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FERMION MASS HIERARCHY AND THE STRONG CP PROBLEM

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Abstract

We present a model which resolves the strong CP problem by the Peccei-Quinn (PQ) symmetry and at the same time provides a natural explanation to the fermion mass hierarchy. Only the third generation acquires mass at the tree-level due to the PQ symmetry present in the model; the second and first generation fermion masses arise as one-loop and two-loop radiative corrections respectively. A distinguishing feature of the model is that the coupling of the axion to the second and third generation quarks is three to four orders of magnitude larger than in conventional invisible axion models.

The smallness of the CP violating θ parameter of the QCD lagrangian ($\bar{\theta} < 10^{-9}$) is a well known problem of the standard $SU_C(3) \times SU_L(2) \times U(1)_Y$ model (the strong CP problem)[1]. Several solutions have been proposed to resolve this [1], among them the most attractive and elegant solution is provided by Peccei and Quinn (PQ)[2]. They observed that an abelian chiral symmetry which possesses a color anomaly could be used to transform the *theta* term away. The strong interaction is therefore automatically CP conserving. A striking consequence of implementing such a chiral symmetry is the existence of a neutral pseudoscalar boson, the axion, which is essentially the Nambu-Goldstone boson associated with the spontaneous breaking of the PQ symmetry.[3] The simplest way of realizing the PQ symmetry results in an axion which couples rather strongly with quarks and leptons. Axion search experiments have ruled out this possibility.[3] However, models where the PQ symmetry is broken at a high energy scale are not in conflict with experiments. There are two consistent ways of implementing this. One is to introduce extra isodoublet and isosinglet Higgs scalars to the theory as discussed by Zhitnitskii and Dine, Fishler and Srednicki (ZDIS).[4] An alternative way is proposed by Kim and Shifman, Vanishtein and Zakharov (KSVZ)[5] in which vector-like isosinglet quarks and an isosinglet Higgs scalar are introduced. In both cases, the PQ symmetry is broken by the vacuum expectation value (VEV) of the isosinglet scalar at a high energy scale. The combined cosmological and astrophysical constraints require the PQ symmetry breaking scale to be between 10^8 and $10^{12} GeV^1$. The resulting axion, is light, long lived and very weakly coupled to the ordinary fermions.[4,5] It is invisible.

A second unresolved puzzle in the standard model which begs for an explanation is the observed hierarchical pattern in the fermion masses. The up, charm and top quark masses, for example, obey $M_u : M_c : M_t \simeq 1 : 10^2 : 10^4$. Similar hierarchy exists in the down quark and the charged lepton sectors as well.[6] Such vast differences in the masses have prompted many to think that only the third generation fermions have tree-level masses, the second generation acquires mass at the one-loop level, whereas the first generation masses are generated by two-loop radiative corrections. Recently, a model implementing this idea has been proposed based on the left-right symmetric gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.[7] The model requires the existence of vector-like isosinglet quarks (from now on we refer them as exotic quarks) for its success. Models which utilize such exotic quarks in order to explain the smallness of the fermion masses (as compared to the W-boson mass) in terms of a generalized seesaw mechanism are also known. [8]

We first note that both the KSVZ solution to the strong CP problem and the atempt to explain the fermion mass require the existence of exotic quarks. This tempts one to speculate that the observed fermion mass hierarchy is perhaps related in some way to the PQ symmetry. In this letter we present a model which realizes such a connection. The model resolves the strong CP problem by the KSVZ mechanism and provides also a natural explanation to the fermion mass hierarchy. Only the third generation acquires mass at the tree-level due to the PQ symmetry present in the model. The second and first generation fermion masses arise as one-loop and two-loop radiative corrections respectively. A distinguishing feature of the model is that the coupling of the axion to the second and third generation quarks is three to four orders of magnitude larger than in conventional invisible axion models. We now proceed to discuss the model in detail.

As mentioned before, isosinglet quarks and isosinglet Higgs scalar are important to the problem at hand; we introduce one generation of exotic isosinglets quarks (P of electric charge $\frac{2}{3}$e, N of electric charge $-\frac{1}{3}e$) and one isosinglet complex Higgs scalar σ. Their gauge group transformation properties and the PQ charges along with those of the ordinary quarks $\Psi_{iL}^T = (u_i, d_i)_L^T, u_{iR}, d_{iR}$ (where i is the generation indices, L and R indicate left and right handedness respectively) and the isodoublet Higgs ϕ are as follows:

$$\begin{aligned}
\Psi_{iL} &: (3, 2, \frac{1}{3})_{-1}, : u_{iR} : (3, 1, \frac{4}{3})_1, d_{iR} : (3, 1, -\frac{2}{3})_1, \\
P_R &: (2, 1, \frac{4}{3})_{-1}, N_R : (3, 1, -\frac{2}{3})_{-1} \\
P_R &: (3, 1, \frac{4}{3})_1, N_L : (3, 1, -\frac{2}{3})_1 \\
\phi &: (1, 2, -1)_0, \sigma : (1, 1, 0)_2 \ .
\end{aligned} \tag{1}$$

The numbers in the parenthesis indicate the transformation under $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the subscript is the PQ charge. The particle spectrum is similar to the KSVZ model except that in our case the ordinary quarks also transform under PQ chiral symmetry. We note that the PQ charges given above leaves a color anomoly which can be used to make the $\bar{\theta}$ term unphysical. With this quantum numbers assignment, as we will show below. The first two generation quark masses will be zero. Additional ingredients have to be added in order to give radiative masses to the first two generation quarks. For this purpose, we introduce two color triplet, isosinglet Higgs scalars η_1 and η_2. Their transformation properties are:

$$\eta_1 = \left(3, 1, -\frac{2}{3}\right)_2 \quad , \eta_z = \left(3, 1, -\frac{2}{3}\right)_{-2} \tag{2}$$

The Yukana coupling invariant under the gauge symmetry as well as the PQ symmetry is:

$$\begin{aligned} \mathcal{L}_y &= \sum_{i=1,2,3} \left(h_i(\bar{u}_i, \bar{d}_i)_L \phi N_R + h'_i(\bar{u}_i, \bar{d}_i)_L i\tau_2 \phi * P_R \right) \\ &+ \sum_{i,j} \left(H_{ij} \Psi^T_{iL} C^{-1} i\tau_2 \eta_1 \Psi_{jL} + H_{i'j} u^T_{Ri} C^{-1} \eta_2 d_{Rj} \right) \\ &+ f P^T_L C^{-1} \eta_2 N_L + f' P^T_R C^{-1} \eta_1 N_R + H.C \end{aligned} \tag{3}$$

Where C is the charge conjugation operator, Hig is a symmetric matrix. In addition bare mass terms connecting the exotic and ordinary quarks' are also allowed.

$$\mathcal{L} = \sum_{i=1,2,3} (m_i \bar{P}_L u_{Ri} + M'_i \bar{N} L d_{Ri}) + hs \bar{P}_L P_R \sigma + h' s \bar{N}_L N_R \sigma + H.C \tag{4}$$

The most general Higgs potential is:

$$\begin{aligned} V &= -\mu_1^2 |\sigma| - \mu_2^2 |\phi|^2 - \sum_{i=1,2} \mu_i^2 |\eta_i|^2 + \lambda_1 |\sigma|^4 + \lambda_2 |\phi|^4 \\ &+ \gamma_i |\eta_i|^4 + \lambda_{12} |\sigma|^2 |\phi|^2 + \sum_{i=1,2} \gamma_{1i} |\sigma|^2 |y_i|^2 + \sum_{i=1,2} \gamma_{2i} |\phi|^2 |\eta_i|^2 \\ &+ \alpha_{12} |\eta_1|^2 |\eta_2|^2 + \alpha'_{12} (\eta_1 \dagger \eta_2)(\eta_2 \dagger \eta_1) \dagger (\lambda(\eta_1 \dagger \eta_2) \sigma^2 + H.C) \end{aligned} \tag{5}$$

When the scalar Higgs ϕ and σ develope VEV's denoted by $< \phi > = v_d$ and $< \sigma >= v_s$

the quarks acquire masses given by the matrix (for the up-quark)

$$(\bar{u}, \bar{c}, \bar{t}, \bar{p})_L \begin{pmatrix} 0 & 0 & 0 & h_1 v_d \\ 0 & 0 & 0 & h_2 v_d \\ 0 & 0 & 0 & h_3 v_d \\ m_1 & m_2 & m_3 & h_s v_s \end{pmatrix} \begin{pmatrix} u \\ c \\ t \\ p \end{pmatrix}_R \tag{6}$$

and similarly for the down-quarks. It is very easy to diagnalize this mass matrix. We find that at this stage, two of the quarks have zero masses; the two non-zero masses are:

$$\begin{aligned} m_\pm^2 &= \frac{1}{2}\Big(h_S^2 V_S^2 + \sum_i h_i^2 v_2^2 + \sum_i m_i^2 \\ &\pm \sqrt{(h_S v_S^2 + \sum_i h_i^2 v_2^2 + \sum_i m_i^2) - 4 \sum_i m_j^2 \sum_j h_i^2 v_2^2}\Big) \end{aligned} \tag{7}$$

It is clear that m_+^2 is superheavy, of order $(h_S^2 v_s^2)$. While $M-$ is possible to have much lower mass, for example, if $h_s v_s >> m_i^2$. We have $m_-^2 \simeq \frac{m_j^2 h_i^2 v_2^2}{h_j^2 v_s^2}$ of course m_j^2 can not be much lower than $h_s^2 v_s^2$, otherwise m_-^2 would be too small. We identify m_- as the top quark mass.

In order to generate non zero mass for the first two generations, we have to consider loop corrections. The one loop corrections to fermion masses are shown in Figure 1. These diagrams are finite and thus the first two generation quark masses are calculable. There are also one loop corrections to the m_{ip} and m_{pi} entries. However, these corrections are proportional to the tree-level entries. They do not change the form of the mass matrix. We will not show them explicitly and simply denote them by δ_m. From figure 1., we understand why two both η_1 and η_2 are needed. Even though, η_1 and η_2 have different PQ charges, they couple differently to quarks before the breaking of the PQ symmetry. However when the PQ symmetry is broken due to VEV of σ, η_1 and η_2 mix and provide the necessary connection between the left and right handed quarks. Evaluating diagrams in Figure 1., we obtain the one loop corrections to the mass matrix:

$$m^{(1)} = \begin{pmatrix} \delta m_{ij}^{(1)} & \delta m_{iP}^{(1)} \\ \delta m_{Pj}^{(1)} & \delta m_{PP}^{(1)} \end{pmatrix} \tag{8}$$

where

$$\delta m_{ij}^{(1)} = -2\frac{\sum_{l,R} H_{Il} H'_{Rj} h_l M'_R \lambda V_2 V_s^2 m_N}{1\sigma\pi^2} \int_0^\infty \frac{dt}{(t+m_N^2)(t+m_{y_1}^2)(t+m_{y_2}^2)} \tag{9}$$

Where $m_N = h_S v_S, m_{\eta i}, m_\phi$ and M_σ are the tree level masses of particles N, η_i, ϕ^0 and σ respectively.

The one loop corrected mass matrix can be written in the form:

$$\begin{pmatrix} a_i b_j & c_j \\ di & B \end{pmatrix} \tag{10}$$

with $a_i = \sum H_{il} h_l, a_i b_j = \delta m_{ij}^{(1)}, d_i = M_i + \delta m_{ip}^{(1)}, c_j = h_j v_d + \delta m_{ij}^{(1)}$ and $B = h_S v_s + \delta m_{PP}^{(1)}$. It is immediately seen from eq. (9) that there are three non-zero mass eigenstates and one zero mass eigenstate. The factorazability of $\delta m_{(ij)}^{(1)}$ implies that two of its eigenvalues are zero. As a result, the determinent of the one-loop corrected mass matrix is zero. This is exactly what we wanted, the first generation remains massless at this stage. Now we have to show that if we go to higher loops the first generation acquires a non-zero mass. This is not difficult job. One of the two loop diagrams, which guarantee non-zero mass for the first generation are shown in Figure 2. For these diagrams, we have:

$$\delta m_{ij}^{(2)} \propto \sum_{k,k',l,l'} H_{ik} H_{kk'} h_{k'} M' l H'_{lj} \lambda^2 I \tag{11}$$

We have not displayed the complicated two loop integral I. With proper choices of the Yukawa Couplings the correct quark masses and quark mixings can be reproduced. We note that the form of the mass matrix is the same as in Reference [7] although its origin is different. The analysis of the mass eigenvalues and eigenstates are similar to Reference [8], we will not go into details here.

We now turn to discuss axion-quark interaction. The axion field in our model is exactly the same as in the KSVZ model, that is $A = Im\sigma$. At the classical level A is massless but acquires a finite mass through the color anomaly: $m_a = m_\pi f_\pi \sqrt{Z}/(1+Z)v_S$,[1] where $Z = m_u/m_d = 0.56$.[6] The interaction of the axion in our model with the ordinary quarks is different from that of the KSVZ model since in our model the exotic quarks mix with the ordinary quarks.

There are several sources which generate the coupling of the axion to the ordinary quarks. The color anomaly generates the familiar coupling of the axion A to the light quark:[1]

$$\mathcal{L}^{(1)} = \frac{1}{vs}\frac{m_u m_d}{m_u + m_d} i\bar{u}\gamma_s u + \bar{d}\gamma_s d)A \tag{12}$$

The coupling of the axion to two photons induces the following FfA coupling.[9]

$$\mathcal{L}^{(2)} = -\frac{3\alpha_{em}^2 Q_j m_j}{2\pi v_s}\left(6_{i=} \sum_{all\ fermions} C_{PQ}^i Q_i^2 \ln\frac{v_s}{m_j} - \frac{2}{3}\frac{4+Z}{1+Z}\ln\frac{\Lambda}{m_j}\right) i\bar{f}_j\gamma_5 f_j A \tag{13}$$

Where C_{PQ}^i is the PQ charge of the ith fermion f_i of electric charge Q_i and mass m_i.

In our model there is additional coupling of A to the ordiinary quarks through their mixing with the exotic quarks. At the tree-level, the axion only couples to the exotic quarks is given by $(ih_S\bar{P}\gamma_S P + ih'_S\bar{N}\gamma_S N)A$ in the weak interacting basis. After diagonalizing the tree-level mass matrix, we find that the coupling of the axion to the super heavy and the third generation mass eigenstates is (for the up-quark)

$$\mathcal{L}^{(3)} \approx i\left(\frac{m_t}{h_S v_S}\bar{t}_L t_R - \frac{m}{h_S v_S}\bar{P}_L t_R - \frac{h}{h_s u_s}\bar{P}_R t_L + \bar{P}_L P_R\right) A + H.C \tag{14}$$

where $M \simeq \sum_{i=1.2.3} m_i^2$ and $h = \sum_{i=1.2.3} h_i^2$. In obtaining eq.(14). We have assumed, for simplicity, that $h_S v_S >> m, h$. Clearly this A to the third generation is about four of magnitude orders larger than that of eq. (13).

To determine the coupling of the axion to the first two generation quarks, we have to consider the loop corrections. At the one-loop level, the such coupling will be generated through diagram similar to Fig. 1 with the $\eta_1\eta_2$ vertex replaced by $\eta_1 \dagger \eta_2 A$. The couplings can be obtained by simply substituting the $\eta_1\eta_2$ mixing coefficient λV_S^2 by $2\lambda V_s$ in eq.(9). There are similar diagrams which generate axion-ordinary quark-exotic quark couplings. We can write the one-loop corrected axion-quark coupling as:

$$i\sum_{I,J}\bar{q}_I m_{IJ}\frac{1+\gamma_5}{2} q_J A + H.C \tag{15}$$

where $I, J = 1, 2, 3, P$ and

$$M_{IJ} \simeq \frac{1}{v_S}\begin{bmatrix} -2\delta m_{ij}^{(1)} & \delta\Delta_{Pj}^{(1)} \\ \delta\Delta_{ip}^{(1)} & h_S v_S + \delta\Delta^{(1)} \end{bmatrix} \tag{16}$$

Here $\delta \bigtriangleup / v_s$ denotes the one-loop correction to the coupling of A to the exotic and ordinary quarks.

We note that M_{IJ} is not proportional to the one-loop corrected mass matrix. So, in the mass eigenstate basis, M_{IJ} is not diagonal and in general all its entries will be non-zero. Higher order corrections are expected to be small and therefore we can neglect them. There is another implication for M_{IJ} not being diagonal in the mass eigenstates basis. The axion couplings to quarks violates CP if the Yukawa couplings are complex. However since the couplings are very small (of order m_f/v_S), the CP violating effects due to the axion are practically "invisible", After having made these comments, we estimate the coupling of A to the first two generation quarks from eq.(16).

Assuming that $V_L^{(1)}$ and $V_R^{(1)}$ are the unitary matrices which diagonalize the one-loop mass matrix, we have:

$$V_L^{(1)} M_{IJ} V_R^{(2)} \simeq -\frac{2}{V_S} \begin{pmatrix} 0 & & & \cdot \\ & m_c & & \\ & & v_t & \\ & & & h_P \end{pmatrix}$$

$$\begin{pmatrix} 0 & h_j v_d + 3\delta\triangle_{pj}^{(1)} \\ m_j 3\delta\triangle_{ip}^{(1)} & j_s v_s + 3\delta\triangle_{pp}^{(1)} \end{pmatrix} + \frac{2}{V_S} V_L^{(1)} \begin{pmatrix} 0 & 0 & 0 & h_1 v_d \\ 0 & 0 & 0 & h_2 v_d \\ 0 & 0 & 0 & h_3 v_d \\ m_1 & m_2 & m_3 & h_S v_S \end{pmatrix} V_R^{(1)} \tag{17}$$

$$+ h_S V_L^{(1)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} V_R^{(1)}$$

From this expression, we find the axion coupling to the first generation quark to be

$$\mathcal{L}_{first} \simeq i\frac{2m_t}{v_s} v_{L13}^{(3)} v_{R31)}^{(1)} \bar{u}\gamma_s u A. \tag{18}$$

The coupling to the second generation quark is:

$$\mathcal{L}_{second} \simeq -i\frac{2m_c}{v_s}\bar{C}\gamma_5 C A + i\frac{2m_t V_{L23}^{(1)} V_{R32}^{(1)}}{v_s} \tau\gamma_s C A \tag{19}$$

The mixing matrix elements $V^{(1)}_{Lij}$ and $V^{(1)}_{Rij}$ are expected to be of the same order of magnitude as the corresponding mixing matrix of the charged current U_{kM}, then $V^{(1)}_{L13}V^{(1)}_{R31} \sim O(U^2_{Ub}) \sim 10^{-5}$ and $V^{(1)}_{L23}V^{(1)}_{R32} \sim O(U^2_{Ub}) \sim 10^{-3}$. Using these numbers, we find that the dominant contribution to the coupling of A to the first generation quark is from eq.(12) and to the second generation quark is from the second term eq.(19).

Let us compare the axion interaction with the ordinary quarks in our model with that of the $KSVZ$ model.In the latter case the axion interact with the ordinary quarks only through eqs. (12) and (13). It is clear that the couplings of A axion to the first generation quarks are the similar in these two models, while its couplings to the second and third generation quarks are enhanced in our model by three to four orders of magnitude which makes it more accesible to experiments.

The mechanism for explaining the quark mass hierarchy problem can be easily generalized to the leptonic sector. However, lacking any information on the neutrino mass spectrum, we do not purseue it here. If we assign tentatively zero PQ charge to the leptons the charged leptons will acquire masses through their Yukarwa couplings to the isodoublet ϕ as in conventional $SU(3)_c \times SU(2)_L \times U(1)_Y$ model and neutrinos will be exactly massless. If neutrinos have finite but small masses, more particles have to be introudced. Models which connect small neutrino masses with the strong CP problem have been studied in Fef.[10].

To summarize, we have constructed a model which resolves the strong CP problem by the PQ symmetry and at the same time provides a natural explanation to the fermion mass hierarchy. Only the third generation acquires mass at the tree-level, and the second and the first generation quark masses arise as one-loop and two-loop radiative corrections respectively. The axion interaction with the ordinary quarks in this model is different from conventional invisible axion models, with its couping to the second and third generation quarks enhanced by three to four orders of magnitude larger than in conventional invisible axion models.

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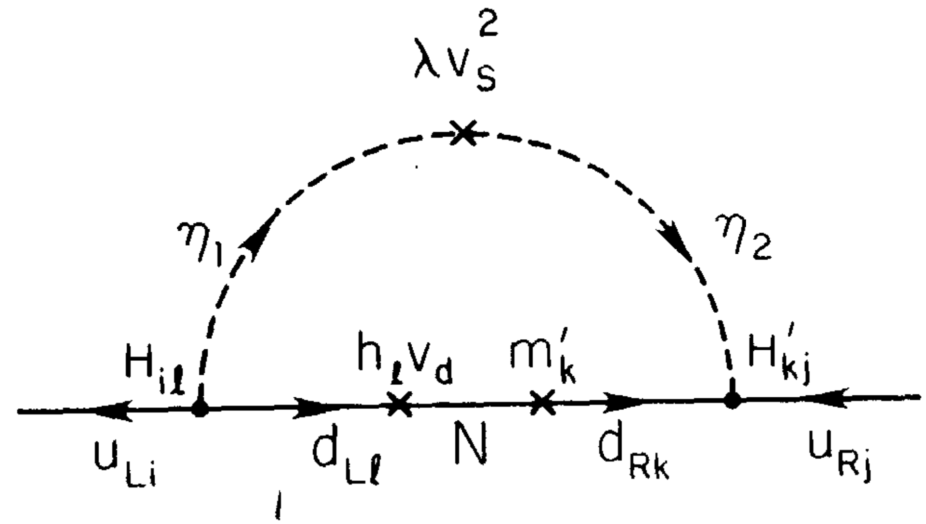
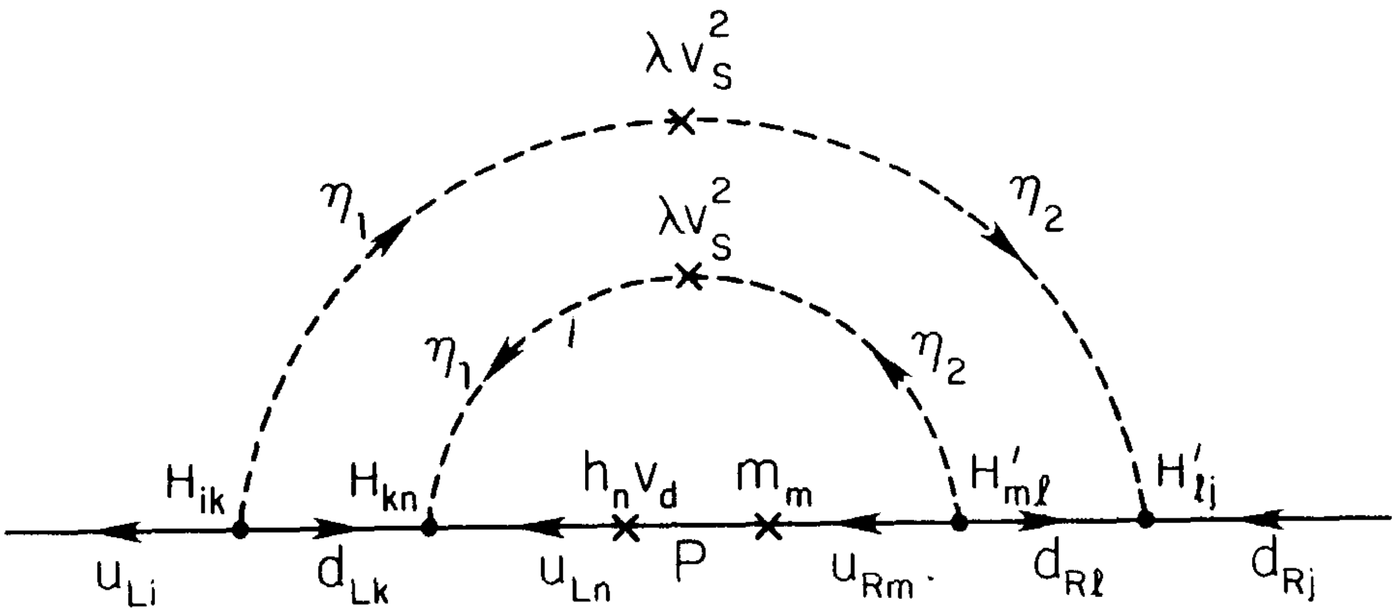
Figure Captions

Figure 1: The one-loop diagram that contribute to the up-quark mass matrix.

Figure 2: Two-loop diagram which generates the first generation quark masses.



λv_S^2
η_1
η_2
$H_{i\ell}$
$h_\ell v_d$
m'_k
H'_{kj}
u_{Li}
$d_{L\ell}$
N
d_{Rk}
u_{Rj}



λv_S^2
λv_S^2
η_1
η_2
η_1
η_2
H_{ik}
H_{kn}
$h_n v_d$
m_m
$H'_{m\ell}$
$H'_{\ell j}$
u_{Li}
d_{Lk}
u_{Ln}
P
u_{Rm}
$d_{R\ell}$
d_{Rj}