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A topological picture of cosmic string self-intersection

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Abstract

We translate the problem of finding the self-intersections of an evolving loop of cosmic string into a topological problem. We use this picture to discuss the relationship between cusps and self-intersections and give a lower bound on the total number of self-intersections any particular loop can have. This bound can be calculated by studying only the kinks and cusps present on the loop. We discuss the ways in which the number of cusps and self-intersections can change under smooth deformations of the initial conditions.

1 Introduction

It has been proposed that gravitational collapse around loops of "cosmic string" could have caused the formation of galaxies, clusters of galaxies and

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other large-scale structure in the universe [1,2,3]. In addition, radiation from superconducting cosmic strings might also have played a role in structure formation [4]. Cosmic strings are a string-like form of matter which appears in some field theories with spontaneously broken symmetries. If such symmetry breaking were to occur at an energy scale of around 10^{16} GeV , a network of cosmic strings would form in the first 10^{-36} sec after the big bang. These strings would have a thickness of about $(10^{16}\mathrm{GeV})^{-1}$ (or 10^{-30}cm) and a mass per unit length of roughly $(10^{16}\mathrm{GeV})^2$ (or 10^{22}gm/cm). (Throughout this article we take $\hbar = c = 1$) As the tangled network of string evolves, a piece of long string can self-intersect and break off a loop [5,6] (Fig. (1)). At any given time this mechanism is expected to produce loops with sizes of order t, the time since the big bang. By today a range of loops up to 10^{10} light-years in size would have been produced.

An important aspect of our understanding involves the self-intersection of a loop of string as it evolves. If a loop continually self-intersects and breaks, it can rapidly shatter into a debris of extremely tiny loops. Loops which do not self-intersect can survive over a much longer time.

In this article we consider the simpler problem of strings in flat spacetime where the strings equations can be solved exactly. This should be a good approximation for loops which are much smaller than the Hubble length. Section 2 is a review of the flat spacetime string equations. We develop a topological picture of the self-intersection process (Section 3) and construct a (loop dependent) lower bound on the number of self-intersections which depends only on the behavior of the string near kinks and cusps (Sections 4 and 5). In Section 6 we discuss effects such as the emission of a self-intersection by a cusp which can occur as the initial conditions are smoothly varied. We expect that our topological point of view can help us gain further insight into the behavior of cosmic strings.

2 The Equations of Motion, the Kibble-Turok Sphere, and Cusps

Loops whose size is substantially less than t and much greater than the string thickness are described by the Nambu equation in flat spacetime [7]:

$$\ddot{\mathbf{r}}(\sigma, t) = \mathbf{r}''(\sigma, t) \tag{1}$$

subject to

$$\dot{\mathbf{r}} \cdot \mathbf{r}' = 0 \tag{2}$$

and

$$(\dot{\mathbf{r}})^2 + (\mathbf{r}')^2 = 1 \tag{3}$$

where $\mathbf{r}$ is position, σ is a parameter which runs along the string, $\dot{\mathbf{r}} \equiv \partial\mathbf{r}/\partial t$ and $\mathbf{r}' \equiv \partial\mathbf{r}/\partial\sigma$. The gauge choice given by Eq. (2) means there are no longitudinal modes, while Eq. (3) chooses a particular parameterization with constant energy per unit σ. The general loop solution is given by:

$$\mathbf{r}(\sigma, t) = \frac{1}{2}\left[\mathbf{a}(\sigma^-) + \mathbf{b}(\sigma^+)\right] \tag{4}$$

Where $\sigma^{\pm} \equiv \sigma \pm t$. Equations (2) and (3) translate into

$$(\mathbf{a}')^2 = (\mathbf{b}')^2 = 1 \tag{5}$$

so $\mathbf{a}'$ and $\mathbf{b}'$ lie on a sphere with radius 1 (known as the Kibble-Turok sphere). Let l be the total length of σ around the loop. The choice of rest frame coordinates and the fact that the loop is closed imply

$$\int_0^l d\sigma\, \dot{\mathbf{r}} = \int_0^l dt\, \mathbf{r}' = 0. \tag{6}$$

which gives

$$\int_0^l d\sigma\, \mathbf{a}' = \int_0^l d\sigma\, \mathbf{b}' = 0. \tag{7}$$

This means the average position or "center of mass" of $\mathbf{a}'$ and $\mathbf{b}'$ is at the center of the sphere. The continuity of $\dot{\mathbf{r}}$ and $\mathbf{r}'$ implies the continuity of $\mathbf{a}'$ and $\mathbf{b}'$. Thus, it is often said that loop solutions correspond to pairs of loops on the Kibble-Turok sphere with the center of mass of each at the center of the sphere. In practice $\dot{\mathbf{r}}$ and $\mathbf{r}'$ can be discontinuous. These discontinuities are introduced when strings cross and reconnect the other way (see Fig. (1)) Although the σ and t derivatives are actually continuous on the scale of the thickness of the string, that scale is extremely small compared to the cosmic sizes of the loops of interest.

The case where $\mathbf{r}' = 0$ (or $\mathbf{a}' + \mathbf{b}' = 0$) is particularly interesting. At these points the string moves with the speed of light (see Eq. (3)). On the Kibble-Turok sphere one can equally well plot $-\mathbf{a}'$ rather than $\mathbf{a}'$. Then the $\mathbf{r}' = 0$ points correspond to points where the $-\mathbf{a}'$ and $\mathbf{b}'$ curves intersect. Given that the center of mass of the $-\mathbf{a}'$ and $\mathbf{b}'$ loops must be at the center of the sphere, such an intersection seems likely to occur if there are no discontinuities (kinks).

The functions $\dot{\mathbf{r}}$ and $\mathbf{r}'$ (and thus $\mathbf{a}'$ and $\mathbf{b}'$) are periodic in σ with period l. The motion of the loop will then be periodic in time with period $l/2$. If the curves $-\mathbf{a}'$ and $\mathbf{b}'$ intersect at some σ_c^- and σ_c^+ respectively there is only one σ_c and corresponding t_c in any given period such that $\sigma_c^- = \sigma_c - t_c$ and $\sigma_c^+ = \sigma_c + t_c$. Thus, the crossing of the $-\mathbf{a}'$ and $\mathbf{b}'$ curves corresponds to a single moment in the period of the loop where $\mathbf{r}' = 0$ at some particular point, which is referred to as a cusp.

3 Self-Intersections and Linking Number

A loop of string intersects itself at a time t_i if $\mathbf{r}(\sigma_1, t_i) = \mathbf{r}(\sigma_2, t_i)$ for some σ_1 and σ_2. This statement is equivalent to

$$\int_{\sigma_1}^{\sigma_2} \mathbf{r}'(\sigma, t_i)\, d\sigma = 0 \tag{8}$$

We have found a useful formalism which uses the Kibble-Turok sphere to study self-intersections. Equation 8 implies

$$-\int_{\sigma_1}^{\sigma_2} \mathbf{a}'(\sigma - t_i)\, d\sigma = \int_{\sigma_1}^{\sigma_2} \mathbf{b}'(\sigma + t_i)\, d\sigma. \tag{9}$$

For every Δ ($= \sigma_2 - \sigma_1$) we define

$$\vec{\alpha}_\Delta(\sigma) \equiv \frac{-1}{\Delta} \int_{\sigma - \frac{\Delta}{2}}^{\sigma + \frac{\Delta}{2}} \mathbf{a}'(\sigma)\, d\sigma \tag{10}$$

and

$$\vec{\beta}_\Delta(\sigma) \equiv \frac{1}{\Delta}\int_{\sigma-\frac{\Delta}{2}}^{\sigma+\frac{\Delta}{2}} \mathbf{b}'(\sigma)\,d\sigma. \tag{11}$$

These quantities are just the averages of $-\mathbf{a}'$ and $\mathbf{b}'$ over a length Δ. Then, defining $\bar{\sigma} = 1/2(\sigma_1 + \sigma_2)$, Eq. (9) is equivalent to

$$\vec{\alpha}_\Delta(\bar{\sigma} - t_i) = \vec{\beta}_\Delta(\bar{\sigma} + t_i). \tag{12}$$

Each of the functions $\vec{\alpha}$ and $\vec{\beta}$ defines a family of closed curves (labeled by Δ) lying inside the Kibble-Turok sphere . When Δ goes to zero $\vec{\alpha}$ and $\vec{\beta}$ approach $-\mathbf{a}'$ and $\mathbf{b}'$ respectively. As Δ increases the curves shrink until $\vec{\alpha} = \vec{\beta} = 0$ when $\Delta = l$. Self intersections of the loop correspond to the curves $\vec{\alpha}_\Delta$ and $\vec{\beta}_\Delta$ intersecting for some value of Δ, as demonstrated by Equation (12) . For a particular choice of Δ there is no reason to expect $\vec{\alpha}$ and $\vec{\beta}$ to intersect. (They have plenty of ways to avoid one another.) On the other hand, as Δ varies from 0 to l it seems reasonable that somewhere along the way they may cross. To each such crossing would correspond an intersection which would cut off a loop of length Δ from the rest of the string.

It is useful to define $Y(\Delta)$ as the linking number of the two curves $\vec{\alpha}_\Delta(\sigma)$ and $\vec{\beta}_\Delta(\sigma)$. For most values of Δ, $Y(\Delta)$ takes on some integer value. At special values of Δ, $Y(\Delta)$ changes up or down discontinuously by one unit. These are the values of Δ where $\vec{\alpha}$ and $\vec{\beta}$ cross, and they correspond to self-intersections of the loop.

4 A Lower Bound on Self-Intersections

The above formalism can be applied to give a lower bound on the number of times a loop self-intersects. It should be made clear that in this discussion we let the loop pass on through when it self-intersects, rather than having it break in two.

For every self-intersection there are two discontinuities in $Y(\Delta)$, one for $0 < \Delta \leq l/2$ and one for $l/2 \leq \Delta < l$ (corresponding to the big half and the little half of the intersecting loop). We limit our discussion to the range $0 < \Delta < l/2$ so that every intersection is counted only once. The loops which break exactly in half have "measure zero" and do not concern physicists.

The linking number of two closed curves can be calculated by constructing a surface bounded by one of the curves and determining the number of times the other curve passes throughout that surface. The curves $\vec{\alpha}$ and $\vec{\beta}$ have an orientation (direction of increasing σ) so the surface also has an orientation. The linking number is the number of times the other curve goes "in" minus the number of times it goes "out" of the surface. The overall sign of Y depends on convention and is of no importance.

We will now show that $Y(l/2) = 0$. We choose as our surface the set of line segments connecting the curve $\vec{\alpha}_\Delta(\sigma)$ to the center of the Kibble-Turok sphere . Since $\mathbf{a}'$ has period l,

$$-\int_0^l d\sigma\, \mathbf{a}' = \vec{\alpha}_{l/2}(\sigma) + \vec{\alpha}_{l/2}(-\sigma) \tag{13}$$

for any σ , and Eq. (7) implies that

$$\vec{\alpha}_{l/2}(\sigma) = -\vec{\alpha}_{l/2}(-\sigma). \tag{14}$$

Similarly

$$\vec{\beta}_{l/2}(\sigma) = -\vec{\beta}_{l/2}(-\sigma), \tag{15}$$

so the curves are symmetric under reflection through the origin. This reflection symmetry means that if one of the line segments that makes up the surface is extended through the origin, it continues to lie in the surface and eventually reaches another point on the curve. Now consider a point where $\vec{\beta}_{l/2}(\sigma)$ pierces the surface for some $\sigma = \sigma_p$. The reflection symmetry assures us that $\vec{\beta}_{l/2}(-\sigma_p)$ is passing through the surface in the opposite sense, and the net result is no contribution to the linking number. In this way all contributions to the linking number come in pairs with opposite signs and $Y(l/2) = 0$.

The above result leads immediately to:

$$\text{Total Intersections} = |Y(0)| + 2N \tag{16}$$

where $N \geq 0$. As Δ varies from 0 to $l/2$, Y must go from $Y(0)$ to 0. It can do so directly by simply taking $|Y(0)|$ unit steps, or it could take $2N$ additional steps. The additional steps must be even in number since they must result in no net change in Y. Each step (regardless of sign) corresponds to a self-intersection, thus Eq. (16).

5 Calculating $Y(0)$

The linking number $Y(\Delta)$ is not defined for values of Δ for which $\vec{\alpha}_\Delta$ and $\vec{\beta}_\Delta$ intersect. In particular, if $\vec{\alpha}_0$ and $\vec{\beta}_0$ ($-\mathbf{a}'$ and $\mathbf{b}'$) intersect — that is, if the loop has cusps — then their linking number is undefined and $Y(0)$ must be understood to mean the limit of $Y(\Delta)$ as Δ goes to zero. We first show how to calculate $Y(0)$ in the case where $\mathbf{a}'$ and $\mathbf{b}'$ are continuous by looking at $\mathbf{a}'$ and $\mathbf{b}'$ near cusps. Then we discuss how to handle discontinuities (kinks).

If we define the surface bounded by $\vec{\beta}_\Delta$ as in Section 4, the points where $\vec{\alpha}_\Delta$ pierces this surface will, for small Δ, be close to crossings of $\mathbf{a}'$ and $\mathbf{b}'$ (cusps). We examine a Taylor series expansion for $\vec{\alpha}_\Delta(\sigma^-)$ around $\Delta = 0$ and $\sigma^- = \sigma_c^-$, the value of σ^- at the cusp. Equation (10) can be integrated to give

$$\vec{\alpha}_\Delta(\sigma) = \frac{-1}{\Delta}\left[\mathbf{a}(\sigma + \frac{\Delta}{2}) - \mathbf{a}(\sigma - \frac{\Delta}{2})\right]. \tag{17}$$

Which to second order is

$$\vec{\alpha}_\Delta(\sigma_c^- + \epsilon^-) = -\left[\mathbf{a}'(\sigma_c^-) + \mathbf{a}''(\sigma_c^-)\epsilon^- + \mathbf{a}'''(\sigma_c^-)(\frac{(\epsilon^-)^2}{2} + \frac{\Delta^2}{24})\right]. \tag{18}$$

and similarly

$$\vec{\beta}_\Delta(\sigma_c^+ + \epsilon^+) = \left[\mathbf{b}'(\sigma_c^+) + \mathbf{b}''(\sigma_c^+)\epsilon^+ + \mathbf{b}'''(\sigma_c^+)(\frac{(\epsilon^+)^2}{2} + \frac{\Delta^2}{24})\right]. \tag{19}$$

From here on we suppress the arguments $\sigma_c^\pm$ in the derivatives of $\mathbf{a}$ and $\mathbf{b}$.

For small Δ, should $\vec{\beta}_\Delta$ pierce the surface bounded by $\vec{\alpha}_\Delta$ it will do so near the cusp (i.e for small ϵ^+ and ϵ^-), and will do so when $\vec{\alpha}$ and $\vec{\beta}$ point in the same direction:

$$\vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}||\vec{\beta}|. \tag{20}$$

This constraint is satisfied to second order by $\epsilon^+ = \epsilon^- = 0$, so to this order $\vec{\alpha}_\Delta(\sigma_c^-)$ and $\vec{\beta}_\Delta(\sigma_c^+)$ are parallel and we may just compare their magnitudes to see whether $\vec{\beta}_\Delta$ pierces the surface bounded by $\vec{\alpha}_\Delta$ or not:

$$\alpha_\Delta^2(\sigma_c^-) = (\mathbf{a}')^2 + 2\mathbf{a}' \cdot \mathbf{a}'''\frac{\Delta^2}{24}. \tag{21}$$

Since $\mathbf{a}'$ lies on the sphere, $(\mathbf{a}')^2 = 1$, and one can differentiate this constraint twice to get $\mathbf{a}' \cdot \mathbf{a}''' = -(\mathbf{a}'')^2$. Equation 21 may be simplified accordingly:

$$\alpha_\Delta^2(\sigma_c^-) = 1 - \frac{1}{12}(\mathbf{a}'')^2\Delta^2 \tag{22}$$

and similarly

$$\beta_\Delta^2(\sigma_c^+) = 1 - \frac{1}{12}(\mathbf{b}'')^2\Delta^2. \tag{23}$$

One can see that near a cusp, for small values of Δ, the curve with the largest magnitude of $\mathbf{a}''$ or $\mathbf{b}''$ lies closer to the center of the sphere.

Using the above analysis one can write down the following expression for $Y(0)$:

$$Y(0) = \sum_i f_i g_i \tag{24}$$

where i counts over all the cusps (intersections of $-\mathbf{a}'$ with $\mathbf{b}'$). The number f_i makes sure that linking number is only counted if $\vec{\beta}$ passes inside of $\vec{\alpha}$:

$$f \equiv \begin{cases} 1 & (\text{if } |\mathbf{b}''| > |\mathbf{a}''|) \\ 0 & (\text{if } |\mathbf{b}''| < |\mathbf{a}''|) \end{cases} \tag{25}$$

The number g_i gives the sign of the contribution to $Y(0)$:

$$g \equiv \operatorname{sign}[\mathbf{a}'' \times \mathbf{b}'' \cdot \mathbf{a}'] = \pm 1. \tag{26}$$

In this way we have defined completely $Y(0)$ for a kinkless loop in terms of first and second σ derivatives of $\mathbf{a}$ and $\mathbf{b}$ at the cusps.

The presence of kinks actually makes the problem simpler. Kinks appear as gaps in the curves on the Kibble-Turok sphere . The curves $\vec{\alpha}$ and $\vec{\beta}$ are just averages over sections of $\mathbf{a}'$ and $\mathbf{b}'$ so they interpolate smoothly across the gap. In the limit where Δ goes to zero, $\vec{\alpha}_\Delta$ and $\vec{\beta}_\Delta$ coincide with $\mathbf{a}'$ and $\mathbf{b}'$ on the Kibble-Turok sphere , but they connect any gaps with straight lines. These straight lines lie *inside* the Kibble-Turok sphere , and they essentially never intersect with one another. The contributions of the straight segments to $Y(0)$ can be calculated using standard methods of analytic geometry. We have explicitly checked Eq. (16) for a variety of specific solutions (with kinks) and we have found it to hold. Typically $N > 0$ in the class of loops we considered.

6 Appearance and Disappearance of Cusps and Self-Intersections

It is interesting to consider continuous deformations of the curves $\mathbf{a}'$ and $\mathbf{b}'$ on the Kibble-Turok sphere . Such deformations may occur due to physical processes such as the expansion of the universe, or gravitational radiation from the loop, which are not accounted for in Eq. (1).

If $\mathbf{a}'$ and $\mathbf{b}'$ are continuous they must cross an even number of times so a kinkless loop must have an even number of cusps. As Thompson [8] has pointed out, there are really two types of cusps, "cusps " and "anticusps". If $\mathbf{a}'$ and $\mathbf{b}'$ are continuously varied the number of cusps can only change by creation or annihilation of cusp-anticusp pairs which accounts for the total number of cusps (i.e. cusps plus anticusps) being even. Here we extend this idea to include processes involving the appearance and disappearance of self-intersections.

The quantity g defined in Eq. (26) is $+1$ for a cusp and -1 for an anticusp. The quantity f, defined in Eq.(25), describes a new property of a cusp which must also be taken into account when considering the relationship between cusps and self-intersections. In this section it will be useful to define f to be $\pm 1/2$ rather than 1 or 0, i.e.

$$f \equiv \begin{cases} +\frac{1}{2} & (\text{if } |\mathbf{b}''| > |\mathbf{a}''|) \\ -\frac{1}{2} & (\text{if } |\mathbf{b}''| < |\mathbf{a}''|) \end{cases} \tag{27}$$

which gives the same result for $Y(0)$ as before. There are then four kinds of cusps which we indicate by $c(+,-)$ for a cusp with $f = +1/2$ and $g = -1$, etc.

Next we consider self-intersections. At values of $\Delta = \Delta_{SI}$ corresponding to self-intersections the linking number $Y(\Delta)$ changes by ± 1 and accordingly we define

$$h \equiv [Y(\Delta_{SI} + \epsilon) - Y(\Delta_{SI} - \epsilon)] = \pm 1 \tag{28}$$

where ϵ is a small positive number. We use the symbols $SI(\pm)$ to indicate self-intersections with $h = \pm 1$. The relation between $Y(0)$ and the number of self-intersections (Eq. (16)) may be written

$$Y(0) + N_{SI(+)} - N_{SI(-)} = Y(l/2) = 0 \tag{29}$$

Where $N_{SI(\pm)}$ is the number of $SI(\pm)$'s in the range $0 < \Delta < l/2$ (to avoid double counting). Equation (29) can be written in terms of f, g, and h as

$$\sum_{cusps} f_i g_i + \sum_{SI's} h_j = 0. \tag{30}$$

Equation (30) tells us something about how the numbers of cusps and self-intersections of each type can change as the $\mathbf{a}'$ and $\mathbf{b}'$ curves are continuously varied. If $Y(0)$ does not change, the number of self-intersections can only change by creation or annihilation of a pair of self-intersections with $h = +1$ and $h = -1$ as shown in Fig. (2). At the moment when such a pair is created or annihilated the self-intersections will be degenerate, having the same Δ , $\bar{\sigma}$, and t.

The number of cusps can only change by cusp-anticusp annihilation, but now we must consider the role of g in such a process. The process of cusp-anticusp annihilation is illustrated in Fig. (3). As $-\mathbf{a}'$ and $\mathbf{b}'$ are varied their crossing points get closer together until they coincide, and then disappear. Just before annihilation $|\mathbf{b}''| - |\mathbf{a}''|$ will have the same sign at both cusps. Equivalently, we may say that for two cusps to annihilate their f's must be the same and their g's must be opposite in sign. Thus, when two cusps annihilate $\sum_{cusps}(f_i g_i)$, and therefore $\sum_{SI's}(h_j)$, must remain the same. If, however, the two cusps in Fig. (3) start with opposite f's we see that at some point, as they come closer together, the value of f $(= \frac{1}{2}\mathrm{sign}(|\mathbf{b}''| - |\mathbf{a}''|))$ at one of the cusps will change sign. When this happens a self-intersection must appear or disappear in order to satisfy Eq. (30). Thompson [8] has stated that the disappearance of a single self-intersection is associated with cusp-anticusp annihilation. Instead, we have shown that the disappearance (or appearance) of a single self-intersection is associated with a change in type of a cusp, which will generally not coincide with cusp-anticusp annihilation.

Consider a cusp with $g = +1$ and $f = +1/2$. If its f now changes to $-1/2$ it can either emit a self-intersection with $h = 1$ or absorb one with $h = -1$, in either case $\sum_{cusps}(f_i g_i) + \sum_{SI's}(h_j)$ is conserved. If a self-intersection is absorbed then as $|\mathbf{b}''| - |\mathbf{a}''|$ approaches zero the Δ of one of the self-intersections will shrink to zero while its $\bar{\sigma}$ and t will approach those of the cusp, and similarly for emission [9]. Whether a cusp absorbs or emits a self-intersection when $|\mathbf{b}''| - |\mathbf{a}''|$ changes sign depends on higher order derivatives of $\mathbf{a}$ and $\mathbf{b}$ at the cusp .

The allowed processes are summarized below.

Cusp-anticusp creation/annihilation:

$$c(\pm,\pm)+c(\pm,\mp)\leftrightarrow \text{nothing} \tag{31}$$

Pairwise creation/annihilation of a self-intersection:

$$SI(+)+SI(-)\leftrightarrow \text{nothing} \tag{32}$$

Self-intersection emission/absorption by a cusp:

$$c(+,\pm)\leftrightarrow c(-,\pm)+SI(\pm) \tag{33}$$

$$c(-,\pm)\leftrightarrow c(+,\pm)+SI(\mp) \tag{34}$$

These are the only ways that the number of cusps or of self intersections can change when $\mathbf{a}'$ and $\mathbf{b}'$ are deformed continuously.

This picture can be extended without too much trouble to handle loops with kinks. One imagines the discontinuities in $\mathbf{a}'$ and $\mathbf{b}'$ to be connected by great circle arcs along which $\mathbf{a}''$ and $\mathbf{b}''$ are very large. Crossings involving these sections ("microcusps" [8]) may then be assigned an f and g as before. Where two such sections cross one must instead examine the straight lines connecting across the gap, as discussed in Section 5, to determine the type of cusp.

7 Conclusions

The formalism we have developed may help provide a better understanding of the evolution of loops of cosmic string. We are presently pursuing the possibilities in several ways.

In this article we have only applied our topological picture to loops which pass on through one another when they self-intersect. We are working on a treatment of the more physical situation in which loops which self-intersect break in two. It would be interesting to know how the $Y(\Delta)$'s of the fragments are related to the original loop. So far all we can say about this is that $\sum_{loops} Y(0)$ is *not* conserved when loops break up (or recombine), but it may be possible to place some constraints on how it can change. We should note, however, that the formalism is already well suited to addressing the physically important question of whether a loop self-intersects or not.

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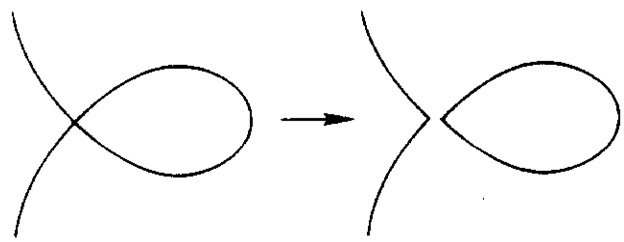

Figure 1: A string can self-intersect and break off a loop. A crossing will introduce discontinuities in the space and time derivatives of the curves

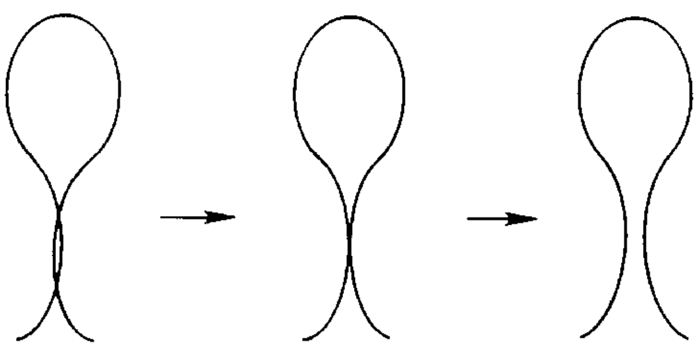

Figure 2: Snapshots of three related loops. At the left the section shown has two self-intersections. Varying **a**′, and **b**′ may lead to degenerate self-intersections (middle) and then no self-intersections (right).

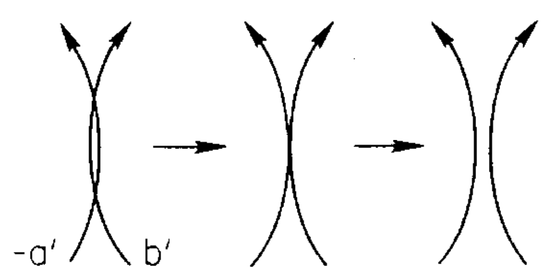

Figure 3: Cusp-anticusp annihilation