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Are Physical Cutoffs required in Spontaneous Gauge Symmetry Breaking?

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ABSTRACT

Triviality of scalar field theories makes the naive version of spontaneous symmetry breaking (SSB) questionable. We study whether the problem of triviality is removed by other sectors of a theory without a need for physical cutoffs (or embedding scales) in the large N limit. The problem for a similar situation with finite N can be understood as being some deviation from the large N limit. This point of view shows the systematic of the problem in a nice way. In consequence the result can be used as an aid for understanding or avoiding such problems in other models. Additionally it is shown that the formal asymmetry of SSB where scalars are just needed to break the symmetry is removed. The right amount of gauge bosons and fermions is needed in balance to stabilize SSB. Upper bounds for fermion and Higgs masses arise naturally.

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Spontaneous gauge symmetry breaking (SSB) is the best known way to obtain gauge boson masses without destroying gauge invariance or renormalizability. SSB can also be used to create fermion masses which are forbidden by symmetry in the unbroken phase. To make SSB work the developement of a vacuum expectation value (VEV) of a scalar sector has to be used. This scalar part of the model is very much like a pure one component $\lambda\Phi^4$ ($\lambda \geq 0$) theory which on the classical level seems to have no problem in taking a nonvanishing VEV if the mass term $\frac{\mu^2}{2}\Phi^2$ has a negative μ^2. On the quantum mechanical level, however, a pure scalar $\lambda\Phi^4$ theory in $d = 4$ dimensions is probably only consistent if the theory is free, i.e. $\lambda_R = 0$ [1,2]. This so called "triviality" would make the central part of SSB questionable as then nonvanishing VEV's would not occur.

The scalar sector is eventually coupled via covariant derivatives and explicitely to other sectors of the full theory. One might hope that a quantum mechanical treatment of the full theory would remove the problem allowing again a naive treatment. Since triviality is connected with the consistency of a renormalized Φ^4 interaction, the strength of the effective Φ^4 interactions induced by the other sectors is important. If the selfcoupling of the scalar sector is too strong, it seems to be justified to neglect other couplings for the moment and this brings the original triviality problem back. Therefore, if there is a rescue from the problem by other couplings at all there should exist upper bounds on λ in terms of the other couplings.

This paper addresses the question whether such a rescue from triviality by other couplings (and without a cutoff) occurs easily or only in special situations. We concentrate on models built in analogy with the standard model of electroweak interactions. The common properties of this question are studied in the large N limit of an N component scalar theory. The answer for finite N can be understood as a perturbation of the large N result and certain common features are understood very systematically this way. Especially, the importance of fermions is nicely exhibited this way.

In a perturbative treatment, the known triviality of a $\lambda\Phi^4$ theory would imply a true Landau singularity [3] of the effective coupling $\lambda(t)$ leading to inconsistencies in the asymptotic scaling behavior of $\lambda(t)$ [2]. In contrast a Landau singularity of a theory in a perturbative treatment is only a hint on possible triviality problems*. But knowing about the nonperturbative properties of a Φ^4 theory one can use the perturbative behavior of the effective coupling $\lambda(t)$ to quantify results by using the perturbative Landau singularity. Observing both the very fast transition of $\lambda(t)$ from perturbative values to its pole (due to selfamplification) and the stability of the pole position under addition of arbitrary small higher order terms in the β-function one can conclude that the scale where $\lambda(t)$ gets strong is known rather precisely. Assuming that there is no fancy nonperturbative behaviour of $\lambda(t)$ on its way to the pole it is possible to make good quantitative statements**. The result is a relation between the initial value $\lambda(0)$ and the corresponding pole position Λ_P.

Knowing that true Landau singularities are inconsistencies, a simple way out of the dilemma is to embed the current theory at a scale Λ_E. All those situations where $\Lambda_E < \Lambda_P$ exists no longer have a triviality problem since new degrees of freedom replace λ and Φ beyond Λ_E.

Another idea which might remove the problem was the coupling to other sectors. In terms of Landau singularities, turning on couplings to other sectors would shift the pole position. Eventually it is possible to move the pole to infinity resulting in a nontrivial theory without the need of an embedding scale. As the mechanism works (if at all) only with sufficiently big additional couplings g^2 it is possible to translate this into an upper bound on $\lambda(0)$ if $g^2(0)$ is fixed.

The combination of both ideas is also possible. This leads to modifications of the allowed range of $\lambda(0)$ for a given embedding scale Λ_E when the additional couplings are turned on [4]. Because of the cutoff, the triviality problem can always be removed. But inventing an embedding scale requires new physics. Therefore the question whether, for a given theory the mechanism in which additional cou-

* In fact, one has to take every Landau singularity as a possible inconsistency of the corresponding theory.

** The agreement of computer simulations with this approach confirms this.

plings can remove the triviality problem works without any need for new physics is important.

In this paper, this question is studied in the framework of the $\frac{1}{N}$ expansion. We first start with an $O(N)$ theory as an equivalent to the scalar sector of theories which make use of SSB and add other fields later. It is possible to sum up all diagrams contributing to the β-function in leading order of $\frac{1}{N}$. The result should be more predictive than one loop results*. First we discuss a second $O(N_2)$ sector coupled to the original $O(N)$ theory. Then we present the $O(N)$ equivalent of SSB by gauging the group and eventually adding fermions.

With the global interaction Lagrangian

$$\mathcal{L}_I = -\frac{\lambda}{8}\left(\Phi_a\Phi_a\right)^2 \tag{1}$$

the β-function in the large N limit (to all orders of perturbation theory) reads [5]

$$\beta = \frac{d\lambda}{dt} = \frac{N}{16\pi^2}\ \lambda^2\ ; \quad t = \frac{1}{2}\ln\left(\frac{q^2}{q_0^2}\right)\ . \tag{2}$$

This result is very similar to the one component Φ^4 theory at one loop.
The effective coupling becomes

$$\lambda(t) = \frac{\lambda(0)}{1-\frac{N}{16\pi^2}\lambda(0)t} \tag{3}$$

with a Landau singularity at the scale $t = \frac{16\pi^2}{N\lambda(0)}$ for every positive finite $\lambda(0) \neq 0$ (vacuum stability excludes the case $\lambda(0) < 0$).

The fact that only $\lambda(0) = 0$ leads to a consistent theory for all scales exactly as in the one loop treatment of a one component Φ^4 theory has been used as an argument for triviality. Since the renormalization properties are the central point in triviality proofs, the summability of all orders of perturbation theory seems to be more important than doing the calculation with finite N and low order of

*The triviality of the large N limit was used as a pro triviality argument in this sense.

perturbation theory. We therefore study the shift of the Landau singularity of the $O(N)$ theory in the large N limit when crosscouplings to other sectors are turned on. The aim is to shift the poles to higher scales beyond a high physical cutoff (which is eventually removed by taking it to infinity). In a first step we add a second $O(N_2)$ sector with a quartic selfcoupling λ_2 and a crosscoupling λ_3 (we rename $\Phi \equiv \Phi_1$ here):

$$\mathcal{L}_I = -\frac{\lambda_1}{8}\left(\Phi_{1,a}\Phi_{1,a}\right)^2 - \frac{\lambda_2}{8}\left(\Phi_{2,b}\Phi_{2,b}\right)^2 - \frac{\lambda_3}{4}\left(\Phi_{1,a}\Phi_{1,a}\right)\left(\Phi_{2,b}\Phi_{2,b}\right) \tag{4}$$

The corresponding β-function in the large N limit are

$$\frac{d\lambda_1}{dt} = \frac{1}{16\pi^2}\left(N_1\lambda_1^2 + N_2\lambda_3^2\right) \tag{5}$$

$$\frac{d\lambda_2}{dt} = \frac{1}{16\pi^2}\left(N_2\lambda_2^2 + N_1\lambda_3^2\right) \tag{6}$$

$$\frac{d\lambda_3}{dt} = \frac{1}{16\pi^2}\lambda_3\left(N_1\lambda_1 + N_2\lambda_2\right) \tag{7}$$

We start with $\lambda_3 = 0$, where we have two decoupled $O(N)$ theories. Without loss of generality we can assume that the pole t_2 of Φ_2 is at a higher scale than t_1, the pole of Φ_1. As both sectors have a Landau singularity, we have to invent a cutoff equal to the lowest pole:

$$t_{cut} \leq t_1 \ . \tag{8}$$

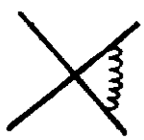
To find out whether the crosscoupling of both sectors helps in shifting t_1 to higher values we turn on $\lambda_3 = \epsilon$. As long as λ_3 is $\mathbf{O}(\epsilon)$, it is clear that the equations for λ_1 and λ_2 are modified by terms $\mathbf{O}(\epsilon^2)$ only. Therefore, λ_1 will grow to stronger values first just as before. Once λ_1 becomes large, the terms of $\mathbf{O}(\epsilon^2)$ are irrelevant and λ_1 starts to produce a pole. But with the growth of λ_1 the derivative of λ_3 gets big and therefore λ_3 will grow to sizable values. The feedback of the bigger values of λ_3 will cause λ_1 to grow even faster. But as the pole of λ_1 develops in a small scale range, the pole position is only mildly affected by this feedback mechanism. Most of the shift of the pole of λ_1 comes from the action of terms $\mathbf{O}(\epsilon^2)$ in the range where λ_1 is small, shifting the onset of the pole slightly.

While the feedback of λ_3 has almost no effect on λ_1, the same mechanism has drastic outcomes for λ_2. The growth of λ_3 through the growth of λ_1 enters the equation for λ_2 and drives λ_2 to grow in the same scale area. Once λ_2 has become big around the same scale as the pole of λ_1 both the selfcoupling of λ_2 and the increase of λ_3 lead to a pole of λ_2 at the Landau singularity of λ_1.

In summary, the Landau singularity of λ_1 is shifted very mildly by turning on the crosscoupling λ_3. On the other hand, the pole of λ_2 immediately collapses to the pole of λ_1. In addition, λ_3 is forced to produce a pole in the same area. All couplings are "trapped" by λ_1*.

Note that this mechanism would also occur if λ_2 had no triviality problem from the beginning. Therefore the addition of a second $O(N_2)$ sector does not help the triviality problem of the $O(N_1)$ theory.

The essential point for this trapping to occur is of course the positive signs of all additional terms in the β-functions. Therefore, we can immediately see what is necessary to avoid the problem. There should be effective Φ^4 interactions with a relative minus sign compared to the scalar fish diagram. On first glance, there appear two ways to achieve this. One possibility is the minus signs coming from fermion loops (statistics). The other possibility is an effective Φ^4 interaction with one more propagator in the loop. This would typically correspond to such a diagram:

 $\sim \; -\lambda g^2$

We now proceed to gauge the $O(N)$ theory and subsequently put in fermions in analogy with the standard model, and we will see how both possibilities produce the desired minus signs. But this is only a necessary condition for a rescue from triviality problems. We have to look at the details of the additional couplings.

In the above discussion of a global $O(N_1) \times O(N_2)$ scalar field theory, we have shown that there exists a single Landau singularity instead of two because of the

* A similar mechanism, in a different context, was discussed by Ref. [6].

trapping mechanism. In the case of a gauged $O(N)$ scalar field theory, there have been various works suggesting that a gauge theory might help evade the triviality problem. We wish to reexamine this question, having in mind possible contributions of light and/or heavy fermions to the scalar sector since this provides a systematic understanding of the general problem for every finite N.

Let us first consider the case in which there are only one scalar multiplet Φ and gauge bosons. The Lagrangian describing interactions among the scalars is given by eq. (4), but now with only one λ. In the large N limit, the contribution of the gauge bosons to the renormalization group equation governing λ gives [5]

$$\frac{d\lambda}{dt} = \frac{N}{16\pi^2}\left[\lambda^2 - 3\lambda g^2 + \frac{3}{4}g^4\right] , \tag{9}$$

for small λ and g^2. Combining with

$$\frac{dg}{dt} = -\frac{b_0}{32\pi^2}\, g^3 , \tag{10}$$

where

$$b_0 = 2\left[\frac{11}{3}C_2(G) - \frac{1}{3}T(R)\right] \overset{\text{for } O(N)}{=} 2\left[\frac{11}{3}(N-2) - \frac{1}{3}\right] \tag{11}$$

with scalars in a vector representation, we obtain

$$\frac{1}{g^2}\frac{d\bar{\lambda}}{dt} = \beta(\bar{\lambda}) = \frac{N}{16\pi^2}\left[\bar{\lambda}^2 + \frac{b_0 - 3N}{N}\bar{\lambda} + \frac{3}{4}\right] \tag{12}$$

$$= \frac{N}{16\pi^2}\left[\bar{\lambda}^2 + B\bar{\lambda} + C\right] \tag{13}$$

where the transformation $\bar{\lambda} \equiv \frac{\lambda}{g^2}$ decouples the RGEs.

If $\Delta = B^2 - 4C < 0$ then there are no zeros of $\beta(\bar{\lambda})$ and, as the RHS of eq. (12) is always positive, $\bar{\lambda}$ increases with increasing t. When $\lambda(t) \gg g^2(t)$, the contribution of the gauge bosons to the scalar sector becomes irrelevant and we are back to the same triviality problem encountered in a pure scalar field theory without the trapping of g.

In the case $\Delta \geq 0$ we can write $\beta(\bar{\lambda})$ as

$$\beta(\bar{\lambda}) = \frac{N}{16\pi^2}(\bar{\lambda} - \bar{\lambda}_-)(\bar{\lambda} - \bar{\lambda}_+) \tag{14}$$

where $\bar{\lambda}_{+,-}$ are two real roots of $\beta(\bar{\lambda}) = 0$ ($\bar{\lambda}_+ \geq \bar{\lambda}_-$). Notice that at $t = 0$, $\bar{\lambda}(0) = \frac{\lambda_R}{g_R^2}$ is the ratio of renormalized couplings. If $\bar{\lambda}(0) > \bar{\lambda}_+$ then $\beta(\bar{\lambda}) > 0$ and $\bar{\lambda}$ increases with increasing t just as in the case $\Delta < 0$. We are again back to the usual triviality problem.

For $\bar{\lambda}_- < \bar{\lambda}(0) < \bar{\lambda}_+$ (or $\bar{\lambda}(0) < \bar{\lambda}_-$), we have $\beta(\bar{\lambda}) < 0$ ($\beta(\bar{\lambda}) > 0$) and $\bar{\lambda}$ decreases (increases) with increasing t until $\bar{\lambda} = \bar{\lambda}_-$, an ultraviolet stable fixed point. The Landau singularity (and hence the triviality problem) is avoided in this case. However, to have a stable vacuum $\bar{\lambda}_- \geq 0$ is required. Since $\bar{\lambda}_+ \geq \bar{\lambda}_-$, the requirement of stability translates into two positive roots. In terms of the general form of the β-function (eq. (13)) this can be fulfilled only by having $B < 0$ and $C \geq 0$ simultaneously.

Altogether we have the following conditions for a stable and nontrivial theory:

a) $b_0 \geq 0$ otherwise g has a Landau singularity

b) $\Delta = B^2 - 4C \geq 0$ to have two real roots

c) $B < 0$

d) $C \geq 0$

Conditions (c) and (d) are necessary for two positive roots. Note that these conditions link group properties with combinatorics of diagrams at the same time in different ways. In the large N limit eq. (12) gives immediately $B = \frac{b_0}{N} - 3 = \frac{13}{3} > 0$ in conflict with condition c) above. From that, we conclude that, in the large N limit, the gauged $O(N)$ scalar field theory without fermions does not help to avoid the triviality problem. This is well known (see for example [7]).

Let us now include fermions which are taken, for simplicity and in analogy with the standard model, to be in the vector representation (just as with the scalars). Let those fermions be chiral as well. Let n_f be the number of fermion multiplets which can be arbitrarily large. For definiteness, we take the left handed fermion fields to be non-singlets under $O(N)$ and their right-handed partners to be singlets. The couplings of Φ to the fermions are given by

$$\mathcal{L}_Y = \sum_i g_{f_i} \bar{\Psi}^i_L \Phi \Psi^i_R + h.c. \quad , \tag{15}$$

where

$$g_{f_i} = \frac{m_{f_i}}{< \Phi >} \quad . \tag{16}$$

The RG equation governing the evolution of λ is in the large N limit

$$\frac{d\lambda}{dt} = \frac{N}{16\pi^2}\left[\lambda^2 - \left(3g^2 - \sum_i g^2_{f_i}\right)\lambda + \frac{3}{4}g^4 - \sum_i g^4_{f_i}\right] \quad . \tag{17}$$

It follows that $\beta(\bar{\lambda})$ now becomes

$$\beta(\bar{\lambda}) = \frac{N}{16\pi^2}\left[\bar{\lambda}^2 + \left(\frac{b_0}{N} + \sum_i \left(\frac{g_{f_i}}{g}\right)^2 - 3\right)\bar{\lambda} + \frac{3}{4} - \sum_i \left(\frac{g_{f_i}}{g}\right)^4\right] \quad . \tag{18}$$

We also have

$$b_0 = \frac{22}{3}(N-2) - \frac{4}{3}n_f - \frac{2}{3} \quad \xrightarrow{\text{large } N} \quad \frac{22}{3}N - \frac{4}{3}n_f \quad . \tag{19}$$

If only light fermions exist, i.e. $\left(\frac{g_f}{g}\right)^2 \ll 1$, conditions a) to d) above give:

a) $\frac{n_f}{N} < \frac{11}{2}$

b) $\frac{n_f}{N} \leq \frac{13 - 3\sqrt{3}}{4}$ or $\frac{13 + 3\sqrt{3}}{4} < \frac{n_f}{N}$

c) $\frac{n_f}{N} > \frac{13}{4}$

Condition (d) is always true. The only allowed range is $\frac{n_f}{N} \in \left]\frac{13+3\sqrt{3}}{4}, \frac{11}{2}\right[$ where the existence of upper and especially lower bounds for the number of fermions is interesting. For these values of $\frac{n_f}{N}$ the roots $\bar{\lambda}_{+,-}$ are to a very good approximation given by

$$\bar{\lambda}_+ = |B| - \frac{C}{|B|} \ ; \quad \bar{\lambda}_- = \frac{C}{|B|} \ . \tag{20}$$

The highest $\bar{\lambda}_+$ and lowest $\bar{\lambda}_-$ both occur for the highest value of $\frac{n_f}{N}$:

$$\bar{\lambda}_+^{max} \simeq 2.72 \; ; \quad \bar{\lambda}_-^{min} \simeq 0.28 \; . \tag{21}$$

In consequence the upper bound on $\bar{\lambda}(0)$ is now $\frac{\lambda_R}{g_R^2} \leq 2.72$. Notice, however, that these upper bounds are representation dependent. Only in the case $\frac{n_f}{N} \in \left] \frac{13+\sqrt{3}}{4}, \frac{11}{2} \right[$ with these upper bounds being respected is the triviality problem avoided.

If heavy fermions are allowed (in analogy with the standard model) then the result is modified. For simplicity, let us assume that there are n_h ($\leq n_f$) degenerate heavy fermions and all others being light. We then have

$$\sum_i \left(\frac{g_{f_i}}{g}\right)^2 = n_h x \; ; \quad \sum_i \left(\frac{g_{f_i}}{g}\right)^4 = n_h x^2 \quad , \tag{22}$$

where $x = (\frac{g_f}{g})^2$. The conditions a) to d) with free parameters x and n_h give now:

a) $\frac{n_f}{N} < \frac{11}{2}$ as before

b) $\left(\frac{13}{3} - \frac{4}{3}\frac{n_f}{N} + n_h x\right)^2 - 3 + 4 n_h x^2 \geq 0$

c) $\frac{13}{3} - \frac{4}{3}\frac{n_f}{N} + n_h x < 0$

d) $\frac{3}{4} - n_h x^2 \geq 0$

In Fig. 1 the resulting bounds in $(\frac{n_f}{N}, x)$ space are shown graphically for various n_h. Note that at this point the scale dependence of x was not taken into account. To do so is easy if we recognize that the condition for a running x is that the parameter does not run out of the original stability region in Fig. 1. Since nonabelian gauge fields are included the Yukawa couplings can be driven to zero for high scales if the initial values are small enough. Different authors have shown (see e.g. [5]) that g_f decreases faster than any gauge coupling, i.e. $x = (\frac{g_f}{g}) \to 0$ for $t \to \infty$. Note that the requirement of this fixed point can induce separate upper bounds on g_f (and therefore m_f) which are not discussed here.

Taking into account that x runs to its fixedpoint the allowed regions in Fig. 1 have to be reduced by those areas where a decreasing $x \to 0$ at constant $\frac{n_f}{N}$

runs out of the allowed region. In Fig. 1 the remaining allowed areas are shown shadowed. As a result $\frac{n_f}{N}$ has to be at least in the same area as for $n_h = 0$. If a finite x is used the lower bound for $\frac{n_f}{N}$ increases to a somewhat higher value. For high n_h the lower bound for $\frac{n_f}{N}$ can be driven for the highest allowed x value to the upper bound of $\frac{n_f}{N}$ (see Fig. 1). If n_h increases (maybe $n_h \sim N$) the allowed range for x is squeezed and finally only $x = 0$, $\frac{n_f}{N} \in \left]\frac{13+3\sqrt{3}}{4}, \frac{11}{2}\right[$ is left over like in the case $n_h = 0$. In all cases condition d) gives an upper bound on x which can be translated into an upper bound on the fermion mass:

$$x = \left(\frac{g_f}{g}\right)^2 \leq \sqrt{\frac{3}{4n_h}} \tag{23}$$

This condition holds for all scales* and in particular at $t = 0$. With the definition $<\Phi>= v$ we obtain**

$$\left(\frac{(g_f)_R}{g_R}\right)^2 = \frac{1}{2}\frac{m_f^2}{m_W^2} \leq \sqrt{\frac{3}{4n_h}}\ , \tag{24}$$

where $m_f = (g_f)_R\ v$ and $m_W^2 = \frac{1}{2}g_R^2\ v^2$. The roots $\bar{\lambda}_{+,-}$ equivalent to eq. (20,21) are now changed. The maximum value for $\bar{\lambda}_+$ is now taken for $C = \frac{3}{4} - n_h x^2 = 0$, where the upper bound for the fermion masses is saturated. Note that in this case $\bar{\lambda}_- = 0$ corresponds to an asymptotically free $\bar{\lambda}$. We have then $\bar{\lambda}_+ = |B| = \frac{4}{3}\frac{n_f}{N} - \frac{13}{3} - n_h\, x$ which is still maximal for the maximal value of $\frac{n_f}{N} = \frac{11}{2}$. For small n_h this can be written as $\bar{\lambda}_+ = |B| = 3 - \sqrt{\frac{3n_h}{4}}$. Overall, the maximum value for $\bar{\lambda}_+$ under variation of n_h is $\bar{\lambda}_+ \leq 3$. Using $m_H^2 = \lambda_R\ v^2$ and $m_W^2 = \frac{1}{2}g_R^2\ v^2$ we obtain

$$\frac{1}{2}\frac{m_H^2}{m_W^2} = \frac{\lambda_R}{g_R^2} \leq 3\ . \tag{25}$$

*For a particular $\frac{n_f}{N}$ condition b) gives for $n_h \geq 2$ sometimes stronger bounds. But if $\frac{n_f}{N}$ is free eq. (23) is an absolute upper bound on x.

** Notice that upper bounds on fermion masses can also be obtained from the requirement of vacuum stability of an effective potential (Ref. [8]). The present bound is tied, in addition, to the non-triviality of the theory.

In summary the problems of the $O(N_1) \times O(N_2)$ example showed how triviality should be avoided and that both gauging the group and coupling to fermions are helpful in this respect. Concentrating on the large N equivalent of SSB in the standard model several conditions had to be met in order for the setup to be consistent and stable. Consistency, i.e. avoiding triviality, and stability lead to conditions where group properties are linked to combinatoric factors in different ways. Having gauge bosons alone did not remove the problem. Yukawa couplings alone are not allowed because their RGE have no ultraviolet fixedpoint. The combined picture with gauge bosons and the right amount of fermions finally works. It is interesting to see that nonabelian gauge bosons are needed for two reasons. The gauge couplings are themselves asymptotically free and they produce asymptotically free Yukawa couplings. The Yukawa couplings are needed since the gauge bosons alone do not remove the triviality problem. In this sense the formal asymmetry of SSB is removed. Scalars are invented to break the symmetry and a balance of gauge bosons and fermions is needed to stabilize this picture. Surprisingly this stabilization also leads to a situation where all gauge, Higgs and Yukawa couplings are simultaneously asymptotically free.

Several bounds can be derived out of the consistency and stability requirements. There are upper bounds for λ in terms of g^2. If all free parameters are allowed to vary the bound $m_H \leq \sqrt{6}\, m_W$ is obtained. The number of fermions is restricted to be at least in the intervall $\left]\frac{13+3\sqrt{3}}{4}, \frac{11}{2}\right[\times N$ allowing an estimate of the necessary n_f for finite N: $n_f \simeq 5N$. Finally there exist upper bounds on the fermion masses determined by stability: $m_f \leq \left(\frac{3}{n_h}\right)^{\frac{1}{4}}\ m_W$. We noted that additional restrictions for m_f typically arise once the Yukawa couplings are studied in detail and their asymptotic behaviour is taken into account.

This treatment allows a systematic understanding of results for similar models with finite N. It especially explains the persistence of triviality problems far away from this bounds. Although $N = 2$ for the standard model is probably not close to a large N situation it is interesting to translate the results to this case.

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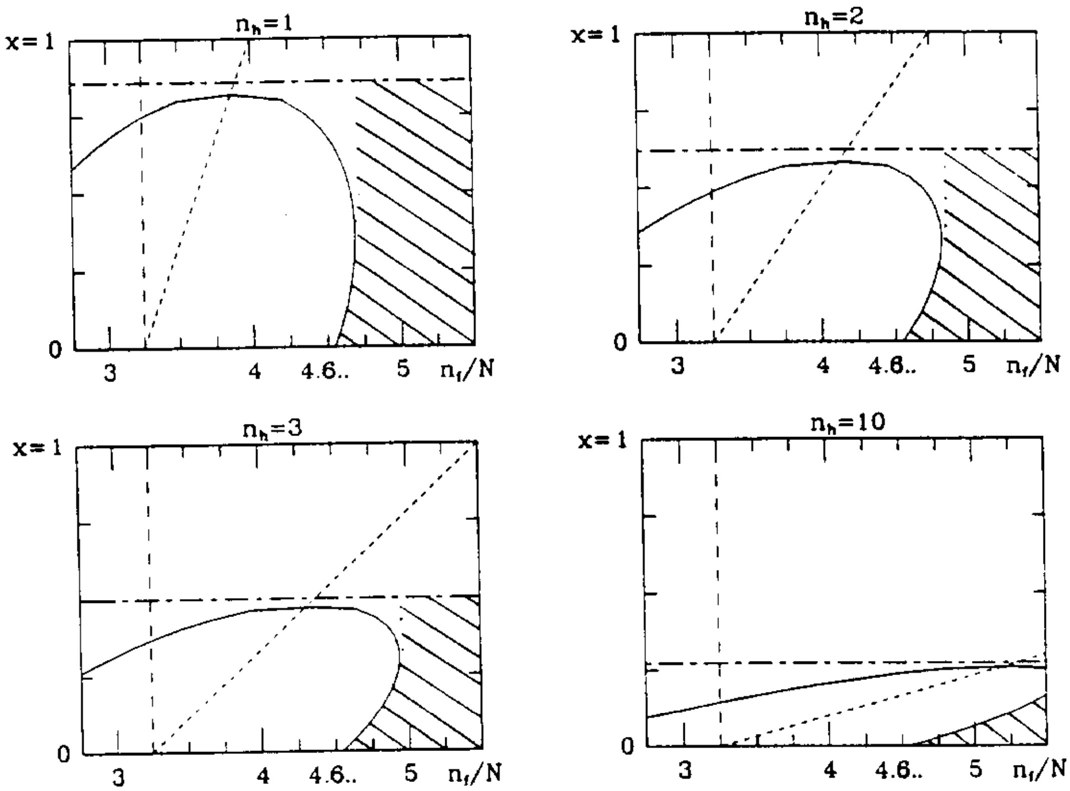

Fig. 1

The bounds in $(x, \frac{n_f}{N})$ space for different n_h. Condition (a) gives $\frac{n_f}{N} < \frac{11}{2}$. (d) gives the horizontal dash-dotted line. The dashed line is a lower bound for $\frac{n_f}{N}$ from (c) with $n_h = 0$. For the actual value of n_h (c) gives the more restrictive dotteded line. Finally (b) excludes the values inside of the solid curve. When the scale dependence of x is included the shadowed areas remain as allowed values.

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