defined in Eq. (8); also note, that as defined, $I(y,y) = I(y)$, where $I(y)$ is defined in Eq. (7). For reference, $64m^{2.5}T^{6.5}m_\pi^{-4}f^4 = 1.66\times 10^{48}\,\mathrm{erg\,cm^{-3}s^{-1}}\,f^4\,T_{\mathrm{MeV}}^{6.5}$.

Given that $f \sim 1$, the validity of the 1-pion exchange approximation is open to question: the inclusion of 2-pion, 3-pion, ... exchange, other meson exchange diagrams, collective nuclear effects, etc. We will not address these issues here.

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11. Mayle, et al. (see Ref. 7) have also computed $\dot{\varepsilon}_a$(ND). However their rate has the wrong temperature dependence: $\dot{\varepsilon}_a$(ND Mayle et al.) $\simeq 3.5 \times 10^{48}$ erg cm^{-3} s$^{-1} f^4 g_{an}^2 (X_n \rho_{14})^2 T_{\rm MeV}^3$, corresponding to $\dot{\varepsilon}_a \propto T^6 F(y)$, cf. Eq. (4) and below. We have taken the liberty of correcting the ND emission rate computed in Ref. 6 for a factor of 2 algebra error.
12. The limits derived in Ref. 8 cannot be easily compared to those of Refs. 6, 7, as the authors of Ref. 8 do not explicitly specify the relationship between the axion-nucleon couplings and the axion mass. However, if infers that relationship, their bound is comparable to that of Ref. 6.
13. The assumption that $|\mathcal{M}|^2$ is constant is strictly only valid in the limit: $3mT >> m_\pi^2$, or $T >> 6.5$ MeV, which is satisfied in the regime of interest ($T \sim 70$ MeV). For further details see Ref. 10.
14. In obtaining Eq. (5c), the expression for axion emission in the ND limit in terms of ρ, X_n, and T, we have used the relationship $e^y \simeq 125 X_n \rho_{14} T_{MeV}^{-3/2}$, which is valid only in the ND limit ($y \ll -1$). One might have been tempted to use the exact expression for y, obtained by solving: $g(y) = 111 X_n \rho_{14} T_{MeV}^{-3/2}$, where $g(y) = \int_0^\infty u^{1/2} du/(e^{u-y}+1)$. In addition to the obvious fact that $\dot{\varepsilon}_a(ND)$ then could not be written in closed form (except for $y \ll -1$), the resulting expression when extrapolated to $y \gtrsim 0$ gives a larger value for $\dot{\varepsilon}_a(ND)$ which overestimates the true emission rate by a larger factor than Eq. (5c) and does not even decrease monotonically with decreasing temperature (increasing y). The simple limiting form chosen in Eq. (5c) extrapolates to the semi-degenerate regime much better, and of course has the same form in the very non-degenerate regime.

Table I - Axion emission rate $\dot{\varepsilon}_a$ (for $n+n \rightarrow n+n+a$): analytical results and numerical results. All numerical results are accurate to better than 1%. Rates are given in units of $T^{6.5}m^{2.5}g_{an}^2 f^4/m_\pi^4$. The integral $I(y)$, defined in Eq. (7), is equal to the numerical results given here divided by 64.

$y \equiv \hat{\mu}/T$	Numerical	Non-degenerate	Degenerate
-10.0	5.53×10^{-13}	5.52×10^{-13}	
-4.0	8.85×10^{-8}	8.99×10^{-8}	
-3.5	2.38×10^{-7}	2.44×10^{-7}	
-3.0	6.36×10^{-7}	6.64×10^{-7}	
-2.5	1.68×10^{-6}	1.81×10^{-6}	
-2.0	4.36×10^{-6}	4.91×10^{-6}	
-1.5	1.10×10^{-5}	1.33×10^{-5}	
-1.0	2.68×10^{-5}	3.63×10^{-5}	
-0.5	6.17×10^{-5}	9.86×10^{-5}	
0	1.32×10^{-4}	2.68×10^{-4}	0
0.5	2.61×10^{-4}	7.28×10^{-4}	2.61×10^{-3}
1.0	4.72×10^{-4}	1.98×10^{-3}	3.69×10^{-3}
1.5	7.79×10^{-4}	5.38×10^{-3}	4.52×10^{-3}
2.0	1.18×10^{-3}	1.46×10^{-2}	5.22×10^{-3}
2.5	1.67×10^{-3}	3.98×10^{-2}	5.83×10^{-3}
3.0	2.22×10^{-3}	1.08×10^{-1}	6.39×10^{-3}
3.5	2.82×10^{-3}		6.90×10^{-3}
4.0	3.43×10^{-3}		7.38×10^{-3}
5.0	4.64×10^{-3}		8.25×10^{-3}
6.0	5.77×10^{-3}		9.04×10^{-3}
8.0	7.75×10^{-3}		1.04×10^{-2}
10.0	9.37×10^{-3}		1.17×10^{-2}
50.0	2.52×10^{-2}		2.61×10^{-2}

Table II - Axion emission rate $\dot{\varepsilon}_a$ (from $n+p \to n+p+a$): numerical results. All rates are accurate to better than 1% and are given in units of $4T^{6.5}m^{2.5}g_{aN}^2 f^4/m_\pi^4$. The quantities $\bar{y}$ and Δy are related to y_1 and y_2 by $\bar{y} = (y_1+y_2)/2$, $\Delta y = |y_1 - y_2|/2$. The integral $I(y_1, y_2)$, defined in Eq. (8), is equal to the numerical results given here divided by 64. Our notation is to be interpreted as follows: '5.53_{-13}' means 5.53×10^{-13}.

$\bar{y}$ \ Δy	0	1	2	3	4	5	6	7	8	9	10
-10	5.53_{-13}	5.53_{-13}	5.53_{-13}	5.53_{-13}	5.53_{-13}	5.52_{-13}	5.49_{-13}	5.41_{-13}	5.21_{-13}	4.75_{-13}	3.88_{-13}
-9	4.09_{-12}	4.09_{-12}	4.09_{-12}	4.08_{-12}	4.08_{-12}	4.05_{-12}	4.00_{-12}	3.85_{-12}	3.51_{-12}	2.87_{-12}	1.98_{-12}
-8	3.02_{-11}	3.02_{-11}	3.02_{-11}	3.01_{-11}	3.00_{-11}	2.95_{-11}	2.84_{-11}	2.59_{-11}	2.12_{-11}	1.46_{-11}	8.41_{-12}
-7	2.23_{-10}	2.23_{-10}	2.23_{-10}	2.21_{-10}	2.18_{-10}	2.10_{-10}	1.92_{-10}	1.57_{-10}	1.08_{-10}	6.21_{-11}	3.06_{-11}
-6	1.65_{-9}	1.64_{-9}	1.64_{-9}	1.61_{-9}	1.55_{-9}	1.42_{-9}	1.16_{-9}	7.99_{-10}	4.59_{-10}	2.26_{-10}	1.00_{-10}
-5	1.21_{-8}	1.21_{-8}	1.19_{-8}	1.15_{-8}	1.05_{-8}	8.54_{-9}	5.90_{-9}	3.39_{-9}	1.67_{-9}	7.39_{-10}	3.05_{-10}
-4	8.86_{-8}	8.78_{-8}	8.47_{-8}	7.73_{-8}	6.31_{-8}	4.36_{-8}	2.51_{-8}	1.23_{-8}	5.46_{-9}	2.25_{-9}	8.91_{-10}
-3	6.36_{-7}	6.21_{-7}	5.69_{-7}	4.66_{-7}	3.22_{-7}	1.85_{-7}	9.12_{-8}	4.03_{-8}	1.66_{-8}	6.58_{-9}	2.54_{-9}
-2	4.36_{-7}	4.13_{-7}	3.42_{-7}	2.37_{-7}	1.37_{-7}	6.74_{-7}	2.98_{-7}	1.23_{-7}	4.86_{-8}	1.88_{-8}	7.12_{-9}
-1	2.68_{-5}	2.40_{-5}	1.72_{-5}	1.00_{-5}	4.69_{-7}	2.20_{-7}	9.08_{-7}	3.59_{-7}	1.39_{-7}	5.26_{-8}	1.98_{-8}
0	1.32_{-4}	1.12_{-4}	7.05_{-5}	3.60_{-5}	1.61_{-5}	6.69_{-7}	2.65_{-7}	1.02_{-7}	3.89_{-7}	1.46_{-7}	5.47_{-8}
1	4.72_{-4}	3.92_{-4}	2.35_{-4}	1.13_{-4}	4.85_{-5}	1.95_{-5}	7.54_{-7}	2.87_{-7}	1.08_{-7}	4.04_{-7}	1.50_{-7}
2	1.18_{-3}	1.01_{-3}	6.35_{-4}	3.17_{-4}	1.37_{-4}	5.47_{-5}	2.11_{-5}	7.96_{-7}	2.98_{-7}	1.11_{-7}	4.13_{-7}
3	2.22_{-3}	1.97_{-3}	1.38_{-3}	7.68_{-4}	3.58_{-4}	1.48_{-4}	5.77_{-5}	2.19_{-5}	8.19_{-7}	3.05_{-7}	1.13_{-7}
4	3.43_{-3}	3.14_{-3}	2.41_{-3}	1.55_{-3}	8.29_{-4}	3.77_{-4}	1.54_{-4}	5.94_{-5}	2.24_{-5}	8.34_{-7}	3.09_{-7}
5	4.64_{-3}	4.35_{-3}	3.60_{-3}	2.61_{-3}	1.63_{-3}	8.60_{-4}	3.88_{-4}	1.57_{-4}	6.04_{-5}	2.27_{-5}	8.44_{-7}
6	5.77_{-3}	5.50_{-3}	4.79_{-3}	3.80_{-3}	2.70_{-3}	1.67_{-3}	8.77_{-4}	3.94_{-4}	1.60_{-4}	6.12_{-5}	2.29_{-5}
7	6.81_{-3}	6.56_{-3}	5.90_{-3}	4.97_{-3}	3.88_{-3}	2.75_{-3}	1.70_{-3}	8.89_{-4}	3.99_{-4}	1.61_{-4}	6.18_{-5}
8	7.75_{-3}	7.52_{-3}	6.92_{-3}	6.06_{-3}	5.05_{-3}	3.93_{-3}	2.78_{-3}	1.72_{-3}	8.98_{-4}	4.02_{-4}	1.63_{-4}
9	8.60_{-3}	8.39_{-3}	7.84_{-3}	7.06_{-3}	6.14_{-3}	5.10_{-3}	3.96_{-3}	2.80_{-3}	1.73_{-3}	9.04_{-4}	4.05_{-4}
10	9.37_{-3}	9.18_{-3}	8.68_{-3}	7.97_{-3}	7.13_{-3}	6.18_{-3}	5.13_{-3}	3.99_{-3}	2.82_{-3}	1.74_{-3}	9.09_{-4}

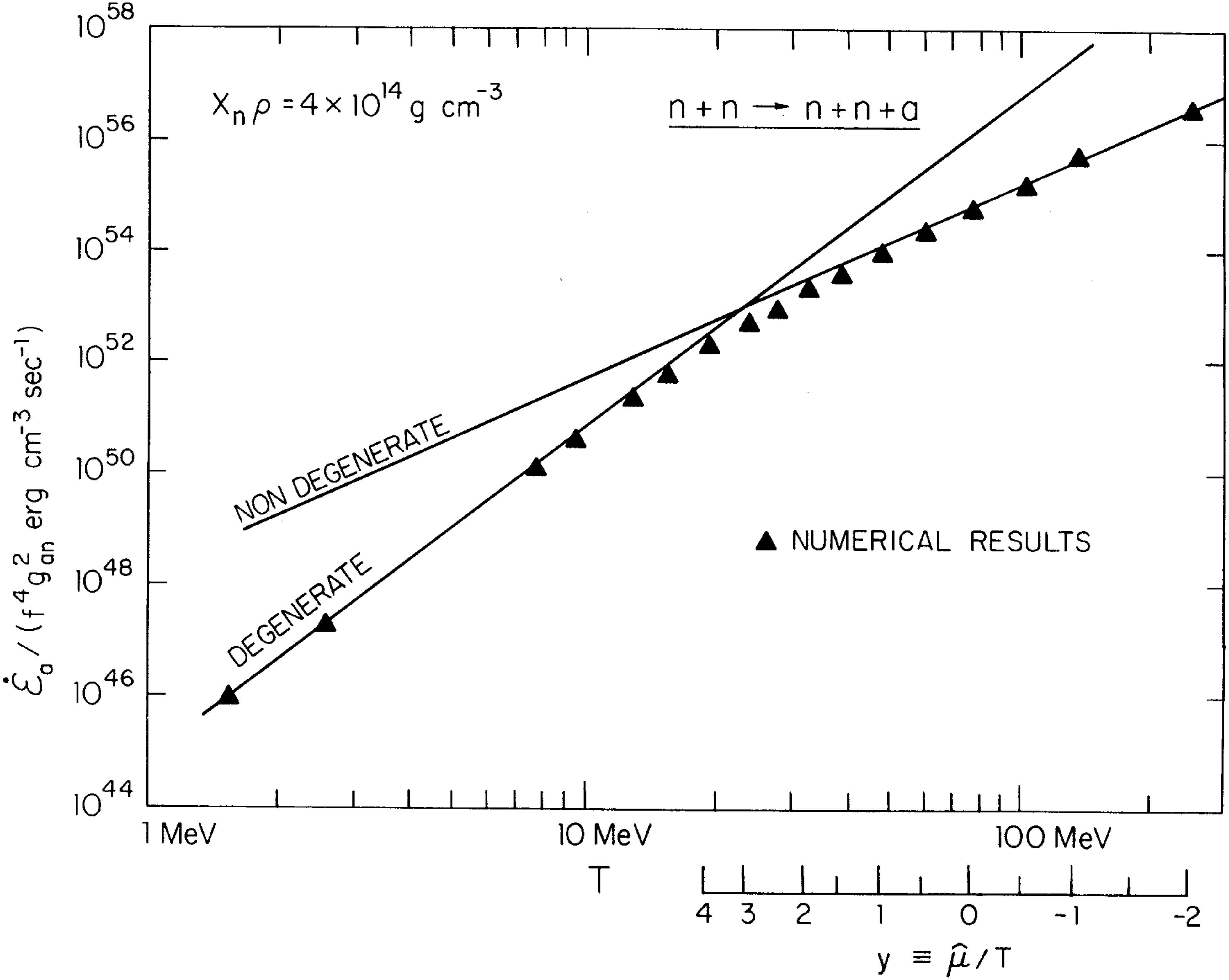
Figure Captions

Figure 1 - The axion emission rate from neutron-neutron, axion bremsstrahlung, for $X_n \rho_{14} = 4$. Shown are the analytical expressions valid in the D and ND limits, cf. Eqs. (1a, b), and our numerical results which are accurate to better than 1% (indicated by the triangles). The temperature which pertains in the core shortly after collpase is ~ 70 MeV. Also shown is $y \equiv \hat{\mu}/T$ as a function of T.

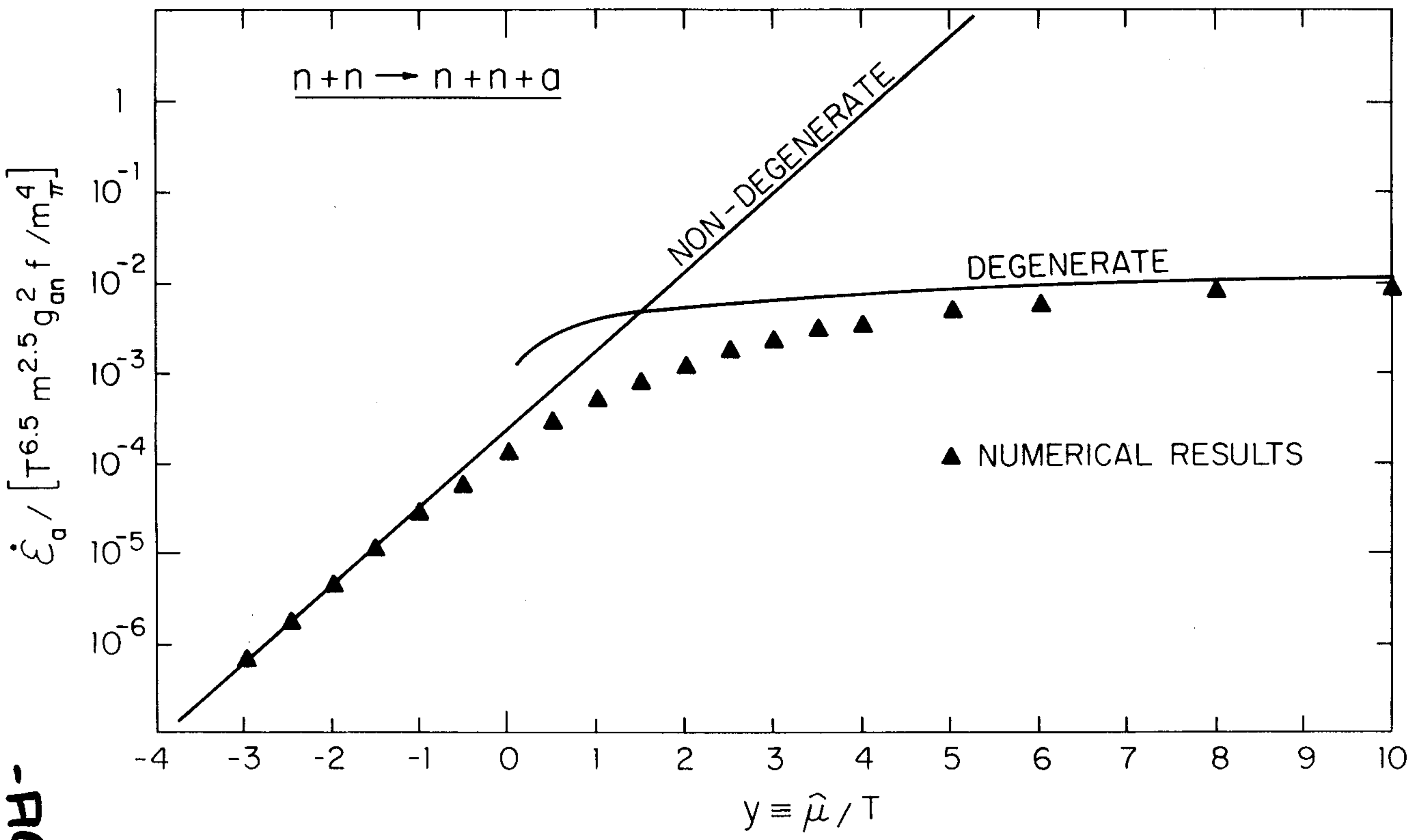
Figure 2 - The axion emission rate from neutron-neutron, axion bremsstrahlung as a function of $y \equiv \hat{\mu}/T$. Shown are the analytical expressions valid in the D and ND limits, cf. Eqs. (5b, 6b), and the results of our numerical integrations which are accurate to better than 1% (indicated by the triangles).

Figure 3 - The axion emission rates from neutron-proton, axion bremsstrahlung, for $\rho = 8 \times 10^{14}\mathrm{gcm}^{-3}$, and the compositions: $X_n = 0.9, X_p = 0.1$ and $X_n = 0.7, X_p = 0.3$. [Note, Fig. 1 applies to the intermediate case $X_n = X_p = 0.5$.] Shown are the analytic expressions valid in the degenerate and nondegenerate limits, as well as our numerical results which are accurate to 1% (indicated by the triangles). [For simplicity, the $(1 - \Delta y/2\bar{y})$ factor has not been included in the degenerate rate.]

Figure 4 - The direct and exchange diagrams for nucleon, nucleon axion bremsstrahlung.

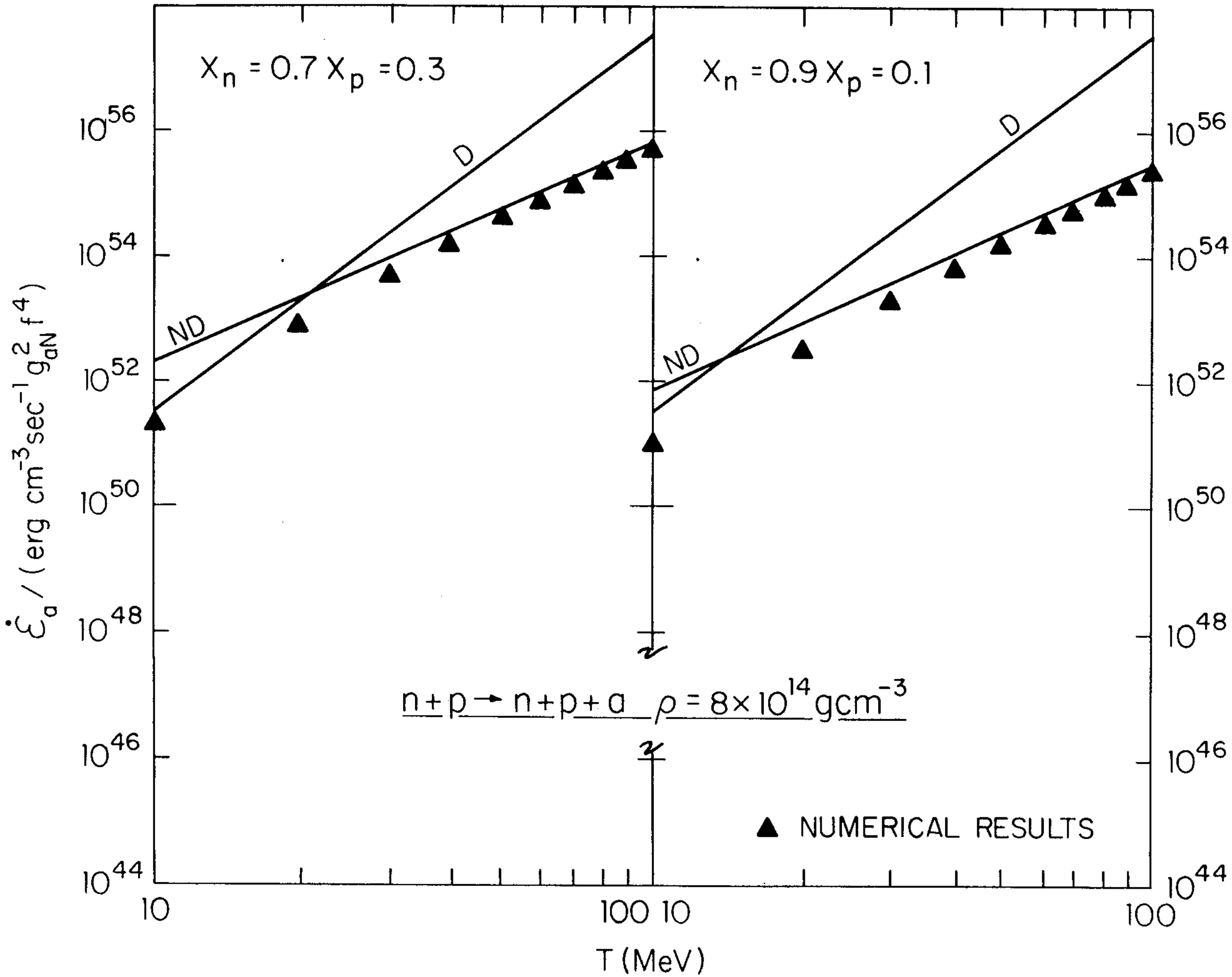

$X_n \rho = 4 \times 10^{14}$ g cm^{-3}
n + n → n + n + a
NON DEGENERATE
DEGENERATE
▲ NUMERICAL RESULTS
$\dot{\mathcal{E}}_a / (f^4 g^2_{an}$ erg cm^{-3} sec$^{-1})$
10^{44}
10^{46}
10^{48}
10^{50}
10^{52}
10^{54}
10^{56}
10^{58}
1 MeV
10 MeV
100 MeV
T
4
3
2
1
0
-1
-2
$y \equiv \hat{\mu}/T$

-FIG 1-

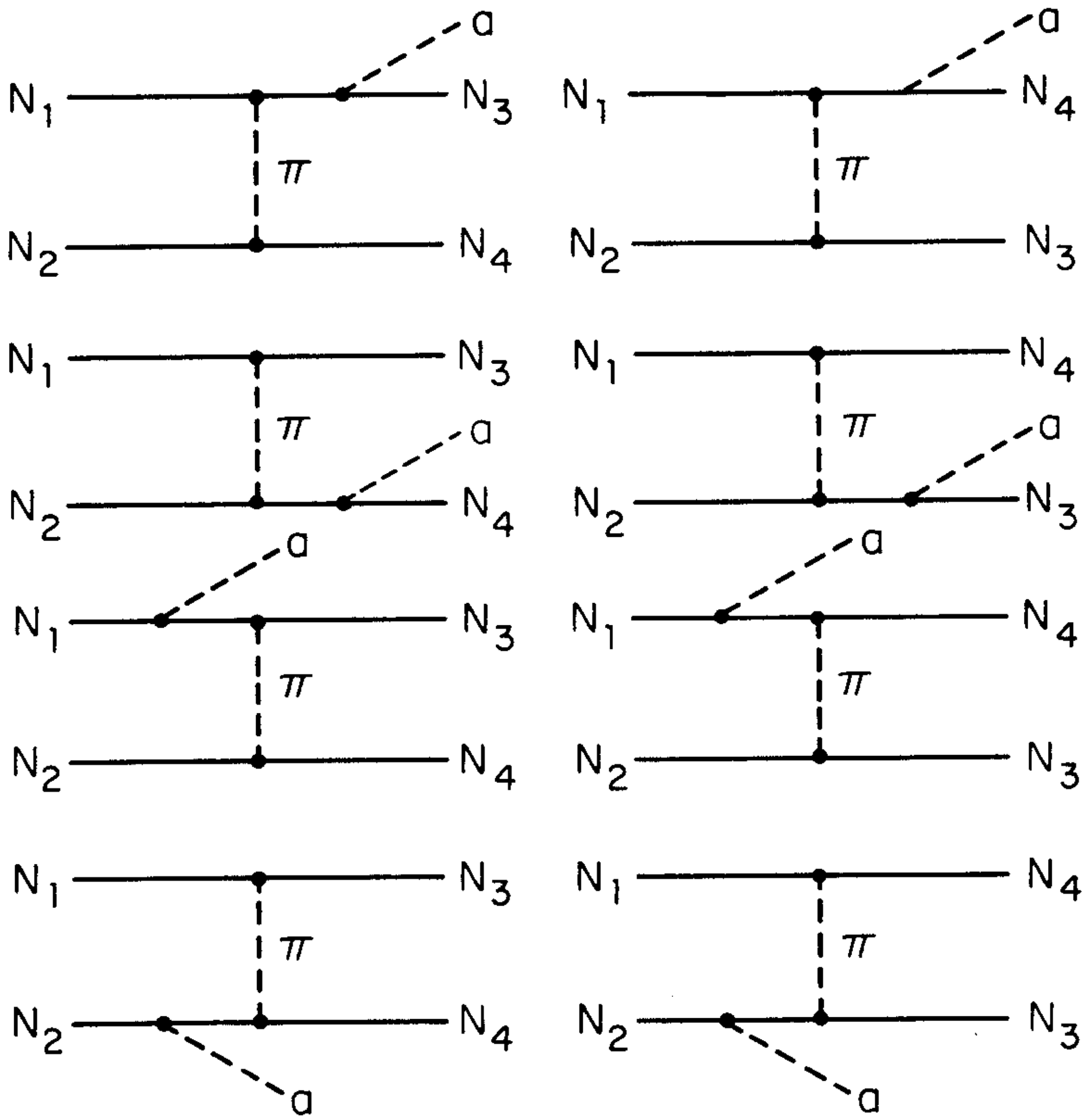


$n+n \rightarrow n+n+a$
NON-DEGENERATE
DEGENERATE
▲ NUMERICAL RESULTS
$\dot{\varepsilon}_a / \left[T^{6.5}\, m^{2.5}\, g_{an}^2\, f / m_\pi^4\right]$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
-4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
$y \equiv \hat{\mu} / T$

-FIG2-


$X_n = 0.7$ $X_p = 0.3$
$X_n = 0.9$ $X_p = 0.1$
D
ND
D
ND
$n + p \to n + p + a$ $\rho = 8 \times 10^{14}\,\mathrm{g\,cm^{-3}}$
▲ NUMERICAL RESULTS
$\dot{\mathcal{E}}_a$ / (erg cm^{-3} sec^{-1} g_{aN}^2 f^4)
T (MeV)
10
100 10
100
10^{56}
10^{54}
10^{52}
10^{50}
10^{48}
10^{46}
10^{44}

-FIG 3-



a
N1
N3
π
N2
N4
a
N1
N4
π
N2
N3
N1
N3
π
a
N2
N4
N1
N4
π
a
N2
N3
a
N1
N3
π
N2
N4
a
N1
N4
π
N2
N3
N1
N3
π
N2
N4
a
N1
N4
π
N2
N3
a

- FIG 4 -