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Isocurvature Baryon Number Fluctuations in an Inflationary Universe

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Abstract

In conventional scenarios of baryogenesis it is extremely difficult, if not impossible, to produce local fluctuations in the baryon-to-photon ratio (so-called isocurvature baryon number fluctuations) on sufficiently large scales to be important for structure formation. Using a recently proposed scenario for baryogenesis called "Spontaneous Baryogenesis", we show that isocurvature baryon number fluctuations with the Zel'dovich spectrum and amplitude $\sim 10^{-3}$ can be produced in inflationary Universe models. It is also possible that non-scale invariant baryon number fluctuations could be produced in more complicated inflationary scenarios, and if so might help to revive the baryon-dominated, or neutrino-dominated scenarios of structure formation.

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1. Introduction

The hot big bang cosmology provides a general framework for explaining the formation of structure in the Universe: when the Universe becomes matter-dominated ($t_{eq} \simeq 3 \times 10^{10}(\Omega h^2)^{-2}$ sec, $T_{eq} \simeq 6.8\Omega h^2$ eV), primeval density inhomogeneities of size $\sim 10^{-3}$–10^{-5} begin to grow via the Jeans (or gravitational) instability, and eventually evolve into the structures that we see in the Universe today—galaxies, clusters of galaxies, superclusters, voids, etc.[1] In a very real sense, the structure formation problem is an 'initial data' problem: given Ω_{tot}, the partial fraction Ω_i of the various particle species, and the spectrum of initial inhomogeneities, the problem is completely specified, and one can numerically simulate the subsequent evolution of structure. Then one can test the model by comparing the simulations with observations of the Universe today.[2] To be viable the proposed model must be consistent with the growing database of observed properties of the Universe: galaxy and cluster correlations; the peculiar velocity field; rotation curves, morphological types, epoch of origin, etc. of galaxies; topology of the large-scale structure—voids, sheets of galaxies, etc. In addition, any model must lead to temperature fluctuations in the microwave background small enough to be consistent with the observed high level of isotropy.

Generically, density inhomogeneities are of two types: curvature perturbations (often called adiabatic perturbations) corresponding to fluctuations in the total energy density and hence wrinkles in space-time; or isocurvature perturbations (also called isothermal perturbations) corresponding to fluctuations in the local number density of some species (e.g., baryons or exotics) but only insignificant fluctuations in the total energy density and background metric. At late times, if a given particle species becomes nonrelativistic and eventually contributes a significant fraction of the energy density of the Universe, then an isocurvature fluctuation in that species will also give rise to fluctuations in the gravitational potential. Such gravitational potential fluctuations will lead to structure formation, as well as fluctuations in the microwave background temperature.

To follow the evolution of fluctuations one must know something about what is fluctuating. Most of the matter in the Universe today is 'dark', that is, does not emit detectable

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[1] Here and throughout $\Omega_i = \rho_i/\rho_{critical}$, $\rho_{critical} = 3H_0^2/(8\pi G) = 1.05h^2 \times 10^4 \mathrm{eV\,cm^{-3}}$, $h = H_0/100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and $\hbar = c = k_B = 1$. For a general review of structure formation, see [1][2].

[2] For recent reviews of numerical simulation of structure formation, see [3].

radiation, and its presence is only inferred by its gravitational effects. (For a review of dark matter in the Universe, see [4][5]). The total amount of matter that clusters with bright galaxies is $\Omega_c \simeq 0.2 \pm 0.1$ [4][5]. Furthermore, the conventional theory of nucleosynthesis constrains the baryonic contribution to the range $0.014 \leq \Omega_b \leq 0.15$ [6], although unconventional scenarios may possibly allow $\Omega_b \simeq 1$ [7]. Assuming the conventional bound on Ω_b and that $\Omega_{\rm tot} = 1.0$ — as theoretical prejudice and inflationary Universe models[8] suggest — then: (1) Most of the mass density is non-baryonic and likely in the guise of relic, weakly-interacting massive particles[9] (WIMPs), such as massive neutrinos, axions, photinos, etc. (i.e., $\Omega_b \sim 0.1$, $\Omega_{WIMP} \sim 0.9$), and (2) There must also be a component to the mass density that does not cluster with bright galaxies, contributing $\Omega_{uc} \simeq 0.8 \pm 0.1$.

Inflation provides a natural mechanism for producing primordial fluctuations.[3]. Typically these are curvature fluctuations[11] with the Harrison-Zel'dovich spectrum[12], and model-dependent amplitude. Inflation also provides a justification for assuming that $\Omega_{\rm tot} = 1$. Assuming an initial spectrum of inflation-produced curvature fluctuations, and WIMP domination today, one arrives at two very definite scenarios for structure formation: hot dark matter (WIMP is a $\sim 91h^2$ eV neutrino species), or cold dark matter (WIMP is just about anything else). At present neither picture is totally satisfactory. In the hot dark matter picture, most of the neutrinos remain unclustered due to their high velocities, accounting for $\Omega_{uc} \simeq 0.8 \pm 0.1$, and the large-scale structure which develops is qualitatively similar to that observed in the Universe today. However, in the neutrino picture galaxy formation occurs very late (redshifts $\lesssim 0.5$), as it only occurs after larger objects (supercluster-sized) have formed and fragmented; this apparently conflicts with the observations of many galaxies and QSO's with redshifts $\gtrsim 1$ (and as high as 4.5). In the neutrino dominated universe galaxies must form by fragmentation because neutrino freestreaming erases density perturbations on scales roughly less than the size of superclusters—see [3].

In the cold dark matter picture galaxies form very nicely, and the theory is definite enough to predict many of their gross properties: rotation curves, number density, masses and internal densities, clustering properties, etc. However, there is no simple means of

[3] There is at least one other interesting early Universe scenario for the production of primeval fluctuations: cosmic strings with string tension $\mu \sim 10^{-6} m_{pl}^2$ [10] The production of cosmic strings in a very early SSB phase transition ($t \sim 10^{-36}$ sec) leads to isocurvature fluctuations, due to the inhomogeneous distribution of string loops.

accounting for the unclustered component—cold dark matter particles should have no difficulty in finding their way into galaxies, and insufficient large-scale structure seems to evolve (cluster–cluster correlations, large-scale streaming velocities, etc.)—though one should keep in mind that our observational view of the large-scale structure of the Universe is far from complete.

Although inflation-induced curvature fluctuations are those most commonly discussed, it is possible to construct inflation models which also lead to isocurvature fluctuations. For example, in inflation models where axions serve as the dark matter, one finds isocurvature axion fluctuations with the Harrison-Zel'dovich spectrum[13]. In this paper we will be discussing a somewhat similar mechanism for producing isocurvature baryon number fluctuations. Such fluctuations may be of interest for structure formation, especially if $\Omega_b \approx 1.0$ or if they are non-scale invariant due to the details of inflation.

While isocurvature baryon number fluctuations used to be a very popular candidate for the primeval fluctuations[2], the advent of scenarios for the dynamical origin of the baryon number of the Universe[14] has put them in disfavor. The reason is simple: in such scenarios the baryon-to-photon ratio is only a function of microphysical parameters, and thereby cannot evolve to be spatially varying[15]. Only by introducing large amounts of primeval shear to the cosmological model is it possible to produce isocurvature baryon number fluctuations[16], and this does not work in inflationary models as any initial shear is damped out during inflation.

In this paper we will show that in the new scenario of baryogenesis proposed in [17], isocurvature baryon number fluctuations arise quite naturally in an inflationary Universe. They are produced in almost the same way that isocurvature axion perturbations are produced[10], and in the simplest models of inflation have the Zel'dovich spectrum. Their amplitude is model-dependent and, if the fluctuations are scale invariant, is constrained by the uniformity of the microwave background to be $\lesssim \Omega_b^{-1} 10^{-4}$. In a neutrino-dominated Universe these isocurvature baryon number perturbations could provide the requisite seed fluctuations for galaxies; in a baryon-dominated Universe, galaxy formation would proceed as with isocurvature axion fluctuations.

2. Spontaneous Baryogenesis

Recently two of the present authors have proposed a qualitatively new scenario for baryogenesis[17]. In the usual scenario, the necessary ingredients are: violation of B, C,

CP; and a departure from thermal equilibrium[18]. In the "Spontaneous Baryogenesis" scenario, baryon number is spontaneously broken at a scale f, as well as being explicitly violated. This results in a pseudo Nambu-Goldstone boson — the *ilion* (from the Greek word for "matter") — a particle much lighter than the scale f. Since the ilion field is a periodic variable, we refer to it as θ, and take the minimum of its potential at $\theta = 0$. Like the axion field, the ilion field, owing to its small mass, is left undetermined after an early phase transition. Provided that this phase transition occurs before or during inflation, the initial value of θ ($\equiv \theta_0$) will be uniform throughout a very large region of the Universe, one which easily encompasses all of the observable Universe today. The baryon asymmetry is produced after the phase transition (and inflation), as the ilion field relaxes to its equilibrium value $\theta = 0$[17].

Unlike the conventional scenario of baryogenesis where C and CP violation are necessary and where the baryon asymmetry is produced out of thermal equilibrium, with Spontaneous Baryogenesis (SBG) C and CP are exact symmetries of the theory and the baryon asymmetry is produced while baryon violating processes are *in thermal equilibrium* due to a dynamical violation of these symmetries, as well as CPT, through non-zero values of θ and $\dot{\theta}$. We will briefly describe the simple SBG model developed in [17]. Let σ be the complex field which develops a vacuum expectation value $\langle\sigma\rangle = f\exp(ib\,a/f)$ during a phase transition which occurs at the scale f. (In general, SBG requires that $f \gtrsim 10^{13}$ GeV.) Here b is the baryon number carried by the σ field, and the ilion field is $\theta(x) \equiv a(x)/f$. Since σ carries baryon number, B is spontaneously broken; in addition, there are other explicit baryon-number violating interactions in the theory, which cause the ilion to develop a small mass ($\equiv m_\theta$). At energies (and temperatures) below the scale f, the effective Lagrangian of the theory will have a derivative coupling of the ilion to the baryon number current: $\partial_\mu \theta J^\mu_B$ (J^μ_B is the baryon number current).

The derivative coupling of the ilion to J^μ_B is the crucial feature of this model for baryogenesis; whenever $\dot{\theta} \equiv \mu$ is different from zero, this term corresponds to a chemical potential for baryons ($\mu_b = -\mu$) and antibaryons ($\mu_{\bar{b}} = +\mu$). As long as $\dot{\theta} \neq 0$, the equilibrium value of the baryon number density will be given by

$$(n_B)_{EQ} \simeq -\mu T^2 \simeq -\dot{\theta} T^2. \tag{2.1}$$

Correspondingly, the equilibrium value of the baryon number-to-entropy ratio is

$$\left(\frac{n_B}{s}\right)_{EQ} \simeq -\frac{\dot{\theta}}{g_* T}, \tag{2.2}$$

where the entropy density $s \simeq g_* T^3$, and g_* as usual counts the number of relativistic degrees of freedom. Note that n_B/s is just the net baryon number per comoving volume.

As $\dot{\theta}$ and T evolve, the equilibrium value of the baryon number per comoving volume changes. This equilibrium value will only be tracked so long as B nonconserving interactions are occurring rapidly compared to the expansion rate, $\Gamma_B \gtrsim H$, and can thereby change the net baryon number per comoving volume on the expansion time scale H^{-1}. Once the B violating interactions become ineffective, say at temperature T_D ($\Gamma_B \lesssim H$ for $T \lesssim T_D$), the baryon number per comoving volume 'freezes in', at a value $n_B/s \simeq -(\dot{\theta}/g_* T)|_{T_D}$, and thereafter remains constant.

The details of this scenario are addressed in [17]; in brief the final baryon asymmetry which evolves depends upon whether the ilion filed is Hubble damped at $T = T_D$ [$m_\theta < 3H(T_D)$], so that $\dot{\theta} \simeq -m_\theta^2 \theta/(3H)$), or if it is already oscillating about $\theta = 0$ [$m_\theta > 3H(T_D)$]. In either case, the net baryon asymmetry which evolves and eventually freezes in is proportional to the initial misalignment of the ilion field: $n_B/s \propto \theta_0$; this is the important feature for the production of isocurvature baryon number fluctuations. We will not dwell on the details of SBG, but rather simply use this fact.

Since the baryon asymmetry produced is proportional to θ_0, SBG requires a period of inflation, to ensure that θ_0 is homogeneous over regions at least as large as today's Hubble volume; otherwise the present Hubble volume would be baryon-symmetric, contrary to observations. Averaging over the whole Universe today, one would find $\langle B \rangle \propto \langle \theta_0 \rangle = 0$ (a result of C, CP conservation), but $\langle B^2 \rangle \propto \langle \theta_0^2 \rangle \neq 0$. That is to say, we live in a large-scale fluctuation of baryon number: there are necessarily causally disconnected regions consisting only of antimatter! The requisite large-scale homogeneity will occur as long as the σ field acquires its vacuum expectation value *during or before* inflation, so that regions in which $\theta(x) \simeq constant = \theta_0$ are inflated to very large size. While the RMS value of θ_0, averaged over the entire Universe is $\pi/\sqrt{3}$, the value of θ_0 in our region of the Universe has some particular value between 0 and π. This is analogous to the axion in inflationary Universe models [19], where the axion number density is proportional to the misalignment angle of the axion field squared. Because of the similarity of the baryon number density in the SBG scenario, $n_B/s \propto \theta_0$, to the axion number density, $n_a \propto \theta_0'^2$, isocurvature baryon number fluctuations will arise just as isocurvature axion fluctuations did. Our discussion of isocurvature baryon number fluctuations will closely follow the work of Seckel and Turner[13] on isocurvature axion fluctuations.

3. Isocurvature Baryon Number Fluctuations

The ilion field $a(x) \equiv f\theta(x)$ behaves like a massless scalar field during inflation since $m_\theta \ll H_I$. Like all massless scalar fields it will have a spectrum of de Sitter produced fluctuations. We define the RMS fluctuation of the ilion field as

$$\langle \delta a(x)^2 \rangle \equiv \int \frac{d^3x}{\mathrm{Vol}}\, \delta a(x)^2 = \int \frac{d^3k}{(2\pi)^3} |a_k|^2 \tag{3.1}$$

where a_k is the Fourier transform of the ilion field

$$a_k = \int d^3x\, e^{ikx} a(x). \tag{3.2}$$

In de Sitter space this quantity is given by

$$|a_k|^2 = \frac{H_I^2}{2k^3} \tag{3.3}$$

where k is the comoving wavenumber ($\equiv 2\pi/\lambda$), normalized so that $\lambda = \lambda_{phys}/R(t) = \lambda_{today}$. $R(t)$ is the cosmic scale factor, here normalized so that $R(t_{today}) = 1$. While a given mode k is sub-horizon sized, $R(t)\lambda \lesssim H_I$, the fluctuations on that scale are dynamically evolving. After a given mode crosses outside the horizon, it can be treated as a classical fluctuation. The classical equation of motion for the mode k is

$$\ddot{a}_k + 3H\dot{a}_k + \left(\frac{k^2}{R^2} - m_\theta^2\right) a_k = 0. \tag{3.4}$$

As long as $m_\theta \lesssim 3H$ and $R(t)/k \gtrsim H^{-1}$, the solution is just[4] $a_k \simeq$ *constant*.

Since the baryon asymmetry which evolves is proportional to $\theta(x) \equiv a(x)/f$ this spectrum of fluctuations in the ilion field will eventually give rise to fluctuations in the baryon asymmetry, n_B/s. Defining the Fourier transform of the asymmetry as above, the RMS fluctuation in the baryon number per comoving volume is

$$\left\langle \left[\frac{\delta n_B}{n_B}\right]^2 \right\rangle = \left\langle \left[\frac{\delta(n_B/s)}{(n_B/s)}\right]^2 \right\rangle \equiv \int \frac{d^3k}{(2\pi)^3} |\delta_k|^2. \tag{3.5}$$

Since n_B/s is proportional to $a(x)$, it follows that

$$|\delta_k|^2 = \frac{|a_k|^2}{f^2} \frac{1}{\theta_0^2} \tag{3.6}$$

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[4] We neglect the decaying mode which becomes negligible at late times.

where θ_0 is the average value of $\theta(x)$ within the inflating patch.

From (3.4) we see that since δ_k is proportional to a_k, δ_k is independent of time, as long as the fluctuation is outside the horizon, and $m_\theta \lesssim 3H$. Evaluating a_k just as the mode crosses outside the horizon during inflation, and begins to behave as a classical fluctuation, we have

$$|\delta_k|^2 \simeq \frac{H_I^2}{2f^2\theta_0^2}\frac{1}{k^3}. \tag{3.7}$$

Fluctuations on a given scale k are often described by the contribution to the RMS fluctuation per logarithmic interval,

$$\left(\frac{\delta n_B}{n_B}\right)_k^2 \equiv \frac{k^3}{2\pi^2}|\delta_k|^2, \tag{3.8}$$

so that

$$\left(\frac{\delta n_B}{n_B}\right)_k \simeq \frac{H_I}{2\pi}\frac{1}{f\theta_0}. \tag{3.9}$$

For the simplest models of inflation both H_I and f are nearly constant during the inflationary epoch, so that the RMS baryon mass fluctuation is scale invariant—the so-called Harrison-Zel'dovich spectrum[12]. In more complicated models of inflation, H_I (and possibly f) can vary significantly during inflation; in this case the quantity H_I/f in (3.9) is evaluated when the mode k crosses the outside the horizon, and the spectrum will not necessarily be scale invariant.

Taking the vacuum energy density during inflation to be $\rho_I \equiv M^4 = (M_{15}\times 10^{15}\mathrm{GeV})^4$ and $f = f_{13} \times 10^{13}\mathrm{GeV}$, it follows that

$$\left(\frac{\delta n_B}{n_B}\right)_k \simeq 3.8 \times 10^{-3}\frac{M_{15}^2}{f_{13}\theta_0}. \tag{3.10}$$

When the Universe becomes matter dominated these isocurvature fluctuations in the baryon number density will give rise to comparable-sized fluctuations in the baryonic (and total) energy density.

4. Microwave Background Fluctuations

Fluctuations in the baryon density will also give rise to fluctuations in the microwave background temperature. Such fluctuations quite naturally separate into small angular scale fluctuations ($\ll 1^\circ$), corresponding to length scales on the surface of last scattering smaller then the horizon at decoupling, and large angular scale fluctuations ($\gg 1^\circ$). The fluctuations on large angular scales arise primarily from the Sachs-Wolfe effect; this can be understood as the gravitational redshift of microwave photons propagating through a perturbed Newtonian potential, $\phi(x)$. Since fluctuations on these scales are super-horizon sized at decoupling, their amplitude is unaffected by microphysics—e.g., the details of recombination, subsequent reionization of the matter, etc. Following Peebles[2],

$$\frac{\delta T(x)}{T} = -\phi(x) = 4\pi G R^2(t) \int d^3x' \frac{\delta\rho(x')}{|x-x'|}. \tag{4.1}$$

From this it follows that the temperature fluctuations which arise due to the isocurvature baryon number fluctuations have the spectrum

$$\delta_T(k) = \frac{3}{2}\Omega_b H^2 R^2 \frac{\delta_k}{k^2} \tag{4.2}$$

where $\delta_T(k) = \int d^3x \exp{(ikx)}\delta T(x)/T$ is the Fourier transform of the temperature fluctuations. Defining the temperature fluctuation per logarithmic interval in analogy with (3.8),

$$\left(\frac{\delta T}{T}\right)_k^2 \equiv \frac{k^3}{2\pi^2}|\delta_T(k)|^2 \tag{4.3}$$

it follows that

$$\left(\frac{\delta T}{T}\right)_k \simeq \frac{3}{2}\Omega_b \frac{H^2R^2}{k^2}\left(\frac{\delta n_B}{n_B}\right)_k. \tag{4.4}$$

One can show that for fluctuations on scales $\gg 1°$, the right hand side of this equation is time independent once the fluctuation has reentered the horizon. Evaluating the fluctuation at horizon crossing, $HR/k \simeq 1$, we obtain

$$\left(\frac{\delta T}{T}\right)_k \simeq \Omega_b \frac{H_I}{2\pi}\frac{1}{f\theta_0} \simeq \Omega_b\left(\frac{\delta n_B}{n_B}\right)_k \tag{4.5}$$

(valid for angular scales greater than $1°$). The observed uniformity of the microwave temperature on large angular scales, $\delta T/T \lesssim 10^{-4}$ or less, implies that

$$\Omega_b\left(\frac{\delta n_B}{n_B}\right)_k \lesssim 10^{-4} \tag{4.6}$$

on scales k corresponding to angular scales greater than a degree (comoving length scales $\gtrsim 30$ Mpc). For the simplest inflationary scenario which predicts a scale invariant spectrum, this constraint will apply to all scales k. If the spectrum is not scale invariant, this constraint does not apply to the small scale fluctuations. A more detailed discussion of the large angular scale microwave fluctuations resulting from isocurvature density perturbations is given in [20].

5. Discussion and Summary

We have shown that with the Spontaneous Baryogenesis scenario[17] it is possible to produce isocurvature baryon number fluctuations in the context of inflationary Universe models. This is in sharp contrast to the standard models of baryogenesis in which it is essentially impossible to produce isocurvature baryon number fluctuations without recourse to special initial conditions[15][16].

These isocurvature baryon number fluctuations may have important consequences for scenarios of structure formation, or provide yet an additional constraint on inflationary models in which the baryon asymmetry arises in the SBG scenario. In particular, if the spectrum of perturbations should be non-scale invariant, as might occur in more complicated inflationary scenarios[21], there could be sufficient power on small scales to provide seed fluctuations for galaxies in the neutrino dominated Universe scenario, making this model potentially viable again.[5].

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[5] Fluctuations in the baryon number of order unity or greater are precluded not only by the smoothness of the microwave background, but also by the fact that we live in a baryon asymmetric Universe: if $\delta n_B/n_B$ were of order unity (or greater), then the fluctuations would be so large that they would wash out the net baryon asymmetry of the Universe and result in a baryon symmetric Universe! Since baryon number fluctuations of order unity or greater are precluded, the fluctuations discussed here should have no impact on primordial nucleosynthesis[7].

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