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The Renormalization of G^2 *

Benjamin Grinstein

Fermi National Accelerator Laboratory
Batavia, Illinois 60510

and

Lisa Randall†

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720, USA

Abstract

The renormalization properties of the operator $G^2 \equiv G^{a\mu\nu}(x)G^a_{\mu\nu}(x)$ are investigated in both pure QCD and in QCD with massive fermions. The computation of the operator's anomalous dimension is simple in background field gauge. The result can be expressed, to all orders in perturbation theory, in terms of the anomalous dimensions of the couplings and fields of the theory.

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†University of California President's Fellow.

The gauge invariant parity conserving product of gluon field strengths, $G^2 \equiv G^{a\mu\nu}(x)G^a_{\mu\nu}(x)$, repeatedly appears in the QCD description of particle interactions. Here, $G^a_{\mu\nu}(x) = \partial_\mu Q^a_\nu - \partial_\nu Q^a_\mu + g f_{abc} Q^b_\mu Q^c_\nu$, and Q^a_μ is the gluon field with color index a and Lorentz index μ, and g is the QCD coupling constant. For example, the vacuum matrix element of G^2 appears in the short distance expansion of the photon propagator, which is related in turn to the cross section for e^+e^- annihilation into hadrons. It is the pure glue color singlet operator of lowest dimension so it is the first nontrivial operator in the short distance expansion of the product of gauge singlet operators (quark bilinears are usually suppressed by their masses).

The G^2 operator has other manifestations as well. QCD sum rules relate processes involving light particles to the vacuum expectation values of the gluon and quark condensates. Even when absolute determination of the nonperturbative matrix element is precluded, expressing matrix elements in terms of these operators can relate different processes.

Furthermore, G^2 contributes to the trace anomaly, the fact that the trace of the energy momentum tensor has a mass independent term. Voloshin and Zakharov [1] have employed this fact in the calculation of hadronic transitions between quarkonium states. Also[2,3], the effective Hamiltonian describing the decay of a light neutral Higgs ϕ into hadrons involves the coupling ϕG^2. Therefore the Higgs decay rate into hadrons involves the G^2 matrix element between hadronic states. Again these matrix elements can be calculated in the manner of Voloshin and Zakharov.

In the context of low energy effective field theories in order to evaluate an operator's contribution to a low energy process, two pieces of information are needed. One must determine both the operator's dependence on the renormalization scale μ, and the matrix element of the operator at a particular scale. In general, the evaluation of the matrix element is a nonperturbative problem which can only be solved when there is a conserved charge (see ref. [3] for an application of such reasoning to the operator G^2) or with sufficiently accurate numerical lattice calculations. We will only address the question of the renormalization group scaling of G^2.

Previous authors have also considered this problem. Kluberg-Stern and Zuber [4] derived the anomalous dimension matrix for gluons and ghosts, in pure QCD in covariant gauge and in axial gauge, in terms of the beta function and the anomalous dimensions of gluon and ghost fields. Moreover, they used BRS invariance to show that the calculation is simplified in background field gauge, where the anomalous dimension matrix takes a block triangular form since gauge variant operators do not mix into gauge invariant ones.

Tarrach [5] employs the fact that calculations are simplified in background field gauge to explicitly verify the result of Kluberg-Stern and Zuber to two loops. He did a further explicit 2 loop calculation with the inclusion of massive quarks. To second order in the loop expansion the operator $\frac{\beta}{g}G^2 - 2\gamma_m m\bar{\psi}\psi$ is renormalization group invariant, where β is the QCD beta function and γ_m is the anomalous mass dimension for the quark mass.

In this letter we will verify Tarrach's result to all orders. Furthermore, we improve on the argument of ref. [4] by employing the explicit gauge invariance of the background field Green's functions in background field gauge. We will see this greatly simplifies the computation of the anomalous dimension matrix, since the effective action is expressible as a gauge invariant function of the gauge fields only, which is a stronger constraint on the structure of the counterterm lagrangian than BRST invariance.

We first review the relevant features of background field (BF) gauge [6]. The BF gauge fixing term is

$$\mathcal{L}_{gf} = \frac{1}{2\alpha}(\partial_\mu Q^{\mu a} - \partial_\mu B^{\mu a} + g f^{abc} B^b_\mu Q^c_\mu)^2 \tag{1}$$

$$= \frac{1}{2\alpha}[D^B(Q-B)]^2 \tag{2}$$

where Q is the quantum gauge field, B is the background field which defines the gauge condition, D^B stands for the B-field covariant derivative and α is the gauge fixing parameter.

From eq. (2) it is clear that $\mathcal{L}_{gf}$ is invariant under the combined transformations

$$B_\mu \rightarrow B^\Omega_\mu \equiv \Omega^{-1}(B_\mu + \partial_\mu)\Omega \tag{3}$$

$$Q_\mu \quad\to\quad Q_\mu^\Omega \equiv \Omega^{-1}(Q_\mu + \partial_\mu)\Omega. \tag{4}$$

If the ghosts are transformed homogeneously, the full lagrangian is invariant under the combined transformation. Now let $W(J)$ be the generator of connected Green's functions in the theory which includes a source term for the quantum field; *i.e.* $\mathcal{L}(J) = \mathcal{L}(J=0)+J\cdot Q$. Then $\Gamma_B(\bar{Q}) = (W_B(J) - \int d^4x J\cdot\bar{Q})|_{\bar{Q}=\delta W/\delta J}$ is the generator of one particle irreducible (1PI) Green's functions. The subscript B in Γ and W reminds us of the implicit dependence of these quantities on the background field. Now $W(J) - \int d^4x J\cdot B = \Gamma(\bar{Q}) + \int d^4x J\cdot(\bar{Q}-B)$ is invariant under $J\to\Omega^{-1}J\Omega$, $\bar{Q}\to\bar{Q}^\Omega$ and $B\to B^\Omega$. Therefore $\Gamma_{inv}(B)\equiv\Gamma(\bar{Q})|_{\bar{Q}=B}$ is invariant under the transformation $B\to B^\Omega$. $\Gamma_{inv}(B)$ is "gauge invariant"; that is, it is invariant with respect to gauge transformations on the field B. This invariance implies nontrivial relations between the coefficients in the expansion of Γ_{inv} in powers of B.

We now proceed to the general argument. We use dimensional regularization and follow the renormalization conventions of Gross [7]. The generating functional for renormalized Green's functions, Γ^R can be expanded either in terms of bare fields and couplings or in terms of renormalized fields and couplings:

$$\Gamma^R(\bar{Q},g) = \Gamma^\circ(\bar{Q}^\circ, g^\circ), \tag{5}$$

where $\bar{Q}^\circ = Z^{1/2}\bar{Q}$, $g^\circ = Z_g g$, and Z and Z_g are defined so that the left hand side of eq. (5) is finite. (Notice that there is a factor of $Z^{n/2}$ when we expand the Γ's in terms of the n-point functions). To renormalize a Green's function with the insertion of the operator $\mathcal{O}$ an additional factor $Z_\mathcal{O}$ is defined by the condition that

$$\Gamma^R_\mathcal{O}(\bar{Q},g) = Z_\mathcal{O}^{-1}\Gamma^\circ_\mathcal{O}(\bar{Q}^\circ, g^\circ) \tag{6}$$

is finite. When operator mixing is incorporated, this is to be interpreted as a matrix equation. We will calculate the operator renormalization factor relating it to the anomalous dimension of the gluon field γ.

The gauge field lagrangian in BFG is

$$\mathcal{L} = \mathcal{L}_\circ + \mathcal{L}_{gf} + \mathcal{L}_{ghosts} + J\cdot Q - \frac{1}{4}J_{G^2}G^2, \tag{7}$$

where $\mathcal{L}_o = -\frac{1}{4}G^2$, $\mathcal{L}_{gf}$ is given in eq. (2), $\mathcal{L}_{ghosts}$ is the corresponding ghost term, J is the source of the gauge field and J_{G^2} is the source for the operator $-\frac{1}{4}G^2$. To define Γ, we Legendre transform only with respect to the source J. This is adequate since we only allow Green's functions with at most a single operator insertion, so that Γ and Γ_{G^2} will serve as the generators of all 1PI Green's functions. Let $W(J, J_{G^2}; g, \alpha)$ be the generator of connected Green's functions and $\Gamma(\bar{Q}, J_{G^2}; g, \alpha)$ be the generator of 1PI functions. Notice that the theory in eq. (7) is invariant if we simultaneously infinitesimally transform the fields, sources and couplings as follows:

$$Q \rightarrow Q(1+\frac{1}{2}\epsilon) \tag{8}$$

$$B \rightarrow B(1+\frac{1}{2}\epsilon) \tag{9}$$

$$g \rightarrow g(1-\frac{1}{2}\epsilon) \tag{10}$$

$$\alpha \rightarrow \alpha(1+\epsilon) \tag{11}$$

$$J \rightarrow J(1-\frac{1}{2}\epsilon) \tag{12}$$

$$J_{G^2} \rightarrow J_{G^2}(1-\epsilon)-\epsilon \tag{13}$$

The ghost fields are not transformed. Notice the inhomogeneous transformation of J_{G^2} was required to cancel the order ϵ G^2 term which would otherwise be generated. Thus we have

$$W_{(1+\frac{1}{2}\epsilon)B}((1-\frac{1}{2}\epsilon)J, (1-\epsilon)J_{G^2}-\epsilon; (1-\frac{1}{2}\epsilon)g, (1+\epsilon)\alpha) = W_B(J, J_{G^2}; g, \alpha). \tag{14}$$

Therefore,

$$\Gamma_{(1+\frac{1}{2}\epsilon)B}((1+\frac{1}{2}\epsilon)\bar{Q}, (1-\epsilon)J_{G^2}-\epsilon; (1-\frac{1}{2}\epsilon)g, (1+\epsilon)\alpha) = \Gamma_B(\bar{Q}, J_{G^2}; g, \alpha). \tag{15}$$

We expand the above expression to order ϵ to conclude

$$\Big[\frac{1}{2}\int d^4x \bar{Q}(x)\frac{\delta}{\delta\bar{Q}(x)} + \frac{1}{2}\int d^4x B(x)\frac{\delta}{\delta B(x)} - \frac{1}{2}g\frac{\partial}{\partial g} + \alpha\frac{\partial}{\partial\alpha} - \int d^4x [J_{G^2}(x)+1]\frac{\delta}{\delta J_{G^2}(x)}\Big]\Gamma_B(\bar{Q}, J_{G^2}; g, \alpha) = 0. \tag{16}$$

Now, $\frac{\delta \Gamma_B}{\delta J_{G^2(x)}}|_{J_{G^2}=0} = -\frac{1}{4}\Gamma_{B,G^2}(x)$. Taking $J_{G^2} = 0$ we obtain

$$-\frac{1}{4}\int d^4x \Gamma_{B,G^2}(x)(\bar{Q}, 0; g, \alpha) = \Big[\frac{1}{2}\int d^4x \bar{Q}(x)\frac{\delta}{\delta \bar{Q}(x)} + \frac{1}{2}\int d^4x B(x)\frac{\delta}{\delta B(x)} - \frac{1}{2}g\frac{\partial}{\partial g} + \alpha\frac{\partial}{\partial \alpha}\Big]\Gamma_B(\bar{Q}, 0; g, \alpha). \quad (17)$$

Applying the above equation to $\Gamma_{inv}(B; g, \alpha) = \Gamma_B(\bar{Q}, 0; g, \alpha)|_{\bar{Q}=B}$, we obtain

$$-\frac{1}{4}\int d^4x \Gamma_{inv,G^2}(x)(B; g, \alpha) = -\frac{1}{4}\int d^4x \Gamma_{B,G^2}(x)(\bar{Q}, 0; g, \alpha)|_{\bar{Q}=B} = \Big[\frac{1}{2}\int d^4x B(x)\frac{\delta}{\delta B(x)} - \frac{1}{2}g\frac{\partial}{\partial g} + \alpha\frac{\partial}{\partial \alpha}\Big]\Gamma_{inv}(B; g, \alpha). \quad (18)$$

One may expand Γ_{inv} in powers of $B^a_\mu(x)$. The coefficient of the term of order $(B)^n$ in such an expansion is $\frac{1}{n!}\Gamma^{(n)\,a_1\ldots a_n}_{inv\;\mu_1\ldots\mu_n}$, *i.e.* the n-point 1PI Green's function. An alternative expansion of Γ_{inv} consists of writing $\Gamma_{inv} = \int d^4x \mathcal{L}_{inv}$ where $\mathcal{L}_{inv}$ is a gauge invariant lagrangian density. $\mathcal{L}_{inv}$ may contain non-local terms. In this expansion the divergences of the theory may be readily identified. The only divergent counterterm of dimension less than or equal to four must be a gauge invariant parity conserving operator, *i.e.* G^2. That is,

$$\Gamma^{\circ}_{inv}(B^{\circ}, g^{\circ}) = \int d^4x C(g^{\circ}) G^{\circ 2}(x) + \cdots \quad (19)$$

for some divergent constant $C(g^{\circ})$. In pure QCD, by choosing Z and Z_g in eq. (5) so as to make this term finite, then 't Hooft's proof of renormalizability [8] insures that the whole of Γ_{inv} is also made finite. With the additional renormalization factor $Z_{\mathcal{O}}^{-1}$, defined in eq. (6), $\Gamma_{inv,\mathcal{O}}$ is finite.

By applying eq. (18) to eq. (19) we can readily obtain Z_{G^2}. The renormalized generating functional Γ^R_{inv} is

$$\Gamma^R_{inv}(B, g) = \Gamma^{\circ}_{inv}(Z^{1/2}B, Z_g g) = \int d^4x C(gZ_g) Z G^2 + \cdots \quad (20)$$

Gauge invariance of Γ_{inv} requires $Z^{1/2}Z_g = 1$ and we have used this in expressing the answer in terms of G^2 only. Since the left hand side is finite, so must be the product of C and Z. We choose the renormalization convention where $CZ = -\frac{1}{4}$. Notice that C is of the form $-\frac{1}{4} + \sum \frac{C_k}{\epsilon^k}$. Therefore Z also takes the form $1+\sum \frac{Z^{(k)}}{\epsilon^k}$, where the $Z^{(k)}$ are such that CZ is finite. Now, from eqs. (6) and (18) we deduce

$$-\frac{1}{4}\int d^4x\Gamma^R_{inv,G^2}(x)(B;g,\alpha) =$$
$$Z^{-1}_{G^2}\Big(\frac{1}{2}\int d^4xB^\circ(x)\frac{\delta}{\delta B^\circ(x)} - \frac{1}{2}g^\circ\frac{\partial}{\partial g^\circ}+\alpha^\circ\frac{\partial}{\partial\alpha^\circ}\Big)\int d^4xC(g^\circ)G^{\circ 2}+\cdots \quad (21)$$

where the right hand side is evaluated at $B^\circ = Z^{1/2}B$ and $g^\circ = Z_g g$. Again, concentrating on the gauge field self-energy we have, after some algebra,

$$-\frac{1}{4}\int d^4x\Gamma^R_{inv,G^2}(x)(B;g,\alpha) =$$
$$Z^{-1}_{G^2}\Big(1-(\frac{1}{2}g^\circ\frac{\partial}{\partial g^\circ}-\alpha^\circ\frac{\partial}{\partial\alpha^\circ})\ln C(g^\circ)\Big)\int d^4xC(g^\circ)G^{\circ 2}(x)+\cdots \quad (22)$$
$$= Z^{-1}_{G^2}\Big(1-(\frac{1}{2}g\frac{\partial}{\partial g}-\alpha^\circ\frac{\partial}{\partial\alpha^\circ})\ln Z^{-1}\Big)\Gamma^R_{inv}(B,g) \quad (23)$$

This is finite if $Z^{-1}_{G^2}(1-(\frac{1}{2}g\frac{\partial}{\partial g}-\alpha^\circ\frac{\partial}{\partial\alpha^\circ})\ln Z^{-1})$ is finite. We choose our renormalization condition so that this product is unity. The result is

$$Z_{G^2} = 1+\frac{1}{2}g\frac{\partial\ln Z}{\partial g}-\alpha^\circ\frac{\partial\ln Z}{\partial\alpha^\circ} \quad (24)$$

We define $\beta(g,\epsilon)=\mu\frac{\partial g}{\partial\mu}$, $\gamma(g)=\frac{1}{2}\mu\frac{\partial\ln Z}{\partial\mu}$. In $4-\epsilon$ dimensions[1], $g^\circ=\mu^{\epsilon/2}Z_g g$, and we define $\beta(g,\epsilon)=-\frac{1}{2}\epsilon g+\beta(g)$[7,9]. Therefore, since $Z^{1/2}Z_g=1$ in BF gauge, $\beta(g)=g\gamma$. Note also that $\gamma(g)=\frac{1}{2}\beta(g,\epsilon)\frac{\partial\ln Z}{\partial g} = -\frac{1}{2}g\frac{\partial Z^{(1)}}{\partial g}$. Similarly, $\gamma_{G^2} = -\frac{1}{2}g\frac{\partial Z^{(1)}_{G^2}}{\partial g}$. Here $Z^{(1)}$ is the residue of the ϵ pole in Z and $Z^{(1)}_{G^2}$ is the residue of the ϵ pole in Z_{G^2}. Therefore, from eq. (24), we can derive

$$Z^{-1}_{G^2} = 1+\frac{2\gamma}{\epsilon}+\mathcal{O}(\frac{1}{\epsilon^2}) \quad (25)$$

and

$$\gamma_{G^2} = g\frac{\partial\gamma}{\partial g} \quad (26)$$

This is the promised result, expressing γ_{G^2} fully in terms of $\gamma(g)=\beta/g$. Notice that the α°-derivative term in eq. (24) does not contribute to γ_{G^2}. This is

[1]The usual factor of $\mu^{-\epsilon/2}$ in the relation between g° and g played no role in the previous discussion and was therefore omitted.

because, in background field gauge, $\gamma(g) = \beta/g$ is independent of the gauge parameter α. For this reason, in what follows we will drop the α-dependence, which amounts to making the choice of gauge $\alpha \to 0$ ('Landau Gauge')[2].

Using this result it is trivial to verify that $\gamma(\bar{g}(\mu))G^2(\mu)$ is μ independent, where by $G^2(\mu)$ we mean the matrix element of the operator G^2 evaluated at a renormalization point μ, and $\bar{g}(\mu)$ is the running coupling constant, a solution to $\mu\frac{d\bar{g}(\mu)}{d\mu} = \beta(\bar{g}(\mu))$.

We now calculate the anomalous dimension matrix in a theory with massive fermions. This is a straightforward extension of the above reasoning. With the inclusion of massive fermions the above argument is modified in three ways. Firstly, the operator $\mathcal{O}_1 = -\frac{1}{4}G^2$ is no longer multiplicatively renormalized. It mixes with the operators $\mathcal{O}_2 = -m\bar{\psi}\psi$ and $\mathcal{O}_3 = \bar{\psi}(i \not{D} - m)\psi$, where D_μ is the covariant derivative acting on fermion fields. Therefore Z_{G^2} is replaced by a matrix Z_{ij}:

$$\Gamma^R_{\mathcal{O}_i}(\psi, B; g, m) = Z^{-1}_{ij}\Gamma_{\mathcal{O}_j}(Z_\psi^{1/2}\psi, Z^{1/2}B; Z_g g, Z_m m). \tag{27}$$

Secondly, by appropriately scaling the fields and parameters, we can derive expressions which relate $\Gamma_{\mathcal{O}_i}$ to Γ, as in eq. (16):

$$\int d^4x \Gamma_{\mathcal{O}_2}(x) = m\frac{\partial}{\partial m}\Gamma \tag{28}$$

and

$$\int d^4x \Gamma_{\mathcal{O}_3}(x) = \frac{1}{2}\int d^4x \left(\psi(x)\frac{\delta}{\delta\psi(x)} + \bar{\psi}(x)\frac{\delta}{\delta\bar{\psi}(x)}\right)\Gamma. \tag{29}$$

Third, the expansion of Γ_{inv} includes two new dimension 4 operators, whose coefficients, as with eq. (19), can be related to Z_ψ and Z_m:

$$\Gamma_{inv}(\psi^\circ, B^\circ; g^\circ, m^\circ) = \int d^4x \left(CG^{\circ 2} + Dm^\circ\bar{\psi}^\circ\psi^\circ + E\bar{\psi}^\circ(i \not{D} - m^\circ)\psi^\circ\right) + \cdots \tag{30}$$

Here C, D and E are divergent functions of the bare couplings. The remaining analysis parallels that presented above. We obtain

$$Z^{-1} = \begin{pmatrix} 1 - \frac{1}{2}g\frac{\partial \ln Z}{\partial g} & -\frac{1}{2}g\frac{\partial \ln Z_m}{\partial g} & -\frac{1}{2}g\frac{\partial \ln Z_\psi}{\partial g} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{31}$$

[2]The anomalous dimension of the fermion field γ_ψ does depend on α. If its effects need to be considered one must properly restore the α dependence in the equations that follow.

Extracting the anomalous dimension matrix is now simple. First, from eq. (27) we have the definition of the anomalous dimension matrix γ_{ij}:

$$\mu \frac{d\Gamma^R_{\mathcal{O}_i}}{d\mu} = -\gamma_{ij}\Gamma^R_{\mathcal{O}_j} \tag{32}$$

That is,

$$\gamma_{ik} Z^{-1}_{kj} = -\mu \frac{\partial Z^{-1}_{ij}}{\partial \mu}, \tag{33}$$

and recalling that $\mu \frac{\partial g}{\partial \mu} = \beta(g, \epsilon) = -\frac{1}{2}\epsilon g + \beta(g)$,

$$\gamma_{ij} = -\frac{1}{2} g \frac{\partial Z^{(1)}_{ij}}{\partial g}, \tag{34}$$

where $Z^{(1)}_{ij}$ is the residue of the ϵ pole of the operator renormalization matrix Z_{ij}. Similarly,

$$\gamma = -\frac{1}{4} g \frac{\partial Z^{(1)}}{\partial g} \tag{35}$$

$$\gamma_m = -\frac{1}{2} g \frac{\partial Z^{(1)}_m}{\partial g} \tag{36}$$

$$\gamma_\psi = -\frac{1}{4} g \frac{\partial Z^{(1)}_\psi}{\partial g} \tag{37}$$

Therefore

$$\gamma = \begin{pmatrix} g\frac{\partial \gamma}{\partial g} & \frac{1}{2} g \frac{\partial \gamma_m}{\partial g} & g \frac{\partial \gamma_\psi}{\partial g} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{38}$$

It is now easy to verify that the linear combination

$$\theta \equiv \frac{1}{2}\gamma(\bar{g}(\mu))G^2(\mu) + \gamma_m(\bar{g}(\mu))\bar{m}(\mu)\bar{\psi}\psi - 2\gamma_\psi(\bar{g}(\mu))\bar{\psi}(i \not{D} - \bar{m}(\mu))\psi \tag{39}$$

is independent of the renormalization point μ, when inserted at zero momentum[3]. Here $\bar{m}(\mu)$, the running mass, is a solution to $\mu \frac{\partial \bar{m}(\mu)}{\partial \mu} = \bar{m}(\mu)\gamma_m(\bar{g}(\mu))$, and $\bar{g}(\mu)$

[3]The method we have used cannot differentiate between the operators $\bar{\psi}(i \not{D} - m)\psi$ and $\bar{\psi}(-i\overleftarrow{\not{D}} - m)\psi$. Because these vanish on-shell, the μ-independence of the matrix elements of θ holds for arbitrary momentum.

is the running coupling constant defined earlier. The reader will recognize the operator θ as the trace anomaly [10], defined by

$$\partial_\mu s^\mu = \theta^\mu_\mu = \theta + m\bar{\psi}\psi. \tag{40}$$

Here s^μ and $\theta_{\mu\nu}$ are the new improved dilatation current and energy momentum tensor. The μ-independence of θ can now be understood. Since space translations are a good symmetry of the quantum theory, the generators $P_\mu = \int d^3x \theta_{\mu 0}$ must satisfy canonical commutation relations and hence remain unrenormalized.

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