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The Scale Problem in Wormhole Physics

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Abstract

The effective potential of second rank antisymmetric tensor or Goldstone boson field arising from wormhole physics constrains the wormhole scale due to the cosmological energy density problem. This upper bound is barely compatible with the lower bound required for the solution of the cosmological constant. In addition, the effect of wormholes on the strong CP problem is discussed.

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The long-standing cosmological constant problem[1] may find its solution[2] in wormhole physics[3-5] in the Euclidian formalism of quantum gravity[6]. In this paper, we point out the possibility that wormholes contribute to the potential energy of Goldstone boson or second rank antisymmetric tensor field, which in turn constrains the scale of wormhole physics.

In pure gravity, there is no nontrivial solution in the Euclidian time with the flat space-time boundary condition[7]. Recently, however, it has been argued that there appear wormhole solutions when Goldstone bosons or second rank antisymmetric tensor fields (both are called as axions from now on) are coupled to gravity[4,5]. (For Goldstone bosons, there is a difference [5] between tunneling and naive Euclidian time physics. Here we are interested in the tunneling physics.) A wormhole carrying a global charge n is an instanton solution changing the total charge of our Universe by $\Delta Q = n$ as the imaginary time τ goes from $-\infty$ to $+\infty$. As axions are massless, a wormhole does not carry any energy or momentum. Its size and action are proportional to $n/\sqrt{vM_p}$ and nM_p/v, respectively, where v is the vacuum expectation value for symmetry breaking. (The axion field a is the Goldstone boson field $v\theta$ with the spontaneous symmetry breaking scale v or the field strength $H_{\mu\nu\rho}$ of second rank antisymmetric tensor via the duality relation $H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma}\partial^\sigma a/v$ in the flat space-time.) The charge n is quantized for Goldstone bosons because the wave function should be single-valued under the 2π rotation of θ for a simply connected background space. The same is true for the antisymmetric tensor field when there is a further topological interaction with non-abelian gauge fields. As we will be interested in the case $v \ll M_p$, the contribution from wormholes with $|n| \geq 2$ will be exponentially small compared to that of unit charge. From now on we will neglect wormholes with $|n| \geq 2$.

Suppose that a wormhole appearing at the scale v is related to the axion field. (We call v the wormhole scale even though that the wormhole size l_{wh} satisfies $l_{wh}^2 = 1/\sqrt{3\pi}\pi v M_p$.) Then, the wormhole contribution to the cosmological constant is of order $K\exp(-S_E)\alpha$ where S_E is the Euclidian action coming from a wormhole configuration, and K is of order $v^2 M_p^2$ and α (say a real number) is the parameter determined by the wave function of the universe[8]. If there is a cosmological constant[1] Λ_0 from other sources, the wormhole contribution will make the total cosmological constant vanish by choosing $\alpha = -\Lambda_0 \exp S_E/K$, which has been shown by Coleman[2] because the wave function of the Universe peaks at that value. However, the interesting wormhole solutions arise when the second rank antisymmetric tensor or the Goldstone boson are coupled to gravity.

Let us proceed to discuss wormholes arising from the coupling of axions with gravity. The effect of wormholes in this case also is obtained by the summing over all possible combinations of wormhole configurations in the dilute gas approximation[8] with conservation of the total charge including baby universes[2]. It is then intuitively obvious that wormhole physics violates the global symmetry corresponding to the axion since baby universes take away some of the global charge out of our Universe. (Baby universes are considered as small universes created from nothing and collapsing into nothing in Minkowski time except by tunneling from or into big universes. A half of a wormhole connects a baby universe with a big universe.)

Let us consider wormholes with $n = \pm 1$. The creation (annihilation) operators of one half of a wormhole are denoted as $a_+^\dagger$ and $a_-^\dagger$ (a_+ and a_-). The contribution to the potential energy of axions from wormholes is then

$$V_{wh} = K e^{-S_E}(e^{ia/v}C + e^{-ia/v}C^\dagger) \tag{1}$$

where $C = a_+^\dagger + a_-$, $C^\dagger = a_+ + a_-^\dagger$. (Note that S_E is a half of the action for a wormhole solution.) V_{wh} is invariant under the nonlinear global $U(1)$ rotation $a/v \to a/v + \epsilon, C \to e^{-i\epsilon}C$ and $C^\dagger \to e^{i\epsilon}C^\dagger$. In addition, $[C, C^\dagger] = 0$ and our Universe is supposed to settle in an eigenstate $|\alpha>$ of C and $C^\dagger$: $C|\alpha>= \alpha|\alpha>$ and $C^\dagger|\alpha>= \alpha^*|\alpha>$. Then, V_{wh} can be written as

$$V_{wh} = -2K\exp(-S_E)|\alpha|\cos(\frac{a}{v} - \theta_0) \tag{2}$$

where $\alpha = |\alpha|e^{-i\theta_0}$. In Eq.(2), we note that one can always shift a such that $a/v - \theta_0 = 2\pi(integer)$ is the minima of V_{wh}.

As wormhole physics turns on below the axion scale v, the massless axion field a will oscillate and settle down at the minimum of the potential, $\cos(a/v - \theta_0) = 1$. (For the antisymmetric tensor field there is no spontaneous symmetry breaking at the scale v. But that is not important for the following discussion.) Then the total cosmological constant is

$$\Lambda_{tot} = -2K\exp(-S_E)|\alpha| + \Lambda_0 \tag{3}$$

It is assumed that Λ_0 is independent of α since it is the contribution arising from the constants in particle physics, vacuum fluctuations of other fields, supersymmetry breaking, etc. Note that Λ_{tot} can be vanishing only if $\Lambda_0 \geq 0$ and $|\alpha| = \Lambda_0 e^{S_E}/2K$. According to Coleman[2], the probability function for $|\alpha|$ is proportional to $\exp[-|\alpha|^2/2 + \exp(3M_p^4/8\Lambda_{tot}(\alpha))]$. Thus, the wave function of the Universe is peaked at $\Lambda_{tot} = 0$. It is assumed that wormhole physics somehow knows the cosmic evolution, like the damped oscillations of axions and the QCD confinement transition long after its turn-on scale v. Unlike wormholes which may arise independently from axion fields, the total cosmological constant in the present case can be zero only if $\Lambda_0 \geq 0$. In addition, Λ_0 much larger than $K\exp(-S_E)$ can not be cancelled by

$\Lambda_{wh} = Ke^{-S_E}|\alpha|$ if α is required to be of order unity. For the dilute gas approximation to be good, the average number density per unit volume, $l_{wh}^4 K\exp(-S_E)|\alpha|$, should be much less than one. This implies that $|\alpha| \ll \exp(S_E)$. Otherwise, the corrections including that due to interaction between wormholes should be understood. Also it seems very unlikely to us that α could be much bigger than of order unity.

Let us now investigate cosmological consequences of the potential for the axion field a. We will not consider cosmic strings or domain walls. Possible consequences of these are model-dependent and may give additional constraints. Our analysis is similar to that for the conventional coherent axion oscillation[9]. Since we anticipate very weakly coupled axions, the finite temperature effects can be neglected. It is also interesting to note that the wormhole size $1/\sqrt{vM_p}$ is smaller than $1/v$, which guarantees the temperature independent treatment even at the symmetry breaking scale v. Note that for the conventional axion case[10] the axion potential depends crucially on QCD physics such that the axion mass turns on below $\sim 1GeV$ scale. In our present case of wormhole physics, the wormhole potential V_{wh} can be treated as temperature-independent below the scale v.

We will comment on three interesting cases; Case (i): a does not couple to the QCD anomaly, Case(ii): a does couple to the QCD anomaly and Case (iii): a decays to neutrinos. In the cases (i) and (ii) a is considered to be almost stable.

Case (i): a does not couple to the QCD anomaly.

In this case, V_{wh} is the only potential violating the global symmetry and hence is the unique source contributing to the mass of axion. In the standard big bang cosmology, one can take the phase difference $\delta a/v$ of order unity over a horizon distance when a starts to oscillate. In the inflationary scenario, if the symmetry breaking occurs before the inflationary epoch, a is uniform in the inflated bubble but we may take a/v to be of order unity to avoid fine tuning. If the symmetry breaking occurs after the inflationary epoch, the phases over horizon distance are not correlated. Thus in any case, we can take a/v to be of order unity where the classical field a starts to oscillate. (The cosmic time t_i for this starting time is always much later than the inflationary epoch.)

The effect of the wormhole potential turns on below the wormhole scale, but the expanding universe[9] will notice it by making the field start to oscillate after t_i where $3H(t_i) \approx m_a$. The dynamical equation for a, $d^2a/dt^2 + 3H da/dt + m_a^2 a = 0$, leads to the conservation of the total number in a comoving volume after t_i. The energy density in a comoving volume will be $m_a^2 A^2/2$. Here A is the amplitude of oscillation. The energy density at the time t_i is $\rho_i = \pi^2 g_i^* T_i^4/30$ where T_i is the temperature at the cosmic time t_i. Noting that $H^2 = 8\pi\rho/3M_p^2$ and $3H(t_1) = m_a$, we obtain

$$T_i = (\frac{5}{4\pi^3 g_i^*} m_a^2 M_p^2)^{1/4} \tag{4}$$

The condition for the constant number density in the a comoving volume

$$\frac{1}{2} m_{ai} A_i^2 R_i^3 = \frac{1}{2} m_{af} A_f^2 R_f^3 \tag{5}$$

and the constant entropy condition

$$g_i^* T_i^3 R_i^3 = g_f^* T_f^3 R_f^3 \tag{6}$$

determine the present time energy density

$$\rho_f \equiv \frac{1}{2} m_{af}^2 A_f^2 = \frac{1}{2} m_a^2 v^2 (\frac{A_i}{v})^2 \frac{g_f^*}{g_i^*} (\frac{T_f}{T_i})^3 \tag{7}$$

where m_a is treated as a constant in the temperature range $T_i \sim T_f$. From a wormhole solution, one gets $K = 6\pi^3 v^2 M_p^2$ and $S_E = \sqrt{3\pi} M_p/4v$. $m_a^2 = 2K \exp(-S_E)|\alpha|/v^2$ from Eq.(2) becomes

$$m_a^2 = 12\pi^3 M_p^2 \exp(-\sqrt{3\pi}M_p/4v)|\alpha| \tag{8}$$

Using Eq.(8), T_i and ρ_f can be expressed in terms of v. Since the critical energy density is $\rho_c = 2.2 \times 10^{-47} h^2 GeV^4$ where $H_0 = 50hkm/sec/Mpc$ and $1 \leq h \leq 2$, we obtain

$$\begin{aligned} \Omega_a &\equiv \frac{\rho_f}{\rho_c} \\ &= \rho_c^{-1}(\frac{48}{125})^{1/4}\pi^3|\alpha|^{1/4}\frac{v^2T_f^3}{M_p}\exp(-\frac{\sqrt{3\pi}M_p}{16v})\frac{g_f^*}{g_1^{*1/4}}(\frac{A(T_1)}{v})^2 \\ &= 2.6 \times 10^{29}(\frac{v}{M_p})^2\exp(-\frac{\sqrt{3\pi}M_p}{16v})h^{-2}|\alpha|^{1/4} \end{aligned} \tag{9}$$

which can be less than $h^{-2}|\alpha|^{1/4}$ for

$$v \leq 4.15 \times 10^{16} GeV \tag{10}$$

Here we have used $g_i^* = g_f^* = 2$, which makes sense because $T_i \leq 5.4|\alpha|^{1/4}KeV$. In this case $m_a \leq 3.9 \times 10^{-19}|\alpha|^{1/2}eV$. Hence $\alpha = O(1)$ and $h = O(1)$ can settle the cosmological constant $\Lambda_0 = (4.0KeV)^4 \approx 10^{-20}\lambda_{QCD}^4$ and still does not cause the energy density problem. However, we expect that supersymmetry is broken at an energy scale larger than that of strong interaction and so Λ_0 should be larger than λ_{QCD}^4. This conflict is the key observation in this letter. (For curiosity, if $|\alpha|$ is stretched to its maximum possible value e^{S_E} instead of $O(1)$, we obtain $v < 2.4 \times 10^4 hGeV$ from $\Omega_a < 1$.)

Case(ii): a is coupled to the QCD anomaly

The potential gets contributions from both QCD instantons and wormholes,

$$V(a) = -2K\exp(-S_E)|\alpha|\cos(\frac{a}{v} - \theta_0) - f_\pi^2 m_{\pi_0}^2 \zeta \cos(\frac{Na}{v} - \theta_1) + \Lambda_0 \tag{11}$$

where $\zeta = m_u m_d/(m_u+m_d)^2$ and N is the vacuum degeneracy in the axion literature[10]. N is related to the domain wall number N_{DW} according to how some of the degenerate vacua are identified. Here we will take $N = 1$ for an estimate of v. θ_0 and θ_1 are independent. θ_0 is the parameter determined from the wave function of the Universe, and θ_1 is the coefficient of the $F\tilde{F}$ gluon anomaly below the electroweak symmetry breaking scale.

If $\theta_0 = \theta_1$ by a unknown reason, we recover the axion bound $v \leq 10^{12} GeV$ because around this scale wormhole physics is unimportant . Then the field a will dynamically settle $\bar{\theta} = a/v - \theta_1$ at zero.

However, the natural relation is $\theta_0 \neq \theta_1$. In this case, $\bar{\theta}$ will not settle at zero but at the point where $\partial V/\partial a = 0$,

$$2Ke^{-S_B}|\alpha| \sin(\frac{a}{v} - \theta_0) + f_\pi^2 m_{\pi_0}^2 \zeta \sin(\frac{a}{v} - \theta_1) = 0 \tag{12}$$

Since we are interested in a small $\bar{\theta}$, $\bar{\theta} \approx \sin(a/v - \theta_1)$. Thus Eq.(12) determines

$$\bar{\theta} \approx \frac{2Ke^{-S_B}|\alpha|}{f_\pi^2 m_{\pi_0}^2 \zeta} \tag{13}$$

To solve the strong CP problem, $\bar{\theta}$ should be within the limit 10^{-9}. Eq.(13) then leads to an upper bound on v,

$$v < 8.5 \times 10^{16} GeV \tag{14}$$

where we used $m_u/m_d = 0.56$.

In the case (ii), there is another bound on v. The axion energy density constraint[9] will give $v < 10^{12} GeV$ as the QCD instanton contribution dominates in the potential. As the bound on v is much lower than the Planck scale, wormhole physics ceases to be important. The cosmological constant due to wormhole physics for $v \approx 8.5 \times$

$10^{16}GeV$ is of order $10^{-9} \times \lambda_{QCD}^4$ from Eq.(13). Thus, the axion for the solution of the cosmological constant problem cannot be the axion for that of the strong CP problem.

Case(iii): a decays into neutrinos.

One may circumvent the argument given in Case(i) if a decays to light particles quickly enough because the cosmological energy problem for axions will not arise. (Of course, these light particles should not raise the same problem, say by decaying to more light particles.) For definiteness, we assume the coupling $(im/2v)(\psi\psi - \psi^\dagger\psi^\dagger)a$ where ψ is a singlet Majorana neutrino and m is its mass. With $m = fv$, the energy conservation implies $m_a > 2m$. If $f = O(1)$, this decay condition gives $v > 0.9 \times 10^{18}GeV$, where Eq.(8) is used. If f is much smaller than 1, v can be somewhat smaller. The decay width of an axion into two fermions is

$$\Gamma = \frac{m^2 m_a}{64\pi v^2}\left(1 - \frac{2m^2}{m_a^2}\right)\sqrt{1 - 4m^2/m_a^2} \tag{15}$$

For the energy density of axions not to interfere with nucleosynthesis, we require $\Gamma^{-1} < 1sec$ for $f = 1$. Eqs. (8) and (15) then give $v > 1.3 \times 10^{16}GeV$, which is softer than the initial bound $v > 10^{18}GeV$. Hence $v > 10^{18}GeV$ seems to be consistent and then a decays quickly in $10^{-42}sec$ for $m << m_a$. In this case the cosmological constant can be much larger without the cosmological energy density problem. But we will not exploit any explicit model for this case.

In conclusion, the contribution from wormhole physics to the potential of the axion field restricts the wormhole scale v (the wormholes size is of order $1/\sqrt{vM_p}$) by the cosmological energy density and strong CP problems. The axion to solve the strong CP problem has its scale bounded by $v < 10^{12}GeV$ and can not be the one to solve the cosmological constant problem. If axions triggering the wormhole creation are stable, the energy density of axions gives the constraint on the wormhole scale $v \leq 4.15 \times 10^{16} GeV$ for $\alpha = O(1)$. Then, the action is so large ($S_E \geq 56.4$) that only a tiny initial cosmological constant $\Lambda < 10^{-20}\lambda^4_{QCD}$ can be settled to zero in the dilute gas approximation. Finally if axions are made to decay into lighter particles, v can be larger than $10^{18} GeV$ which contains the interesting region for the solution of the cosmological constant for $\alpha = O(1)$.

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