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LIGHT NEUTRINOS AS COSMOLOGICAL DARK MATTER – A CRUCIAL EXPERIMENTAL TEST

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Abstract

Cosmological dark matter allegedly dominates the energy of the universe. Among all dark matter candidates, the light neutrino is the only particle actually known to exist in nature. The most likely light neutrino candidate is ν_τ with mass $m(\nu_\tau) \approx 15 - 65\ eV$. The only practical way to show that $m(\nu_\tau)$ is in that range, is to search for $\nu_\mu - \nu_\tau$ oscillations reaching values of $\sin^2 2\theta_{\tau\mu}$ as low as $4 \cdot 10^{-4}$. This calls for an improvement of the best existing experiment by one order of magnitude. A dedicated accelerator experiment with an emulsion followed by a spectrometer, detecting at least 40,000 neutrino interactions, can settle the issue. Such an experiment does not seem impossible. A positive result would prove that most of the energy of the universe consists of ν_τ particles.

It is widely believed that the universe is filled with cosmological dark matter. The dark matter accounts for most of the energy of the universe. It probably leads to a "flat" universe, *i.e.* neither "open" nor "closed". A flat universe has an energy density ρ_0 which equals the "critical density" $\rho_c = \frac{3H^2}{8\pi G}$, where H and G are, respectively, the Hubble parameter and Newton's constant. Theoretically, we prefer $\rho_0 = \rho_c$ (in other words $\Omega \equiv \frac{\rho_o}{\rho_c} = 1$). Experimentally, it seems that Ω may still be somewhat larger or smaller than one, but is nevertheless dominated by the cosmological dark matter.

There are many candidates for the cosmological dark matter[1], the leading among them being weakly interacting neutral particles which do not emit observable radiation and are difficult to detect in terrestrial experiments. Among these candidates, the three leading classes are:

(*i*) Light neutrinos with masses around $15 - 65\ eV$. The dominant neutrino could be, in principle, ν_e, ν_μ, ν_τ or a hypothetical fourth neutrino ν_σ.

(*ii*) WIMPS (Weakly Interacting Massive Particles) with masses of several GeV's. These could be a heavy neutrino around $4 - 8\ GeV$, a photino, another supersymmetric particle (if it is lighter than the photino) or other neutrino-like or photino-like objects.

(*iii*) Axions or other Goldstone particles.

There are other, even more exotic, candidates. There are also other kinds of dark matter such as the dark matter *inside* galaxies. We do not discuss these here.

Among all the above candidates, all class (ii) and class (iii) particles may or may not exist. At present, there is no shred of evidence for the existence of any of them. On the other hand, the three light neutrinos of class (i) definitely exist, although we do not know if any of them have masses in the range $15 - 65\ eV$.

To our best knowledge, the only argument *against* the light-neutrino dark-matter hypothesis is based on attempts to understand galaxy formation[2]. Some such calculations have indicated that light neutrinos may not have the right "clumping" properties. More recent calculations, in which cosmic strings are "thrown in", yield more optimistic results. We cannot express any opinion on this issue, except to suggest that the theory of galaxy formation, even according to its most enthusiastic practitioners, is far from reaching a stable, mature, status. It can hardly be used as a *decisive* argument for or against a specific dark matter candidate.

Any unbiased observer who has not been "brainwashed" by recent speculations concerning supersymmetry, axions or galaxy formation would undoubtedly conclude that the leading "suspect" in the dark matter puzzle must be the light neutrino, the only candidate actually known to exist in nature. Among the three known neutrinos, the tau neutrino ν_τ, is the most likely candidate. It is, therefore, extremely important to search for neutrinos in general, and tau neutrinos in particular, at the relevant mass range of $15 - 65\ eV$.

In this note we consider the phenomenological situation related to this problem. We argue that a conceptually simple neutrino oscillation experiment can play a crucial role in establishing the tau neutrino as the dark matter of the universe. *A positive result will solve the cosmological dark matter problem!.* A negative result in such an experiment, will make the light-neutrino dark-matter hypothesis extremely unlikely.

We first note that the total energy density of the universe can be written as:

$$\rho_0 \equiv \Omega\rho_c = \Omega\frac{3H^2}{8\pi G} = \Omega h^2 \cdot 11\ \frac{keV}{cm^3}$$

where h is defined by $H \equiv h \cdot 100\frac{km/sec}{mpc}$. The accepted observational bounds[3] on Ω and h are $\Omega < 2$ and $\frac{1}{2} < h < 1$. However, Ω and h are related to the present age of the universe t_0. For instance, for $\Omega < 2$ and $t_0 > 10^{10} yrs$, we obtain $h < 0.57$ and $\Omega h^2 < 0.65$. For $\Omega = 1$ (the preferred theoretical value) and $t_0 = 1.5 \cdot 10^{10} yrs$ in a matter-dominated universe, we obtain $\Omega h^2 = 0.2$. We can safely assume:

$$0.15 \leq \Omega h^2 \leq 0.65.$$

The number density n_ν of any flavor of light stable neutrinos is related to the known number density of photons n_γ by:

$$n_\nu = \frac{3}{11} n_\gamma \approx 110\ cm^{-3}.$$

Hence, if ρ_0 is *entirely* dominated by one flavor of light neutrinos, we must have[4]:

$$m_\nu = 100\Omega h^2\ eV.$$

For $\Omega = 2$ and $t_0 > 10^{10} yrs$ we obtain the most conservative upper limit $m_\nu < 65\ eV$. For the "favored" values of $\Omega = 1,\ t_0 = 1.5 \cdot 10^{10} yrs$ we obtain $m_\nu = 20\ eV$. For other reasonable values of Ω, h and t_0 we always obtain masses around $15 - 65\ eV$.

Which of the three known neutrinos might have a mass around $15 - 65\ eV$? The upper limits on $m(\nu_e)$, obtained from direct measurements and from $SN1987A$ are around $10 - 20\ eV$. The two other neutrinos, ν_μ and ν_τ, could be heavier than $65\ eV$ only if they decayed fast enough. There are very good reasons to believe that this is not the case[5]. We therefore assume here that $m(\nu_\mu) < 65\ eV$; $m(\nu_\tau) < 65\ eV$.

It is probable that $m(\nu_\tau) \gg m(\nu_\mu) \gg m(\nu_e)$. This would be the case in most models[5] and particularly in almost any theory in which neutrino masses are obtained via the "see-saw" mechanism[6]. We therefore assume here that ν_μ is much lighter than ν_τ. The most likely ratio in a "see-saw" mechanism is:

$$\frac{m(\nu_\mu)}{m(\nu_\tau)} \approx \left[\frac{m(\mu)}{m(\tau)}\right]^2 \approx 3.5 \cdot 10^{-3}$$

and we may probably safely assume[5] that the above mass ratio is somewhere between 10^{-1} and 10^{-3}. However, as long as it is well below one, our arguments are essentially independent of the precise ratio.

If both ν_μ and ν_τ are lighter than $65\ eV$ and if $m(\nu_\tau) \gg m(\nu_\mu)$, *the tau neutrino becomes the leading dark matter candidate.*

We are discussing here a ν_τ mass value which is six orders of magnitude below the best direct limit[7] $m(\nu_\tau) < 35\ MeV$. The only way to probe this mass region are neutrino oscillations involving ν_τ. If $\nu_e,\ \nu_\mu,\ \nu_\tau$ have nonvanishing masses, it is essentially inevitable that neutrino oscillations occur. Such oscillations between two species ν_i and ν_j depend only on $\Delta m^2 = m_i^2 - m_j^2$ and on $\sin^2 2\theta_{ij}$ where $m_i,\ m_j$ are the masses and θ_{ij} is the mixing angle. Since we assumed that $m(\nu_\tau) \gg m(\nu_\mu) \gg m(\nu_e)$, and we are interested in the range $15\ eV \leq m(\nu_\tau) \leq 65\ eV$, we must consider only $\nu_\tau - \nu_\mu$ and $\nu_\tau - \nu_e$ oscillations and we know that, to a good approximation, $\Delta m^2 \approx [m(\nu_\tau)]^2 \approx (200 - 4500)\ eV^2$.

What can we say about the $\nu_\tau - \nu_e$ and the $\nu_\tau - \nu_\mu$ mixing angles $\theta_{\tau e}$ and $\theta_{\tau\mu}$?

The angle $\theta_{\tau e}$ mixes non-adjacent generations. It is analogous to $\theta_{13}^{(q)}$ in the quark sector, which is known to be smaller (but probably not *much* smaller) than 10^{-2}. If $\theta_{\tau e} \approx \theta_{13}^{(q)}$ we expect $\sin^2 2\theta_{\tau e} \leq 4 \cdot 10^{-4}$. The best $\nu_\tau - \nu_e$ oscillation data[8] (as well as the best ν_e "disappearance" data) reach only much larger values of $\sin^2 2\theta_{\tau e}$ and therefore tell us nothing about $m(\nu_\tau)$.

This leaves us with $\nu_\tau - \nu_\mu$ oscillations as the last resort. The angle $\theta_{\tau\mu}$ mixes *adjacent* generations. It is analogous to $\theta_{23}^{(q)}$ in the quark sector. Experimentally, $\sin\theta_{23}^{(q)} = 0.043 \pm 0.008$. If we had $\theta_{\tau\mu} = \theta_{23}^{(q)}$ we would expect $\sin^2 2\theta_{\tau\mu} \approx 0.005 - 0.010$. In the quark sector, we have another mixing angle which connects neighbouring generations: the original Cabibbo angle, obeying $\sin\theta_{12}^{(q)} = 0.22$ or $\sin^2 2\theta_{12}^{(q)} = 0.18$. We do not really know why $\theta_{12}^{(q)} \gg \theta_{23}^{(q)}$. We also do not know the actual value of $\theta_{\tau\mu}$, but on the basis of the above analogy to the quark sector, it might be anywhere, say, between 0.03 and 0.22. the pattern of the charged lepton mass ratios is not very different from that of the quark mass ratios. Most theoretical models expect mixing angles to be somehow related to fermion mass ratios. We may therefore "guess" that the $\theta_{\tau\mu}$ is not far from the above range, possibly below it, but not too far below. Since $\theta_{13}^{(q)}$ is probably near 0.01, and the mixing of "distant" generations is expected to be smaller, we propose a very conservative lower bound $\theta_{\tau\mu} \geq 0.01$. This would mean $\sin^2 2\theta_{\tau\mu} \geq 4 \cdot 10^{-4}$. This bound seems safe although, in principle, arbitrarily small values of $\theta_{\tau\mu}$ cannot be excluded. What we need is, therefore, a $\nu_\tau - \nu_\mu$ oscillation experiment probing the region of Δm^2 between 200 and 4500 eV^2 *and reaching* $\sin^2 2\theta_{\tau\mu}$ *values which are at least as low as* $4 \cdot 10^{-4}$, *preferrably even lower.*

The relevant range in Δm^2 is easily accessible. How far can we go in the other crucial variable, $\sin^2 2\theta_{\tau\mu}$? The best ν_μ "disappearance" experiments reach only[8] $\sin^2 2\theta_{\mu x} \approx 0.05$, far above the required range. By far the best $\nu_\mu - \nu_\tau$ data comes from[9] Fermilab experiment E531, using a hybrid combination of an emulsion and a spectrometer. This experiment, at the 90% confidence level, reached $\sin^2 2\theta_{\tau\mu} \approx 4 \cdot 10^{-3}$, just enough to exclude $\theta_{\tau\mu} = \theta_{23}^{(q)}$. What we now need is an improved experiment that can reach *at least* down to $\sin^2 2\theta_{\tau\mu} \approx 4 \cdot 10^{-4}$, hopefully below it. Such an experiment will provide us with an excellent probe of the possibility that the cosmological dark matter is due to tau-neutrinos.

The E531 experiment[9] was not originally designed to search for ν_τ oscillations. It was a by-product of a charm lifetime experiment. It still achieved, by far, the best $\nu_\mu - \nu_\tau$ oscillation data. In that experiment, approximately 4000 neutrino interactions were detected. A τ candidate was defined as an event with a kink (having $p_T > 125\ MeV$) or a three-prong secondary vertex, no prompt muon (to eliminate standard $\nu_\mu \rightarrow \mu$ events), a negative charged track (to eliminate charm events) and a minimum momentum for the τ ($p_\tau > 2.5\ GeV$, to avoid confusion with other background). With these cuts, most τ events should survive, but no candidate events were found. The experiment, with these cuts, had no background at all. On the basis of zero τ candidates and 1870 ordinary charged current events with an identified μ, the range of $\sin^2 2\theta_{\tau\mu} \leq 4 \cdot 10^{-3}$ was obtained.

Improving the bound by at least an order of magnitude would require a new dedicated experiment using similar techniques. The emulsion seems necessary in order to observe τ tracks with a typical length of a few hundred microns. The spectrometer is needed in order to point towards the suspected vertex. Conceptually, the simplest method would be to repeat the essential features of experiment E531 with a larger number of events. One needs *at least* $20,000$ charged current neutrino interactions with identified muons, preferrably more. Depending on the efficiency and the acceptance for muon identification, this would require a total of at least $30,000$ and probably $40,000$ neutrino interactions.

This can be achieved by any combination of more emulsion, higher beam intensity and longer running time. Assuming that the transverse size of the detector covers most of the width of the neutrino beam, the number of neutrino interactions can be roughly estimated by the following crude formula:

$$\left[\frac{N_{\nu-events}}{1000}\right] = \eta \cdot \left[\frac{E_p}{100\ GeV}\right] \left[\frac{n_p}{10^{18}}\right] \left[\frac{M_{target}}{1\ ton}\right]$$

where E_p and n_p are, respectively, the energy and the number of protons on target and M_{target} is the active target mass. The coefficient η is always of order one and it contains all the details of the beam, detector, etc. In a sample of CERN and Fermilab experiments over the last few years, η-values between 0.6 and 3.5 are obtained. For our purposes, we need to generate a factor of 40 on the left hand side of our equation.

For a single realistic run at Fermilab with 800 GeV protons and 10^{18} protons on target, we therefore have:

$$\left[\frac{N_{\nu-events}}{1000}\right] = 8\eta \cdot \left[\frac{M_{target}}{1\ ton}\right].$$

For $\eta = 1$ we therefore need, say, two runs with at least 2.5 *tons* of emulsion. The situation for the CERN SPS is somewhat better. Because of the higher beam intensity and the higher repetition rate of the machine, and in spite of the lower energy, one obtains for a typical realistic run $E_p = 400\ GeV$, $n_p = 6 \cdot 10^{18}$, yielding:

$$\left[\frac{N_{\nu-events}}{1000}\right] = 24\eta \cdot \left[\frac{M_{target}}{1\ ton}\right].$$

With $\eta = 1$, two such runs with *800 kg* (or 200 liters) of emulsion would do the job. Some of the above numbers could be modified by factors of two, depending on the quality of the neutrino beam, the length of the run, the percentage of machine protons dedicated to the experiment, the distance of the detector, the acceptance and efficiency, etc. In fact, we believe that by optimizing all of these parameters, it may be possible to obtain the required sensitivity with a somewhat smaller amount of emulsion, possibly below 100 liters. For $\eta \approx 3$ (a value which have been achieved in past experiments), one needs approximately 70 liters.

With so many events, scanning the emulsion becomes a difficult and lengthy procedure. Almost all scanned events would involve a muon which is detected by the spectrometer and traced back to a primary vertex in the emulsion. Rejecting these events is a fairly rapid procedure. Selecting the serious candidates and scanning them is the heart of the experiment. A dedicated ν_τ experiment which is not a by-product of something else, may allow a more efficient procedure of selecting candidate events before the cuts.

It may be worthwhile to concentrate on specific decay modes of τ (*e.g.* single hadron or three prongs or electron) and in this way considerably reduce the necessary amount of scanning. The price paid would, of course, be the necessity of having a higher total number of events and therefore a proportionately larger amount of emulsion.

It seems that the best method would be to concentrate on events containing an energetic negative electron and no muon. Such events would include 17% of all τ-leptons, necessitating a total number of events which is six times larger, *i.e.* a total of $250,000$ neutrino interactions. However, such a procedure would eliminate all normal charged current events and almost all neutral current events. The main *physics* background here would come from ν_e contamination in the neutrino beam, usually estimated at 1%. This would yield approximately $1,500$ ν_e-initiated charged current events. Most of the scanned events would be of this type. If the electron comes from the primary vertex in the emulsion, the event should be rejected. If a kink is observed for an e^-, it is a τ^- candidate. In spite of the sixfold increase in the total number of neutrino interactions, the absolute number of scanned events will be reduced by more than an order of magnitude, relative to the case in which one searches for all τ decay modes.

The total amount of emulsion needed for performing this version of the experiment at CERN will have to be of the order of 500 liters (assuming $\eta \approx 3$). The typical effective transverse area of the neutrino beam at a distance of 1 km is a few squared meters (say, $3m^2$), leading to a total emulsion thickness of the order of 15 cm or five radiation lengths. In order to overcome showers, conversions and other facts of life, it would be advantageous to use several layers of emulsion (say, each with a depth of 1 cm) separated by tracking chambers which can help identify the electrons and distinguish them from various types of background. The combined electronic information from the detector behind the emulsion and the chambers between the emulsion plates could help identify true electron events, reducing the total number of scanned events to a few thousands, a number similar to that of experiment E531. Scanning will consist of searching for the relatively simple signature of a kink involving a short track of a few hundred microns followed by a single negative electron.

It is conceivable that the experiment can also be performed with other detectors containing a track-sensitive target. It might be interesting to pursue this possibility. However, the requirement of hundreds of kilograms of active target and the necessity of observing τ-tracks of a few hundred microns are not easily reconciled in other methods. A particularly attractive possibility along these lines is the idea of using scintillating optical fibres in order to detect τ-tracks in a neutrino beam[10].

It is, in principle, also possible to detect τ leptons without explicitely observing their tracks, using much larger active targets and higher event rates. However, at the level of sensitivity required here, background becomes an extremely serious problem in such experiments.

If τ events are discovered, we must be certain that they come from $\nu_\mu' s$ which oscillated into $\nu_\tau' s$ rather than from a ν_τ-contamination which exists in the neutrino beam as a result of direct hadronic decays. The prime candidate for such decays is the $c\bar{s}$ meson, known as F or D_s. The decay of F is the dominant mechanism for producing ν_τ in beam dump experiments. However, for the type of experiment discussed here, at a distance of, say, 1 km, the number of τ events originating from F-decay is expected to be negligible. It *may* become the limiting factor if the $\nu_\mu - \nu_\tau$ oscillation experiment is ever pushed to even lower values of $\sin^2 2\theta_{\tau\mu}$. The background due to "direct ν_τ" can, in principle, be measured by turning down, removing or diverting the focused neutrino beam. At lower energies (such as at CERN), the F background is smaller than at higher energies (such as at Fermilab).

We conclude that the proposed experiment is difficult, but not impossible. The potential reward is, in our opinion, extremely significant.

If the experiment is performed and oscillations are found, it will provide us with information on $m(\nu_\tau)$. A *precise* determination of $m(\nu_\tau)$ may require additional, more complicated, experiments at different distances and/or energies. However, the existence of any $\nu_\mu - \nu_\tau$ oscillations in an experiment of the type discussed here, would indicate that $m(\nu_\tau)$ is *at least* a few $eV's$, making it a very likely candidate for the dark matter. *If $m(\nu_\tau)$ is found to be in the appropriate mass range, it is probably the cosmological dark matter of the universe and it becomes the dominant contributor to its energy!*

If the result is negative down to $\sin^2 2\theta_{\tau\mu} \approx 4 \cdot 10^{-4}$ and if, like E531, the experiment is sensitive to $m(\nu_\tau)$-values as low as a few eV, we face two possibilities: The most likely one is that $m(\nu_\tau)$ is at, or below, few eV and it does not form the cosmological dark matter of the universe. In that case, $m(\nu_\mu)$ is most likely to be at, or below, 10^{-2} eV, just the range required for explaining the solar neutrino puzzle by $\nu_\mu - \nu_e$ oscillations[11]. The dark matter could then be a fourth light neutrino ν_σ at the $15 - 65$ eV mass range or, more likely, an axion or a WIMP.

The second possibility (in the case of a negative result) is that ν_τ is still around $15 - 65$ eV, but for some peculiar reason $\theta_{\tau\mu} < 0.01$, well below the analogous quark angles and possibly even below the angle $\theta_{13}^{(q)}$. This would be a very small angle and it is not suggested by any known model. However, such a situation cannot be ruled out and the only way to cope with it would be to push the experiment even further, to lower values of $\sin^2 2\theta_{\tau\mu}$.

If $m(\nu_\tau)$ is in the $15 - 65$ eV range, $m(\nu_\mu)$ is likely to be approximately around 0.1 eV. In such a case, $\nu_\mu - \nu_e$ oscillations at $\Delta m^2 \approx 10^{-2}$ eV^2 become relevant. Such experiments are being now contemplated. However, even if $\nu_\mu - \nu_e$ oscillations are discovered at $m(\nu_\mu) \approx 0.1$ eV, we still cannot be sure that ν_τ is the cosmological dark matter. Only a direct observation of $\nu_\tau - \nu_\mu$ oscillations will be convincing.

Needless to say, the purpose of this note is not to design an experiment. Any experimental method which would lead to the neccessary values of $\sin^2 2\theta_{\tau\mu}$ and to the discovery of ν_τ at the dark-matter mass range, will be welcome. The above discussion serves only to emphasize the great importance of the proposed measurement and to indicate that the experiment appears to be feasible.

We summarize: among all dark matter theories, only the light neutrino possibility is based on a particle which is known to exist; the most likely light neutrino as a dark matter candidate is ν_τ; if ν_τ is the cosmological dark matter, we must have $m(\nu_\tau) \approx 15 - 65\ eV$; the only practical way to probe this mass is to search for $\nu_\mu - \nu_\tau$ oscillations at $200 < \Delta m^2 < 4500\ eV^2$ down to low values of $\sin^2 2\theta_{\tau\mu}$; a conservative estimate requires $\theta_{\tau\mu} \geq 0.01$ or $\sin^2 2\theta_{\tau\mu} \geq 4 \cdot 10^{-4}$; this calls for an improvement of the best existing experiment by at least one order of magnitude; a dedicated accelerator experiment with an emulsion followed by a spectrometer, detecting at least 40,000 neutrino interactions, should settle the issue; such an experiment does not seem impossible.

We urge experimentalists to perform this crucial experiment, hoping that it can prove that the cosmological dark matter of the universe consists of tau-neutrinos. A positive result will, of course, also be the first experimental observation of a ν_τ, the first observation of neutrino oscillations and the first evidence for non-vanishing neutrino masses. It should be exciting to be the first to observe a new particle and, at the same time, to show that it dominates the mass of the universe!

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