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AXIONS, SN 1987A, and ONE PION EXCHANGE

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Abstract

Nucleon-nucleon, axion bremsstrahlung is the primary mechanism for axion emission from the nascent neutron star associated with SN 1987A, and the matrix element for this process has been calculated in the one pion exchange approximation (OPE). The axion mass limit which follows from SN 1987A is the most stringent astrophysical bound, $m_a \lesssim 10^{-3}$eV, and has received much scrutiny. It has been suggested that by using OPE to calculate the cross section for the related process, $pp \to pp + \pi^o$, and comparing the result to experimental data one can test the validity of this approximation, and further, that such a comparison indicates that OPE leads to a value for this cross section which is a factor of 30-40 too large. *If* true, this would suggest that the axion mass limit should be revised *upward* by a factor of ~ 6. We carefully calculate the cross section for $pp \to pp + \pi^o$ using OPE and find excellent agreement (to better than a factor of 2) with the experimental data.

There has been a great deal of interest in axion emission from SN 1987A.[1-5] And indeed, consideration of the effect of axion emission on the neutrino burst observed by the KII[6] and IMB[7] detectors seems to provide strong evidence against the existence of an axion with mass in the range $10^{-3} - 2$ eV; for the DFS type axion this improved the existing astrophysical bound by a factor of ~ 10, while for the hadronic type axion, the improvement was more than a factor of 10^3.[8] If the axion exists, then the dominant emission process from SN 1987A should have been nucleon-nucleon, axion bremsstrahlung. The matrix element squared for this process has been computed in the OPE approximation;[2] the 4 direct and 4 exchange diagrams are shown in Fig. 1. Given that the pion-nucleon coupling is of order unity one might question the accuracy of such an approximation—of course, the pion-nucleon coupling is derived by a comparison between an OPE treatment of pion-nucleon scattering and experimental data.[9] In addition, since the densities that existed at the core of the nascent neutron star associated with SN 1987A shortly after collapse were $\sim 2\rho_{\text{nuclear}} \sim 8\times 10^{14}$ g cm^{-3}, one should also worry about collective nuclear effects. Here, we will restrict our discussion to the validity of the OPE approximation itself.

Choi, et al[5] recently suggested a clever way of checking the accuracy of the OPE approximation in computing nucleon-nucleon, axion bremsstrahlung. Their idea is to compute the cross section for the related process, nucleon-nucleon, pion bremsstrahlung using OPE and to compare to the body of existing experimental data. Both the axion and the pion are pseudo Nambu-Goldstone bosons, and as such couple derivatively:

$$\mathcal{L}_{\text{int}} = ... + i\frac{g_N}{2m}\partial_\mu a\bar{N}\gamma^\mu\gamma^5 N + i\frac{\lambda}{2m}\partial_\mu\pi^o\bar{N}\gamma^\mu\gamma_5 N \; + ...$$

where N is the nucleon field, π^o is the neutral pion field, a is the axion field, $m \simeq 0.94$ GeV is the nucleon mass, $g_N \sim m/(f_a/N_a)$ is the axion-nucleon coupling, and in the OPE approximation the pion-nucleon coupling $\lambda \simeq 2m/m_\pi \simeq m/f_\pi$, where $m_\pi \simeq 135$ MeV is the neutral pion mass and $f_\pi \simeq 95$ MeV is the pion decay constant. The axion mass, m_a, and PQ symmetry breaking scale, f_a, are related by

$$m_a = 0.62\,\text{eV}\,[10^7\,\text{GeV}/(f_a/N_a)]$$

where N_a is the color anomaly of the PQ symmetry. For details about the axion and its couplings to matter, see Refs. 10. Because of the similarity of the pion and axion couplings we see that by substituting $\lambda \to g_N$ and $m_a \to m_\pi$, the matrix element for $NN \to NN+\pi^o$ can be obtained from that for $NN \to NN + a$.

The matrix element squared for nucleon-nucleon, axion bremsstrahlung has been calculated in Ref. 2; for the process $pp \to pp + a$ it is

$$\sum_{SPIN} |\mathcal{M}|^2_{\text{axion}} = \frac{256}{3}\frac{g_p^2 m^2}{m_\pi^4}(3-\beta)$$

where $|\mathcal{M}|^2_{\text{axion}}$ has been summed over *both* initial and final proton spins, and β is related to the average of the cosine squared of the angle between the direction of the momentum transfer in the direct and exchange diagrams: for degenerate matter $\beta = 0$; for our purposes here $\beta \simeq 1$ (see Ref. 2). In addition, several approximations were made in calculating $|\mathcal{M}|^2_{\text{axion}}$: the nucleons were assumed to be non-relativistic, the axion mass was taken to be zero, and $3mT$ was assumed to be much larger than m_π^2 (here T = temperature) so that the pion mass in the pion propagator can be neglected—all good approximations in the core of the neutron star associated with SN 1987A. However, we must keep them in mind to understand the realm of validity for using $|\mathcal{M}|^2_{\text{axion}}$ to compute the corresponding matrix element squared for $pp \to pp + \pi^o$.

Once $|\mathcal{M}|^2_\pi$ is at hand it is straightforward to obtain the cross section for $pp \to pp + \pi^o$ (Ref. 11):

$$d\sigma = \frac{1}{4} S \sum_{SPIN} |\mathcal{M}|^2_\pi (2\zeta_{ab})^{-1} (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2 - p_3) d\Pi_1 d\Pi_2 d\Pi_3$$

where a, b denote the incoming protons, 1,2 denote the outgoing protons, 3 denotes the pion, $d\Pi_i = d^3p_i/2E_i(2\pi)^3$, $S = 1/2$ is the usual symmetry factor for identical particles in the final state, the factor of 1/4 is inserted to average over initial proton spins, and the kinematical factor (Moeller flux factor)

$$\zeta_{ab} = s^{1/2}(s - 4m^2)^{1/2} = s(1 - 4m^2/s)^{1/2}$$

The quantity s is the center-of-mass (CM) energy squared, which is related to the KE of the incoming proton in the lab ($\equiv T_L$) by

$$s = 4m^2 + 2mT_L$$

The momenta of the incoming protons in the CM frame is

$$|\vec{p}_a|^2 = |\vec{p}_b|^2 = mT_L/2$$

The threshold for pion production is

$$T_L|_{\text{threshold}} = 2m_\pi + m_\pi^2/2m \simeq 2m_\pi \simeq 0.27\,\text{GeV}$$

From these last two formulae we see that for the approximations made in obtaining $|\mathcal{M}|^2_\pi$ from $|\mathcal{M}|^2_{\text{axion}}$ to be valid (relativistic pion, non-relativistic nucleons) T_L must satisfy:

$$2m_\pi \ll T_L \ll 2m \quad \text{or} \quad 0.3\,\text{GeV} \ll T_L \ll 2.0\,\text{GeV}$$

To actually compute the cross section it is most convenient to use the Dalitz representation for the 3-body phase space factor:[11]

$$(2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2 - p_3) d\Pi_1 d\Pi_2 d\Pi_3 = (16s)^{-1}(2\pi)^{-3} dm_{12}^2 dm_{13}^2$$

where we have taken advantage of the fact that the matrix element squared is constant to perform most of the integrals, and the invariant mass variables are

$$m_{12}^2 = (p_1 + p_2)^2 \qquad m_{13}^2 = (p_1 + p_3)^2$$

It is then straightforward to obtain $\sigma(pp \to pp + \pi^o)$:

$$\sigma(\mathrm{pp} \to \mathrm{pp} + \pi^o) = \frac{1}{2048\pi^3} \sum_{SPIN} |\mathcal{M}|_\pi^2 \frac{I}{s^2(1 - 4m^2/s)^{1/2}}$$

$$I = \int_{4m^2}^{(\sqrt{s}-m_\pi)^2} x^{-1/2}(x - 4m^2)^{1/2}[(s - x - m_\pi^2)^2 - 4xm_\pi^2]^{1/2} dx$$

where $x = m_{12}^2$. Using the matrix element squared which is obtained from $|\mathcal{M}|^2_{\mathrm{axion}}$ by substituting $g_p \to m/f_\pi$ we find

$$\sigma = 2.46 \times 10^{-25}\,\mathrm{cm}^2 \frac{I}{s^2(1 - 4m^2/s)^{1/2}}$$

Note, that although approximations have been made in evaluating $|\mathcal{M}|_\pi^2$, the phase space integrals have been evaluated exactly. The cross section as a function of the lab energy of the incoming proton, T_L, is shown in Fig. 2.

[In the limit that the 2 outgoing protons and pion are non-relativistic the integral I can be evaluated in closed form (see Byckling and Kajantie[11]),

$$I = \frac{\pi}{\sqrt{2}} \frac{(m_\pi/m)^{1/2}}{(1 + m_\pi/2m)^{3/2}} s^2(1 - 2m/\sqrt{s} - m_\pi/\sqrt{s})^2$$

While the assumption that the outgoing protons are non-relativistic is probably reasonable, the assumption that the outgoing pion is relativistic is not a good one; moreover, it is contrary to the approximation made in computing $|\mathcal{M}|_\pi^2$. However, over the lab energy range, $0.3\,\mathrm{GeV} \le T_L \le 1.0\,\mathrm{GeV}$, this approximate formula for I is suprisingly accurate, to about 5%. Using this expression for I, we obtain

$$\sigma = \frac{1}{2048\sqrt{2}\pi^2} \frac{(m_\pi/m)^{1/2}}{(1 + m_\pi/2m)^{3/2}} \sum_{\mathrm{SPIN}} |\mathcal{M}|_\pi^2 \frac{(1 - 2m/\sqrt{s} - m_\pi/\sqrt{s})^2}{(1 - 4m^2/s)^{1/2}}$$

$$\simeq 1.86 \times 10^{-25}\,\mathrm{cm}^2 \frac{(1 - 2m/\sqrt{s} - m_\pi/\sqrt{2})^2}{(1 - 4m^2/s)^{1/2}}$$

a form similar to that of Choi, et al[5].]

There exists a wealth of experimental data for the process $pp \to pp + \pi^o$ at energies $T_L \gtrsim 300$ MeV. The data is summarized in Refs. 12 and 13, and a convenient analytical fit is given in Ref. 12. The fit, which is consistent with all the experimental data (typical standard errors of $\lesssim 10\%$) at lab energies $T_L \lesssim 1.5\,\mathrm{GeV}$ is also shown in Fig. 2, and is given by the following expression:

$$\sigma = \frac{\pi}{2|\vec{p}_a|^2}\alpha(p_r/p_0)^\beta \frac{m_0^2\Gamma^2(q/q_0)^3}{(<M>^2 - m_0^2)^2 + m_0^2\Gamma^2}$$

where

$$p_r^2(s) = \frac{[s-(m-<M>)^2][s-(m+<M>)^2]}{4s}$$

$$q^2(<M>^2) = \frac{[<M>^2 - (m-m_\pi)^2][<M>^2 - (m+m_\pi)^2]}{4<M>^2}$$

$$q_0^2 = q^2(m_0^2)$$

$$p_0^2 = (m+m_0)^2/4 - m^2$$

$$<M> = M_0 + (\tan^{-1} Z_+ - \tan^{-1} Z_-)^{-1}\frac{\Gamma_0}{4}\ln\frac{1+Z_+^2}{1+Z_-^2}$$

$$Z_+ = 2(\sqrt{s} - m - M_0)/\Gamma_0 \qquad Z_- = 2(m + m_\pi - M_0)/\Gamma_0$$

$$\alpha = 3.772 \qquad \beta = 1.262$$

$$M_0 = 1.22\,\mathrm{GeV} \qquad m_0 = 1.188\,\mathrm{GeV}$$

$$\Gamma_0 = 0.12\,\mathrm{GeV} \qquad \Gamma = 0.099\,\mathrm{GeV}$$

Recall that the range of validity for our calculation above was: $0.3\,\mathrm{GeV} \ll T_L \ll 2\,\mathrm{GeV}$. In that region the experimental data, represented by the analytical fit, agree very well with our calculation, to better than a factor of 2. As one moves outside this range our calculation begins to overestimate the cross section significantly. At small T_L our calculation is not valid as the pion is not highly-relativistic: Because the pion couples derivatively $|\mathcal{M}|_\pi^2$ should vanish at threshold, and thus our approximation to $|\mathcal{M}|_\pi^2$, which does not vanish at threshold, should *overestimate* $|\mathcal{M}|_\pi^2$. Likewise, at large T_L, where the nucleons are becoming relativistic, we would expect our approximation to overestimate σ because the momentum dependence of nucleon propagators has been neglected. Thus, the deviation of the OPE treatment of the cross section from the experimental data, at both small and large T_L, is as expected.

[In order to estimate the effect of pion production threshold on the OPE calculation of $\sigma(pp \to pp + \pi^0)$ we have calculated $|\mathcal{M}|_\pi^2$ at threshold ($T_L = 2m_\pi$) with $m_a = m_\pi \neq 0$. We find

$$\sum_{\mathrm{SPIN}} |\mathcal{M}|_\pi^2 = 64\lambda^2 \frac{m}{m_\pi^3},$$

or a factor of $3m_\pi/8m \sim 1/20$ smaller than the value obtained taking $m_a = 0$.[14] Moreover, slightly above threshold ($T_L \gtrsim 300$ MeV), we find that the square of any given diagram is modified from the $m_a = 0$ result by a factor

$$1 - m^2 m_\pi^2/(p_1 \cdot p_3)^2 \propto |\vec{p}_{3CM}|^2 \propto (T_L/2m_\pi - 1)$$

The effect of correcting the OPE cross section by this approximate threshold factor is shown in Fig. 2: The agreement between the OPE result and the experimental data improves dramatically for small values of T_L. Of course, to properly take into account the fact that $m_\pi \neq 0$ one should compute the complete matrix element squared with $m_\pi \neq 0$—an ambitious project which is currently in progress.[15]]

Choi, et al[5] basically carried out the same calculation as we did above, and compared to similar experimental data at energies $T_L = 400, 500$ MeV. They found that the OPE approximation overestimated the experimental cross section by factors of 30-40. Our disagreements with the calculation of Choi, et al[5] are manifold. Firstly, the value of $S \sum_{\rm SPIN} |\mathcal{M}|^2_\pi$ that they used is larger than ours by a factor of $12(f_\pi/m_\pi)^2 \simeq 6.6$. Of this, a factor of $4(f_\pi/m_\pi)^2 \simeq 2.0$ traces to their using $\lambda = 2m/m_\pi$ for the pion nucleon coupling vs. our using $\lambda = m/f_\pi$; the remainder, a factor of 3.3, apparently has to due with spin averaging and the symmetry factor S. With regard to the value of λ; one can make arguments for either choice, and in any case the discrepancy due to this is only a factor of 2. Secondly, they have apparently used the non-relativistic approximation to evaluate I. However, the numerical factor in their expression for I differs from ours by a factor of

$$\frac{2\sqrt{2}}{\pi}(m/m_\pi)^{1/2}(1 + m_\pi/2m)^{3/2} \simeq 2.63$$

Taken together, these factors account for an overall factor of $6.58 \cdot 2.63 \simeq 17.3$, which is the entire discrepancy between our expressions for $\sigma(pp \to pp + \pi^o)$ and their expression. Further, they have only compared to the experimental data at two energies, $T_L = 400, 500$ MeV, energies which are not too far from the threshold for pion production, where the approximation made in using $|\mathcal{M}|^2_{\rm axion}$ to obtain $|\mathcal{M}|^2_\pi$ (i.e., relativistic pion) is breaking down. As mentioned earlier, near threshold one expects the estimate for $\sigma(pp \to pp + \pi^o)$ to be high. As we saw in Fig. 2, the agreement between the predicted cross section and the experimental data is very good at energies well above threshold.

In the supernova, where the nucleons have thermal distributions characterized by temperatures of order 20-80 MeV, the thermally averaged CM energy is given by

$$< s >=< (p_a + p_b)^2 >\simeq< 4m^2 + |\vec{p}_a|^2 + |\vec{p}_b|^2 - 2\vec{p}_a \cdot \vec{p}_b >\simeq 4m^2 + 6mT$$

so that the average value of s corresponds to lab energies $T_L \simeq 3T \sim 60 - 240$ MeV. Such energies are below the threshold for pion production, and so direct comparison at the relevant energies is not possible. However, one can be very encouraged by the excellent agreement in the region where comparison is possible, and the fact that that region is not so far from the energies of interest. Moreover, there is no reason to expect a surprise at the lower energies relevant to SN 1987A. Given the circumstances, the agreement is much better than one might have expected: OPE does not take into account resonances—the threshold for $\Delta(1232)$ production is only $T_L \simeq 629$ MeV. [Of course, the energies in the supernova are well below the threshold for any baryon resonance.] Contrary to the claims of Choi, et al[5] this phenomenological comparison seems to validate the OPE approximation. Since the axion emission rate $\dot{\varepsilon}_a \propto m_a^2$, the axion mass limit derived scales as $\dot{\varepsilon}_a^{-1/2}$—a factor of 2 uncertainty translates into a factor of $\sqrt{2}$ uncertainty in the mass limit. While it appears that there should be little worry about using OPE to compute $\dot{\varepsilon}_a$, one must still worry about collective nuclear effects. Because of the high densities at the core of the supernova, we have no similar laboratory data to compare with. Modulo this important uncertainty, it appears that the rate of axion emission from the supernova has been calculated to adequate accuracy, especially given the other uncertainties.

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Figure Captions

FIGURE 1—The 4 direct and 4 exchange diagrams for nucleon-nucleon, axion (or pion) bremsstrahlung, in the OPE approximation.

FIGURE 2—The cross section for $pp \rightarrow pp + \pi^o$ as a function of the lab KE of the incoming proton, T_L. The solid curve is an accurate fit to the experimental data[12], and the broken curve is the cross section as calculated in the OPE approximation (valid for $0.3\,\mathrm{GeV} \ll T_L \ll 2\,\mathrm{GeV}$). The OPE result corrected by the approximate threshold factor $(= 1 - m^2 m_\pi{}^2/(p_1 \cdot p_3)^2)$ is indicated by the triangles.

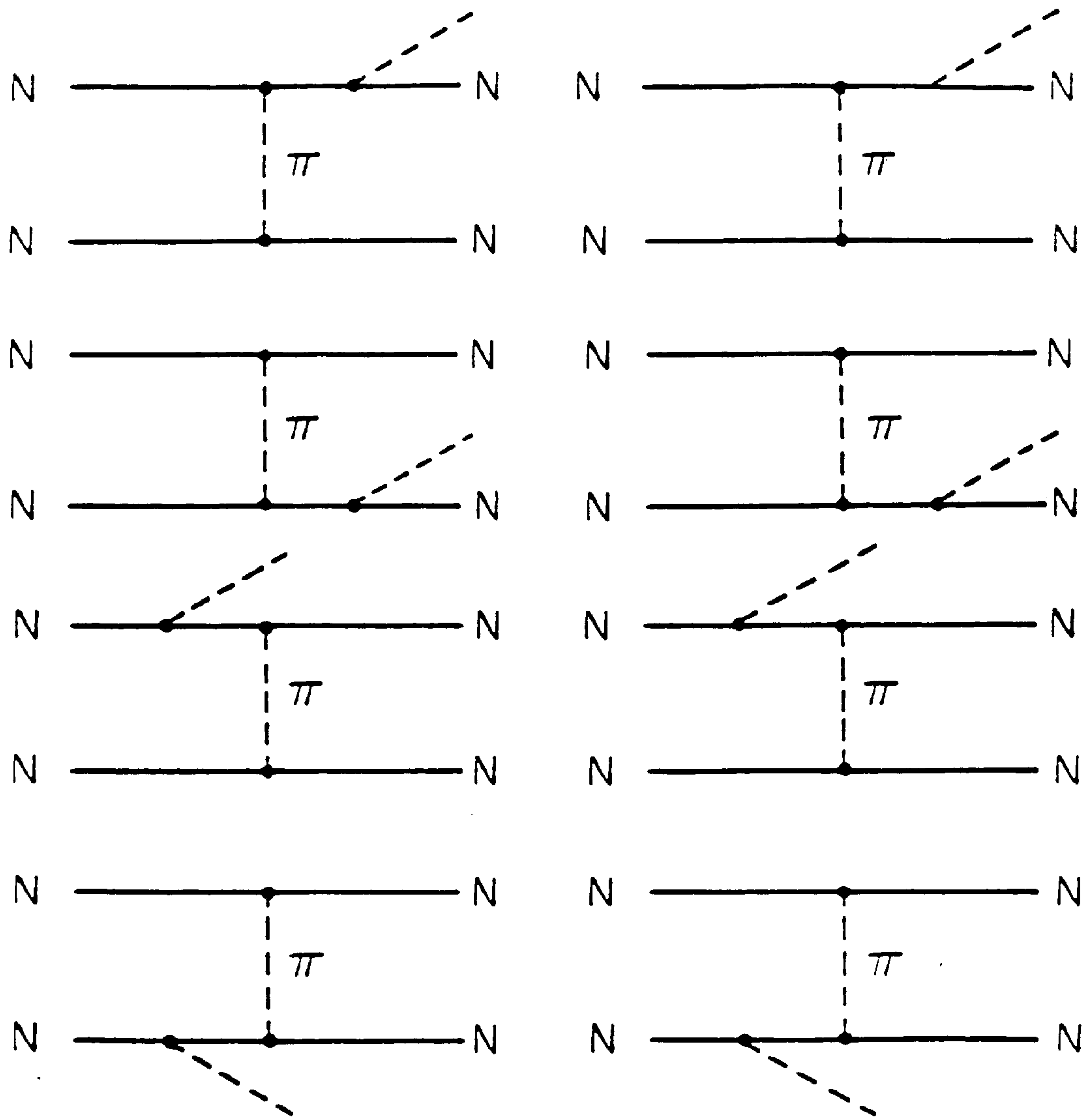
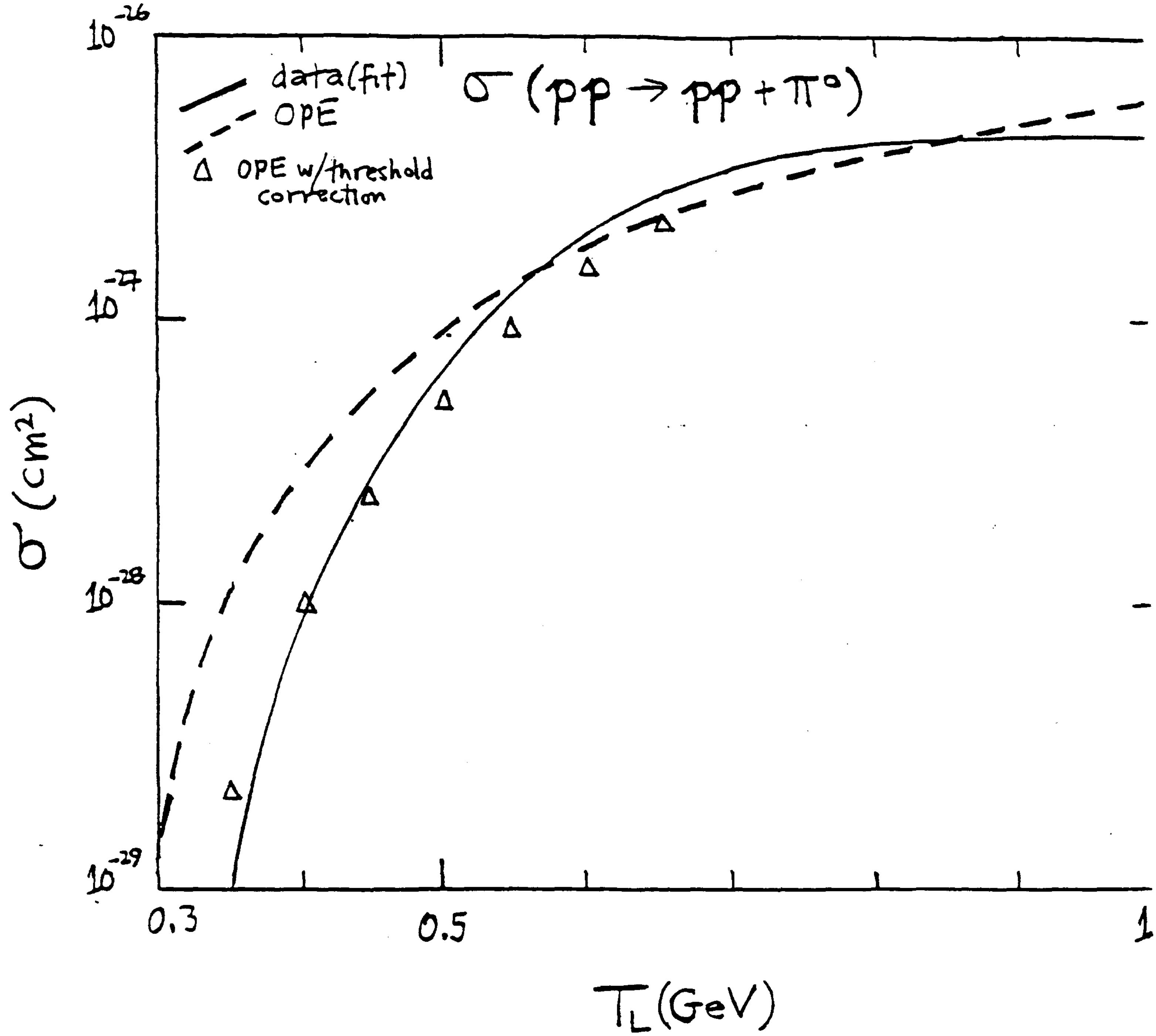


FIGURE 1



$\sigma\,(pp \to pp + \pi^0)$
data(fit)
OPE
Δ OPE w/threshold correction
10^{-26}
10^{-27}
10^{-28}
10^{-29}
$\sigma\,(cm^2)$
0.3
0.5
1
T_L(GeV)

- FIGURE 2 -