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Nonlinear Evolution of the 3-Family Fritzsch Mass Matrices

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Abstract

Renormalization of the Fritzsch mass matrices is studied to compare the predictions of the ansatz imposed at a high scale with low energy data. The evolution effects are expected to be moderate, but their impact can be compared with rather precise mixing data. The viability of the Fritzsch model is found to be little changed by renormalization: with standard Higgs structure it remains marginally acceptable, but with two Higgs doublets and a charged Higgs mass $\simeq 50\ GeV$, agreement with KM mixings and $B-\bar{B}$ mixings is retained with a slightly lowered top mass of $\simeq 90\ GeV$. Remarks concerning the 4-family extension imply that the renormalization effects are very significant and can not be ignored.

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The quark masses, mixings and number of families remain completely unspecified in the standard model (SM) of strong and electroweak interactions, but beyond the SM both the structure and size of the mass matrices should be determined. Even without a detailed model, one can speculate about the mechanisms that generate certain structures for the mass matrices, and typically the number of parameters will be constrained. If there are fewer degrees of freedom than physical quantities to be explained, relations between the masses and mixings can be established. In this spirit, such mechanisms should set the structure of the mass matrices at the scale where contact with the new physics is to be made. Hence any phenomenologically-inspired ansatz for the mass matrices has to be evolved from that scale where it is postulated down to a scale where the experimental data is presently known, 1 GeV, for example. Of special importance are such changes that affect the structure of the ansatz.

One popular choice for the 3-family quark mass matrices $\mathbf{M}_U$ and $\mathbf{M}_D$ is that suggested by Fritzsch[1] on the basis of chiral-symmetry breaking in stages and "nearest-neighbor" interactions:

$$\mathbf{M}_U = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}, \qquad \mathbf{M}_D = \begin{pmatrix} 0 & A' & 0 \\ A'^* & 0 & B' \\ 0 & B'^* & C' \end{pmatrix} \tag{1}$$

where the Hermitian mass matrices are expressed in terms of six real parameters and two phases (the up mass matrix can be taken to be real). Since there are only eight parameters, there must be two constraints for the six quark masses and three mixing angles plus one $CP-$violating phase. Knowledge of the four independent Kobayashi - Maskawa[2] (KM) mixing parameters plus five light quark masses then leads to information about the top quark mass. Imposition of additional constraints from the recent $B_d - \bar{B}_d$ mixing results[3] suggests, however, that the Fritzsch model with standard Higgs structure is marginally viable, unless two Higgs doublets are included.[4]

The analysis referenced above,[4] however, like all others does not take into account the renormalization effects that arise when the mass matrices are evolved from the high chiral-symmetry breaking scale Λ_{SB} down to the 1 GeV scale where observations are made. In this paper, we take into account the nonlinear terms in the renormalization group equations (RGE), which in fact change the form of the $\mathbf{M}_U$ and $\mathbf{M}_D$ matrices, and present detailed numerical results for the experimental comparisons.

At the one loop level there are three types of corrections to the Yukawa coupling vertices: there are loop contributions involving virtual gauge particles (G_Y), fermion loops appearing in the scalar leg (T_Y), and Higgs exchange contributions (S_Y) as shown in Fig. 1. In terms of these three types of corrections, the general RGE's can be written as[5]

$$-16\pi^2 \frac{d\mathbf{M}_Y}{dt} = \left(G_Y\,\mathbf{1} - T_Y\,\mathbf{1} - \frac{3}{2}\mathbf{S}_Y\right)\mathbf{M}_Y\ , \quad Y = U,\ D \ \text{ for up and down} \tag{2a}$$

where

$$t = ln\left(\frac{\mu}{1\ GeV}\right)\ , \qquad \mathbf{M}_Y = \frac{\mathbf{m}_Y}{v}\ , \qquad v \simeq 175\ GeV \tag{2b}$$

and in the SM with minimal Higgs structure, for example,

$$\begin{aligned} G_U &= G_D + g_1^2 = 8g_3^2 + \tfrac{9}{4}g_2^2 + \tfrac{17}{12}g_1^2 \\ T_U &= T_D = 3\ Trace\left(\mathbf{M}_U\mathbf{M}_U^\dagger + \mathbf{M}_D\mathbf{M}_D^\dagger\right) \\ \mathbf{S}_U &= -\,\mathbf{S}_D = \mathbf{M}_U\mathbf{M}_U^\dagger - \mathbf{M}_D\mathbf{M}_D^\dagger \end{aligned} \tag{2c}$$

Note that the first two terms in (2a) are proportional to unity in flavor space and their effect is to rescale the matrices $\mathbf{M}_Y$. Only the third term, $\mathbf{S}_Y$, evolves the matrix structure. Since the over all scale is not explained, we can drop the first two terms and absorb their effect in a rescaling of the matrices. If we implicitly assume this to be done, we are left with the (non-Hermitian) equations that evolve the structure:

$$\frac{32\pi^2}{3}\frac{d\mathbf{M}_Y}{dt} = \mathbf{S}_Y \mathbf{M}_Y \tag{3}$$

The elements of S_Y are quadratic in elements of the Yukawa coupling matrices which are proportional to masses normalized by the vacuum expectation value. This makes clear that renormalization effects do not change the structure if the spectrum is very light compared to the vacuum expectation value,† $v = 175\ GeV$. In this sense the simple treatment without renormalization is then the right procedure. On the other hand, we know that the top mass has to be rather heavy, and these effects can then no longer be ignored.

The whole task then consists of two non-commuting subtasks. One is the diagonalization of a given structural ansatz and extraction of the masses and mixings at a given scale; the other is the solution of the RGE's. If one can solve the general form of the evolution equations for the physical quantities without reference to any particular ansatz, it is possible to relate the low energy data to the higher scale. The phenomenological ansatz such as (1) is diagonalized in the usual way, and the two pieces are put together at the high scale. Another, of course equivalent, way is to solve the RGE's for the phenomenological ansatz which converts the original matrices (since there is no symmetry that protects them) to new matrices at 1 GeV. With the usual techniques, the physical quantities are then extracted from the modified matrices and compared with data. The advantage of the first approach is that the diagonalization of the ansatz is the same as before; only the data are modified by the evolution compared to the original values. The second approach has the advantage of showing the way in which the original matrices are modified.

We concentrate on the first method for the moment. With minimal Higgs structure

†The relevant v is defined without the usual factor of $\sqrt{2}$.

and the definition

$$\beta(t) = exp\left(\frac{3}{16\pi^2}\int_0^t [m_t(t')/v]^2\, dt'\right) \simeq exp\left[\frac{3}{16\pi^2}\frac{m_t^2}{v^2} ln\left(\frac{\mu}{1\ GeV}\right)\right] \tag{4}$$

the results of Ref. 5 can be used to express the KM mixing matrix to a very good approximation by

$$\mathbf{V}(t) = \begin{pmatrix} \bar{V}_{ud} & \bar{V}_{us} & \bar{V}_{ub}\beta^{\frac{1}{2}} \\ \bar{V}_{cd} & \bar{V}_{cs} - \frac{1}{2}\bar{V}_{cb}(\beta^{\frac{1}{2}} - 1) & \bar{V}_{cb}\beta^{\frac{1}{2}} \\ \bar{V}_{td}\beta^{\frac{1}{2}} & \bar{V}_{ts}\beta^{\frac{1}{2}} & \bar{V}_{tb} - \frac{1}{2}\bar{V}_{ts}(\beta^{\frac{1}{2}} - 1) \end{pmatrix} \tag{5}$$

and

$$\begin{aligned} m_u(t) &= r_U\bar{m}_u\ , \quad m_c(t) = r_U\bar{m}_c\ , \quad m_t(t) = r_U\bar{m}_t\beta^{\frac{1}{2}} \\ m_d(t) &= r_D\bar{m}_d\ , \quad m_s(t) = r_D\bar{m}_s\ , \quad m_b(t) = r_D\bar{m}_b\beta^{-\frac{1}{2}} \end{aligned} \tag{6}$$

where a bar denotes quantities at $\mu = 1$ GeV and the factors r_Y correspond to the overall rescalings from G_Y and T_Y, which we shall drop immediately. In the model with two Higgs doublets and $v_1 = v_2 = v/\sqrt{2}$, $\beta^{\frac{1}{2}}$ in the matrix of (5) is replaced by $\beta^{-\frac{1}{2}}$, while the factors $\beta^{\frac{1}{2}}$ and $\beta^{-\frac{1}{2}}$ in (6) are replaced by β and $\beta^{\frac{1}{2}}$, respectively. If we pick a certain set of masses with errors, the top mass selected and the scale $\mu = \Lambda_{SB}$ will determine β and, therefore, all data and errors at Λ_{SB}.

Next we solve the Fritzsch ansatz in (1) by using well known matrix techniques to express the elements of the mass matrices in terms of eigenvalues and some free parameters. Since known masses and the top mass are input, we look for the KM mixing predictions in terms of the masses and other free parameters, i.e., the phase angles $\phi_{A'}$ and $\phi_{B'}$. Comparison with the range of experimentally-allowed mixings reveals that the V_{us} and V_{cb} elements give the strongest constraints on the allowed masses.[‡] For a first discussion[6] we expand the exact results[§] for $|V_{us}|^2$ and $|V_{cb}|^2$ in

[‡]There is a very tiny area where the V_{cs} element is a bit more restrictive, but we shall ignore this.

[§]See ref. 4. Alternatively, we have also used a method based on Sturm sequences which leads systematically to very compact results.

the small quantities $\frac{m_u}{m_c}, \frac{m_c}{m_t}, \frac{m_d}{m_s}, \frac{m_s}{m_b}$ and find in leading order

$$|V_{us}|^2 \simeq \frac{m_u}{m_c} + \frac{m_d}{m_s} - 2\sqrt{\frac{m_u}{m_c}\frac{m_d}{m_s}}\cos\phi_{A'} \tag{7a}$$

$$|V_{cb}|^2 \simeq \frac{m_c}{m_t} + \frac{m_s}{m_b} - 2\sqrt{\frac{m_c}{m_t}\frac{m_s}{m_b}}\cos\phi_{B'} \tag{7b}$$

The simple geometric interpretations of such equations are triangles where the sides have lengths $|V_{us}|$, $\sqrt{\frac{m_u}{m_c}}$, $\sqrt{\frac{m_d}{m_s}}$ (or $|V_{cb}|$, $\sqrt{\frac{m_c}{m_t}}$, $\sqrt{\frac{m_s}{m_b}}$ for the second equation) and the angle opposite the side proportional to $|V_{us}|$ ($|V_{cb}|$) is $\phi_{A'}$ ($\phi_{B'}$).

For the first of these two equations all numbers are fixed (including some errors), and a solution for $\phi_{A'}$ exists. The range of $\phi_{A'}$ is completely determined by the errors of the quantities that enter this equation. The second equation has m_t as a free variable; therefore, we find a range of allowed top masses as a function of $\phi_{B'}$, even without experimental errors. Adding the errors will make the allowed range wider. From the geometrical interpretation, it is clear that with the two sides $|V_{cb}|$ and $\sqrt{\frac{m_s}{m_b}}$ fixed in length and the angle between them arbitrary, the third side of length $\sqrt{\frac{m_c}{m_t}}$ ranges between

$$\sqrt{\frac{m_s}{m_b}} - |V_{cb}| \lesssim \sqrt{\frac{m_c}{m_t}} \lesssim \sqrt{\frac{m_s}{m_b}} + |V_{cb}| \tag{8}$$

It is then straightforward to replace the unevolved quantities by their evolved values according to (5) and (6) above to determine the bounds on m_t with the nonlinear renormalization taken into account. As a result (7a) will not change at all, while (7b) will get corrections deforming all three sides of the triangle. In the discussion to be presented later on we shall quantify these results by giving a more exact treatment and extract the physical top mass.

As a second, alternative approach to the renormalization issue, we consider the evolution of the Fritzsch mass matrices in (1) down to the 1 GeV scale. The KM mixing matrix is then computed from the evolved mass matrices at 1 GeV and compared directly with the experimental information existing at that scale.

For this purpose, we rewrite the RGE's in terms of the explicitly Hermitian matrices $\mathbf{H}_Y = \mathbf{M}_Y \mathbf{M}_Y^\dagger$ according to

$$\frac{32\pi^2}{3}\frac{d}{dt}\mathbf{H}_Y = \mathbf{S}_Y \mathbf{H}_Y + \mathbf{H}_Y \mathbf{S}_Y \tag{9}$$

since, as we shall see, the mass matrices can evolve into non-Hermitian forms. To a good approximation we can hold $\mathbf{S}_Y$ constant and evaluate it at t_{SB}, corresponding to $\mu = \Lambda_{SB}$, according to

$$\begin{aligned} \mathbf{S}_Y(t_{SB}) &= b_Y \mathbf{M}_U \mathbf{M}_U^\dagger + c_Y \mathbf{M}_D \mathbf{M}_D^\dagger \\ &\simeq b_Y \left(C^2 \mathbf{E}_3 + BC\mathbf{E}_{23} + BC\mathbf{E}_{32} \right) \end{aligned} \tag{10a}$$

with

$$\begin{aligned} b_U &= -b_D = 1 \qquad \text{minimal Higgs model} \\ b_U &= 3b_D = 1 \qquad \text{double Higgs model} \end{aligned} \tag{10b}$$

where $\mathbf{M}_U$ and $\mathbf{M}_D$ are given in (1), and $\mathbf{E}_3, \mathbf{E}_{23}$ and $\mathbf{E}_{32}$ are the projection matrices on the 33, 23 and 32 elements, respectively. The last form is obtained by observation in retrospect that $C^2 \sim 0.33, BC \sim 0.03$ and the omitted terms such as C'^2, etc. are at least one order of magnitude smaller. We can then successively decouple and solve the differential equations in (9) and find to leading order in each element

$$\mathbf{H}_U(t) \simeq \begin{pmatrix} A^2 & \frac{B^2}{C}A(\gamma-1) & AB\gamma \\ \frac{B^2}{C}A(\gamma-1) & B^2\gamma^2 + A^2 + \frac{B^4}{C^2}(\gamma-1)^2 & BC\gamma^2 + \frac{B^3}{C}\gamma(\gamma-1) \\ AB\gamma & BC\gamma^2 + \frac{B^3}{C}\gamma(\gamma-1) & (C^2+B^2)\gamma^2 \end{pmatrix} \tag{11a}$$

$$\mathbf{H}_D(t) \simeq \begin{pmatrix} |A'|^2 & \frac{B}{C}B'A'(\gamma'-1) & A'B'\gamma' \\ \frac{B}{C}B'^*A'^*(\gamma'-1) & \begin{array}{c} |B' + \frac{B}{C}C'(\gamma'-1)|^2 + |A'|^2 \\ +\frac{B^2}{C^2}|B'|^2(\gamma'-1)^2 \end{array} & \begin{array}{c} B'C'\gamma' \\ +\frac{B}{C}(C'^2 + |B'|^2)\gamma'(\gamma'-1) \end{array} \\ A'^*B'^*\gamma' & \begin{array}{c} B'^*C'\gamma' \\ +\frac{B}{C}(C'^2+|B'|^2)\gamma'(\gamma'-1) \end{array} & (C'^2+|B'|^2)\gamma'^2 \end{pmatrix} \tag{11b}$$

in terms of

$$\gamma(t),\ \gamma'(t) \simeq \exp\left\{\frac{3}{32\pi^2} b_{U,D} \int_{t_{SB}}^{t} dt' C^2(t')\right\}, \qquad t_{SB} = ln\left(\frac{\Lambda_{SB}}{1GeV}\right) \tag{11c}$$

It is then a simple matter to check that the renormalized structure of the up and down mass matrices is of the form

$$\mathbf{M}_U(t) \simeq \begin{pmatrix} 0 & A & 0 \\ A & \frac{B^2}{C}(\gamma - 1) & B\gamma \\ 0 & B\gamma & C\gamma \end{pmatrix} \tag{12a}$$

$$\mathbf{M}_D(t) \simeq \begin{pmatrix} 0 & A' & 0 \\ A'^* & \frac{B}{C}B'^*(\gamma' - 1) & B' + \frac{B}{C}C'(\gamma' - 1) \\ 0 & B'^*\gamma' & C'\gamma' \end{pmatrix} \tag{12b}$$

in the approximations made in the determination of $\mathbf{H}_U$ and $\mathbf{H}_D$. Note that $\mathbf{M}_U$ remains Hermitian while $\mathbf{M}_D$ becomes non-Hermitian due to the asymmetrical nature of the approximations in (10), reflecting the fact that the top quark mass is by far the largest quark mass in the 3-family scenario.

By diagonalizing the renormalized Hermitian matrices $\mathbf{M}_U$ and $\mathbf{H}_D$ at $\mu = 1$ GeV and identifying the mass eigenvalues as $\lambda_u = diag(m_u, -m_c, m_t)/v$ and $\lambda_D^2 = diag(m_d^2, m_s^2, m_b^2)/v^2$, respectively, we can relate the matrix elements in $\mathbf{M}_U(\Lambda_{SB})$ and $\mathbf{M}_D(\Lambda_{SB})$ of (1) to the quark masses determined at 1 GeV. The invariant traces of $\mathbf{M}_U, \mathbf{M}_U^2, \mathbf{H}_D$ and $\mathbf{H}_D^2$ and determinants of $\mathbf{M}_U$ and $\mathbf{H}_D$ then lead to

$$A^2 = m_t m_c m_u/(C\gamma v^3) \tag{13a}$$

$$B^2\gamma = (m_t m_c + m_c m_u - m_t m_u)/v^2 - A^2 \tag{13b}$$

$$C = \text{solution of cubic equation} \simeq (m_t - m_c + m_u)/(\gamma v) \tag{13c}$$

$$|A'|^2 = m_b m_s m_d/(C'\gamma' v^3) \tag{13d}$$

$$|B'|^2 = \frac{1}{2\gamma'^2}\left[\frac{B^2}{C^2}|A'|^2(\gamma' - 1)^2 + \left\{\frac{B^4}{C^4}|A'|^4(\gamma' - 1)^4 + 4\gamma'^2\left[(m_b^2 m_s^2 + m_b^2 m_d^2\right.\right.\right.$$

$$+\ m_s^2 m_d^2)/v^4 + |A'|^4 - |A'|^2 (C'^2 v^2 \gamma'^2 + m_b^2 + m_s^2 + m_d^2)/v^2 \Big] \Big\}^{1/2} \Big] \qquad (13e)$$

$$C' = \text{solution of 12th order polynomial equation} \simeq (m_b - m_s + m_d)/(\gamma' v) \qquad (13f)$$

Note that we recover the unevolved results in the limit $\gamma = \gamma' = 1$. It is important to recognize the roots C and C' of the polynomial equations referred to in (13c) and (13f) must be extracted with high precision.

We can then apply the projection operator technique of Jarlskog[7] to the renormalized matrices $\mathbf{H}_U$ and $\mathbf{H}_D$ to calculate the squares of the KM matrix elements at $\mu = 1$ GeV,

$$|V_{\alpha j}|^2 = Tr\left[P_\alpha P'_j\right] \qquad (14a)$$

where the projection operators P_α and P'_j are related to Vandermonde matrices and determinants

$$P_1 = \left(\lambda_2^2\, \mathbf{1} - \mathbf{H}_U\right)\left(\lambda_3^2\, \mathbf{1} - \mathbf{H}_U\right) / \left[(\lambda_2^2 - \lambda_1^2)(\lambda_3^2 - \lambda_1^2)\right] \ , \quad \text{etc.} \qquad (14b)$$

as in Ref. 7.

Finally for both approaches we must take into account the additional evolution of the running mass of the top quark from $m_t(1GeV)$ to $m_t(m_t)$. This can be expressed by the relation

$$m_t(m_t) = \gamma_{m_t} m_t(m_t)\ |_{linear\ evolution} \qquad (15a)$$

where

$$\gamma_{m_t} \simeq \exp\left\{\frac{3}{32\pi^2} b_U \int_0^{t_{m_t}} dt' C^2(t')\right\} \qquad (15b)$$

The expression for m_t plays an important role in the calculation of the allowed $B - \bar{B}$ mixing region for which

$$m_t^2 |V_{tb} V_{td}^*|^2 R(z_t, z_\eta, v_2/v_1) \simeq (2.0 \pm 0.5) \frac{(0.140)^2}{B_B f_B^2} \qquad (16)$$

as determined by the ARGUS results.[3] The physical mass of the top quark, as computed from the running mass with the first order QCD correction

$$m_t^{phys} = m_t(m_t)\left[1 + \frac{4}{3\pi}\alpha_s\right] \tag{17}$$

then receives a similar correction. For more detailed discussion of the issues raised in this paragraph we refer the interested reader to Ref. 4.

We have now introduced the tools needed to compare the Fritzsch model predictions with low-energy data when renormalization effects are taken into account. This is of considerable interest, for the simple treatment without renormalization calls into question the validity of the model, at least when only the minimal Higgs structure is present.[4] For our renormalization studies, we have applied two different approaches as described above. In the first method where the diagonalization is carried out at the high scale, the errors introduced by the approximations arise mainly from the lowest-order expansions (which have been carried out to higher order although the simplicity is lost immediately) in Eq. (7), while those leading to Eqs. (5) and (6) are negligibly small. In the second method where the diagonalization occurred at the low scale, the basic approximation retained only the large elements in $\mathbf{S}_Y$ as indicated in (10b) and held them fixed in order to find an approximate analytical solution to the RGE's in (9). A full numerical simulation of the problem, where both integration and diagonalization were carried out numerically, confirmed the reliability of those approximations.

To obtain detailed numerical results, we use the Gasser - Leutwyler[8] determination of the quark masses at 1 GeV and the KM mixing matrix evaluation of Schubert.[9] For purposes of illustration, in Fig. 2 we plot the phase angle $\phi_{B'}$ of $\mathbf{M}_D$ vs. $m_t(1GeV)$ and m_t^{phys} for the KM-allowed annulus and $B - \bar{B}$ mixing band for the standard Higgs and double Higgs model, with and without evolution. Figure 2a corresponds to a special case considered earlier in Ref. 4, where the quark masses selected at 1

GeV and scalar Higgs mass are indicated in the figure caption.

The KM-allowed region without evolution is independent of the particular Higgs structure chosen, but the $B-\bar{B}$ mixing bands depend on the Higgs structure through the underlying box diagrams[10] involving W boson exchange and also scalar Higgs exchange in the case of the double Higgs model. With renormalization taken into account, the KM-allowed region now depends on the particular Higgs structure through the β-dependence of (5) and (6) and through the γ- and γ'-dependence on b_Y in (10). The decrease of $[m_t(1GeV)]_{max}$ from 152 GeV in Fig. 2a to 140 GeV in Fig. 2b for the SM and to 135 GeV in Fig. 2c for the DHM can be understood qualitatively if one compares the approximate upper bounds obtained from the evolved form of (8)

$$\bar{m}_t \lesssim \begin{cases} \bar{m}_c\beta^{-1}\left|\sqrt{\frac{\bar{m}_s}{\bar{m}_b}} - |\bar{V}_{cb}|\beta^{\frac{1}{4}}\right|^{-2}, & \text{(SM)} \\ \bar{m}_c\beta^{-\frac{2}{3}}\left|\sqrt{\frac{\bar{m}_s}{\bar{m}_b}} - |\bar{V}_{cb}|\beta^{-\frac{1}{6}}\right|^{-2}, & \text{(DHM)} \end{cases} \tag{18}$$

with that for the unevolved form with $\beta = 1$. The $B-\bar{B}$ mixing bands also move downward, since the running mass of the top quark which enters Eq. (16) is increased by the extra factor of γ_{m_t} in (15). The m_t^{phys} scales at the bottom of Figs. 2b and 2c are also contracted relative to that for Fig. 2a.

We see that the net effect of the mass renormalization of the Fritzsch matrices is to modify the large 33, 23 and 32 elements and to introduce nonvanishing 22 elements, which can be comparable in magnitude to the 12 and 21 elements for the case of $\mathbf{M}_U$. These modifications result in changes to the KM-allowed region and $B-\bar{B}$ mixing bands as indicated in Fig. 2 and the paragraph above, such that the overlap of the two experimental regions is not improved for the Fritzsch model. Unless the experimental results obtained for $B-\bar{B}$ mixing decrease, or the theoretical estimates for the product $B_B f_B^2$ involving the bag parameter and decay constant increase, the 3-family Fritzsch model with minimal Higgs structure will be ruled out. With two Higgs doublets and a charged Higgs scalar mass ~ 50 GeV, the model is viable with a top mass $m_t^{phys} \sim 90$ GeV.

It is perhaps surprising that the renormalization effects due to a large top quark mass ~ 90 GeV do not alter the situation more. The reasons that this is not so can be traced to the facts that $m_t^{phys}/v \sim 0.5$ is still relatively small, i.e., $\beta \sim 1.15$ for $\Lambda_{SB} = 10^5$ GeV, and that the β and γ_{m_t} corrections to (16) and (18) are correlated and tend to rescale the unevolved plots without greatly influencing the validity of the model. The latter is an artifact of the Fritzsch model with its hierarchical ansatz and will not be true in general. In particular, extensions to 4 families should lead to strong nonlinear renormalization effects, for the fourth family top and bottom masses are expected to be $O(v)$, if they exist. Even without a detailed analysis, it is clear that the evolution of heavy quarks is then totally dominated by the nonlinear fixed points. Additionally, the correlations found for the 3-family Fritzsch ansatz will undoubtedly be lost. Therefore, analyses of 4-family models without renormalization considerations must be regarded as highly suspect.¶

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¶A first attempt to include the renormalization effects in 4-family models has been made in Ref. 11.

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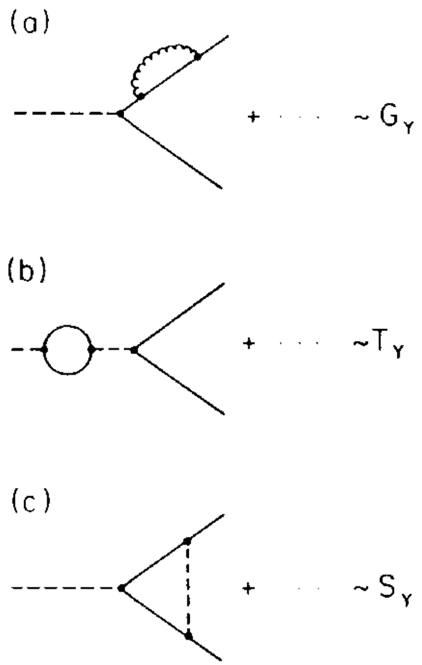
Figure Captions

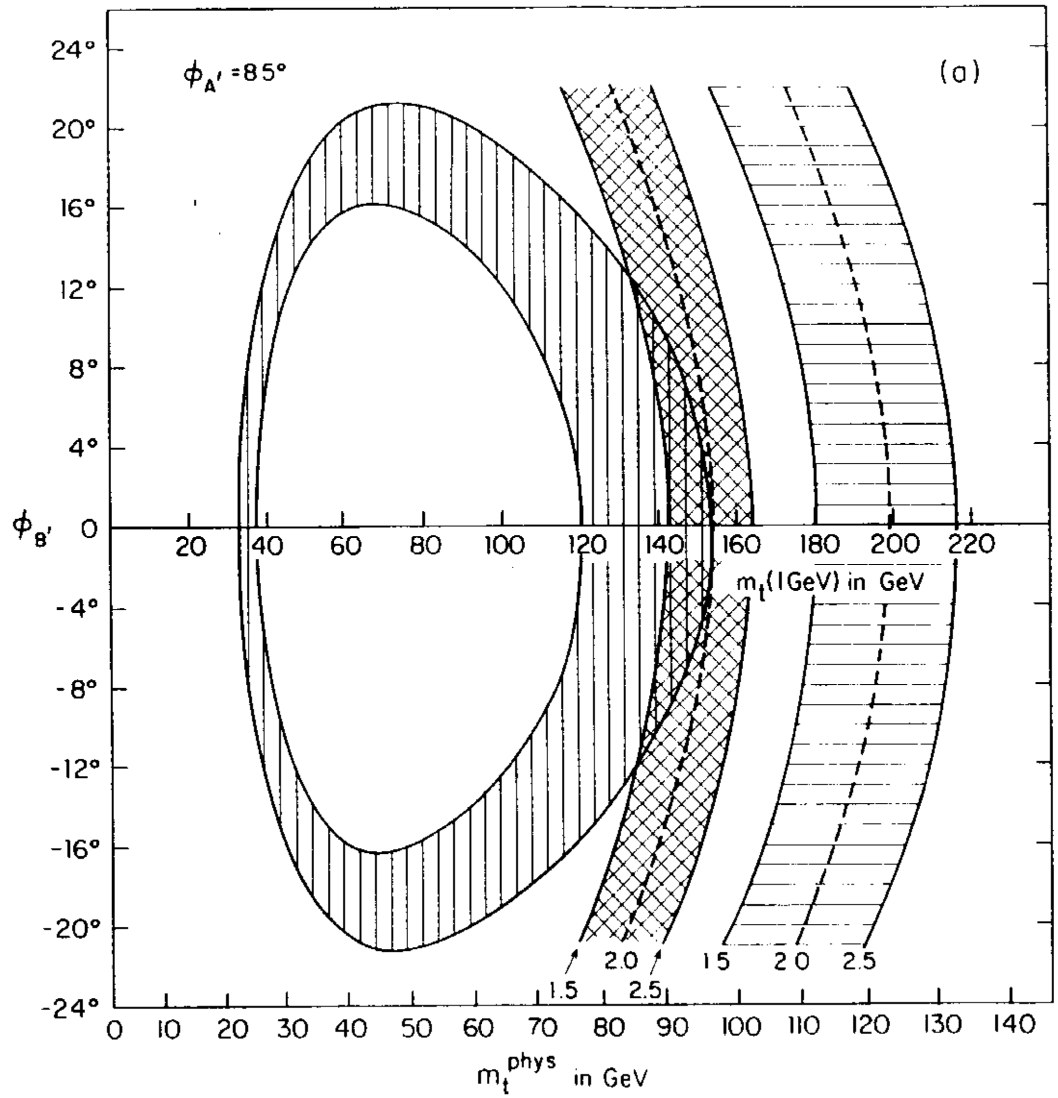
Figure 1: Loop corrections to the Yukawa coupling vertices involving (a) virtual gauge particles, (b) fermions in the scalar leg and (c) scalar Higgs exchange.

Figure 2: Phase angle $\phi_{B'}$ vs. $m_t(1GeV)$ and m_t^{phys} plots for the Fritzsch model showing the physically- allowed KM annular region and the $B_d - \bar{B}_d$ mixing bands single-hatched for the standard Higgs model and double-hatched for the two-doublet Higgs model. Here (a) refers to no evolution, (b) to evolution with standard Higgs structure and (c) to evolution with two-doublet Higgs structure. The 1 GeV quark masses chosen for the graphs are $\bar{m}_u = 3.5$ MeV, $\bar{m}_d = 6.1$ MeV, $\bar{m}_s = 120$ MeV, $\bar{m}_c = 1.35$ GeV and $\bar{m}_b = 5.3$ GeV along with a charged scalar Higgs mass of 50 GeV.



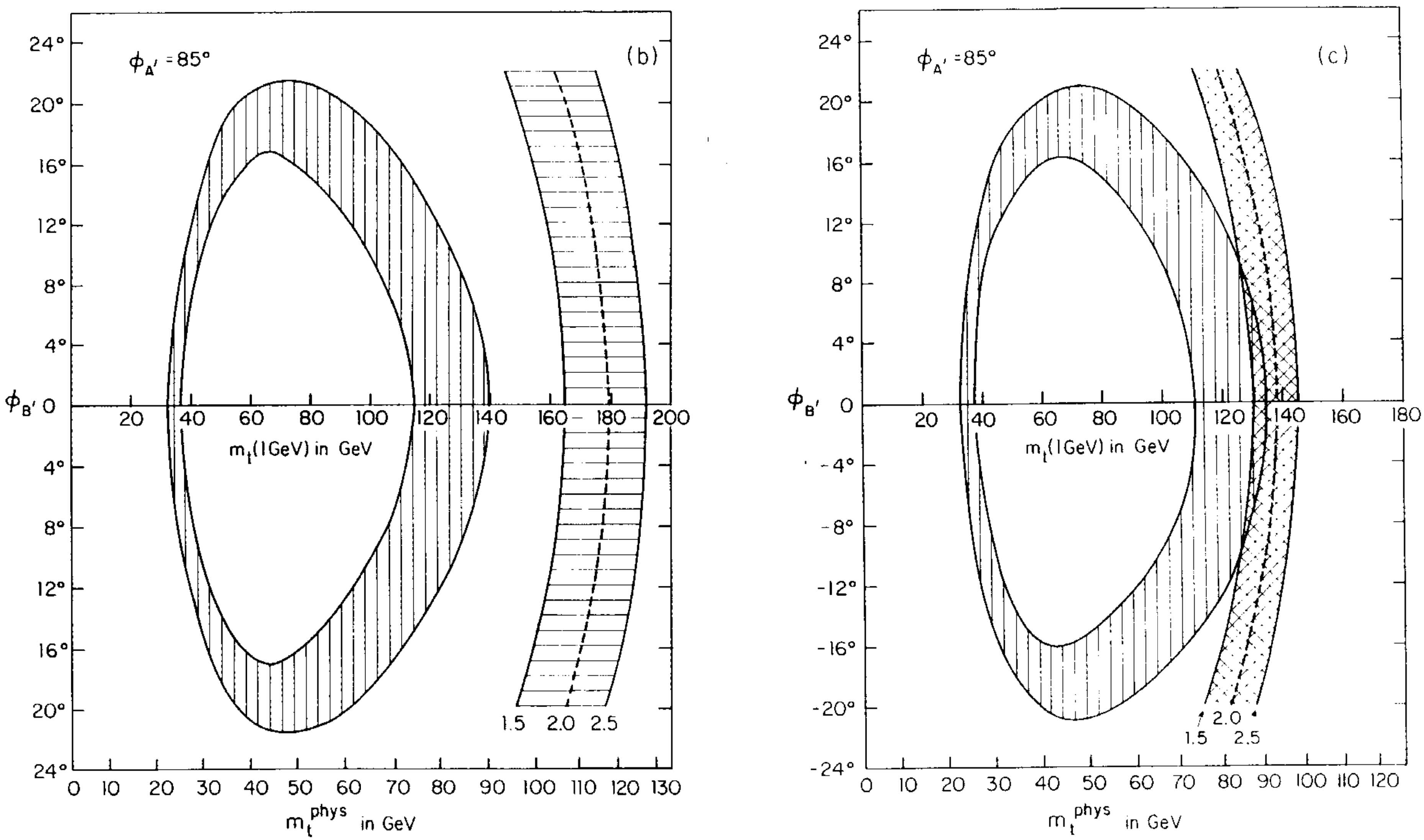
(a)
$\sim G_Y$
(b)
$\sim T_Y$
(c)
$\sim S_Y$

Fig. 1



$\phi_{A'} = 85°$
(a)
$\phi_{B'}$
24° 20° 16° 12° 8° 4° 0 -4° -8° -12° -16° -20° -24°
20 40 60 80 100 120 140 160 180 200 220
m_t(1GeV) in GeV
1.5 2.0 2.5
1.5 2.0 2.5
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140
m_t^{phys} in GeV

Fig. 2



(b)
$\phi_{A'} = 85°$
$\phi_{B'}$
$m_t(1 GeV)$ in GeV
m_t^{phys} in GeV
1.5 2.0 2.5
(c)
$\phi_{A'} = 85°$
$\phi_{B'}$
$m_t(1 GeV)$ in GeV
m_t^{phys} in GeV
1.5 2.0 2.5

Fig. 2