Fermi National Accelerator Laboratory

FERMILAB–Conf–88/162–T
November, 1988

Large transverse momentum and higher twist phenomena[1]

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Abstract

A brief summary of the physics of large transverse momentum and higher twist phenomena is given.

[1]Summary talk at the Conference on Higher Twists and High p_T Physics, Paris, September, 1988.

1. Higher twist effects in deep inelastic scattering

A large part of this conference has been dedicated to the discussion of higher twists, so it is opportune to begin this summary talk with a brief reminder of how this rather peculiar terminology entered the subject.

The word 'twist' was introduced in the work of Gross and Treiman[1]. It is used to classify the importance of operators which appear in the light cone expansion of two currents. In totally inclusive lepton-hadron scattering, a photon of momentum q scatters off a hadron of momentum p. In the Bjorken limit, ($Q^2 = -q^2$ and $\nu = 2p.q \to \infty$ at fixed $x_B = Q^2/2/\nu$), the two currents are kinematically constrained to have a light-like separation, $x^2 \sim 0$. In this region we obtain[2],

$$J(x)J(0) \sim \sum_\tau E_\tau(x^2) \sum_s C_s(x^2\mu^2, g)\, x_{\alpha_1} \dots x_{\alpha_s}\, O^{\alpha_1 \dots \alpha_s}[\tau]. \tag{1.1}$$

This light cone expansion is a property of the interacting field theory which is extracted from perturbation theory. C is a dimensionless function calculable in perturbation theory. The singular function has dimension $E(x^2) \sim x^{-(2d_J-\tau)}$. The operators $O[\tau]$ are classified by their twist τ, which is defined as the mass dimension of the operators d minus their maximum spin s,

$$\tau = d - s \tag{1.2}$$

Thus, for example, we may write,

$$\begin{aligned}
\overline{\psi}\gamma^{\alpha_1}\psi, &\qquad \tau = 3 - 1 = 2 \\
\overline{\psi}\gamma^{\alpha_1}D^{\alpha_2}\dots D^{\alpha_s}\psi, &\qquad \tau = (s+2) - s = 2 \\
F^{\mu\alpha_1}D^{\alpha_2}\dots D^{\alpha_{s-1}}F^{\alpha_s}_{\mu}, &\qquad \tau = (s+2) - s = 2 \\
\overline{\psi}\gamma^{\alpha_1}\psi\overline{\psi}\gamma^{\alpha_2}\psi, &\qquad \tau = 6 - 2 = 4.
\end{aligned} \tag{1.3}$$

In calculating the dimension of these operators the fermion field ψ has mass dimension $\frac{3}{2}$, whereas the gluon field A^α has a mass dimension of unity. The covariant derivative D^α has twist zero. The operators of twist 2 are responsible for the approximate scaling which is observed in deeply inelastic lepton hadron scattering. The matrix elements of the operators of twist two predict the moments of the structure functions,

$$\langle p|O^{\mu\nu\alpha_1\ldots\alpha_{2n}}[\tau=2]|p\rangle = \frac{(-1)^n}{2n!} p^\mu p^\nu p^{\alpha_1}\ldots p^{\alpha_{2n}} \int_0^1 dx x^{2n} F_2(x). \qquad (1.4)$$

Operators of higher twist lead to corrections which are suppressed by powers of Q^2, the scale which characterises the hardness of the virtual photon hadron scattering. By extension this terminology has come to be used for effects which are suppressed by powers of the large scale Q in any hard process. Thus the term higher twist is nothing more than a jargon expression for power suppressed effects.

The light cone expansion gives a complete description of deep inelastic scattering both at the level of leading twist ($\tau = 2$) and at the higher twist level ($\tau > 2$). Nevertheless it has to a large extent been superseded in the description of leading twist effects by the QCD improved parton model[3] which was introduced some ten years ago. The QCD parton model combines the attractive space-time picture of the parton model with the solid theoretical basis provided by perturbative QCD. Its great advantage is that it is applicable not only to light cone dominated processes but also to other hard processes. At leading twist level the QCD improved parton model for a process with one incoming parton can be written as,

$$\sigma(P) = \sum_i \int dx\ \hat{\sigma}_i(xp)\ F_i(x,\mu). \qquad (1.5)$$

The short distance partonic cross-section is denoted by $\hat{\sigma}$ and the distribution of partons of type i by F_i. The moments of the parton distribution functions are determined by the matrix elements of the leading twist operators.

Beyond the leading twist level, there is no *simple* partonic interpretation of power suppressed effects. For a recent discussion and references to earlier literature see ref. [4]. The impulse approximation, inherent in the parton picture is no longer correct when one considers power suppressed effects. At the $1/Q^2$ level there are real interference effects which must be taken into account. There is a complete classification of the operators which occur at the $1/Q^2$ level and a calculation of the coefficient functions with which they appear[5]. However the operator basis can be chosen in several different ways[6]. In the parton language the matrix elements of these operators translate into parton correlation functions which, depending on the basis chosen, describe either the longitudinal or transverse correlations of the partons inside the hadron.

I shall now give a brief description of power suppressed effects in deep inelastic scattering (DIS) following the treatment of ref. [6], although I would stress that there are other equivalent treatments. To begin the discussion of power suppressed effects in DIS it is convenient to consider a *model* in which the distribution of partons in longitudinal and transverse momentum is specified by the distribution $F(x,k_T)$. The incoming massless partons are put on their mass-shell $k^2=0$. The resulting structure function for a single species of quark with unit charge is,

$$\begin{aligned} F_L(x_B,\frac{\Lambda^2}{Q^2}) &= \frac{4}{Q^2}\int d^2k_T k_T^2 F(x_B,k_T) = \frac{4\langle k_T^2\rangle}{Q^4} \\ F_T(x_B,\frac{\Lambda^2}{Q^2}) &= \int d^2k_T F(x_B,k_T) + \frac{1}{Q^2}\int d^2k_T k_T^2\left[4F(x_B,k_T) + x_B\frac{\partial}{\partial x_B}F(x_B,k_T)\right] \end{aligned} \tag{1.6}$$

F_T and F_L are related to the standard structure functions by $F_T = F_2/x_B$ and $F_L = F_2/x_B - 2F_1$. The presence of the intrinsic transverse momentum has generated a longitudinal structure function. The transverse structure function contains both scaling and power suppressed pieces. The power suppressed piece of F_T contains two terms and is not *a priori* of definite sign.

This model has several deficiencies which are related. For example, the Adler sum rule, which is an exact current algebra sum rule, is not satisfied. This is a consequence of the ansatz for the parton distribution function. In order to define the transverse direction with $p^2=0$ we have to introduce an extra vector which is not present in the original problem. If we restore the Lorentz invariance with $p^2=0$ the model becomes trivial ($k_T=0$). The model is non-trivial only if the target mass is non-zero. In this case Lorentz invariance requires that F have the form, ($k^2=0$),

$$F(x,k_T^2) = \frac{1}{\pi M^2}\Phi\left(\frac{2pk}{M^2}\right)\theta((p-k)^2) = \frac{1}{\pi M^2}\Phi\left(x+\frac{k_T^2}{xM^2}\right)\theta(x(1-x)M^2-k_T^2) \tag{1.7}$$

The x and k_T dependences are related. Working out the kinematics exactly[7] we see that this form reproduces ξ scaling[8], where the average value of the transverse momentum in Eq. (1.6) is related to an integral over the longitudinal degrees of

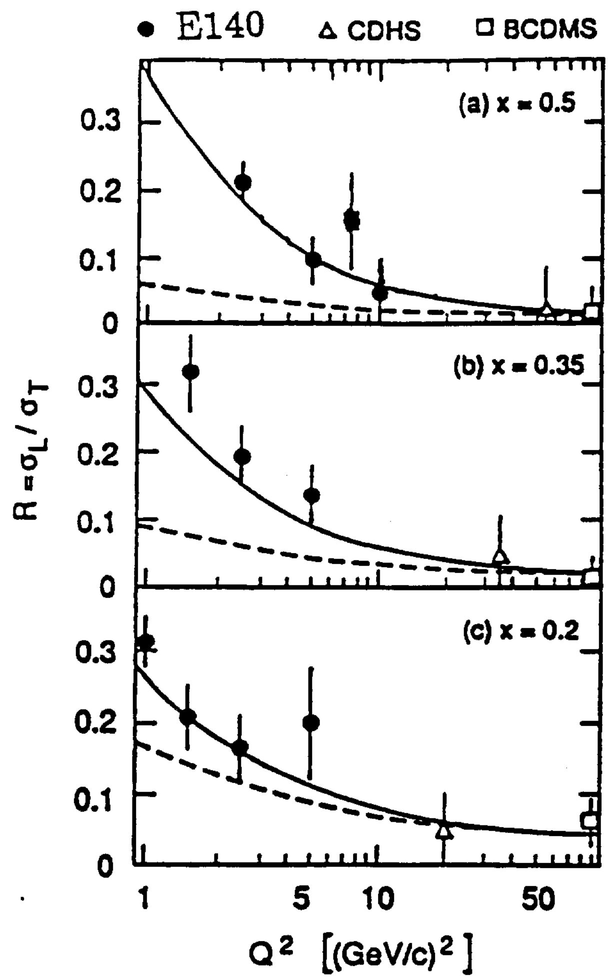

Figure 1: R as a function of Q^2 for various x.

freedom,

$$\left\langle k_T^2 \right\rangle = M^2\xi^2 \int_\xi^1 \frac{d\eta}{\eta} F(\eta), \quad \xi = \frac{2x_B}{(1+\sqrt{1+4M^2x_B^2/Q^2})}, \quad F(\eta) = \xi \int_\xi^1 d\eta \Phi(\eta) \quad (1.8)$$

Depite the fact that it is wrong, the model given by Eq. (1.6) has many of the features of the interacting QCD theory[6]. Even in the full interacting theory, the higher twist description of the structure function F_L is simpler than the description of F_T and the higher twist corrections to F_L satisfy a positivity constraint just as in Eq. (1.6). But without a reliable way of evaluating the hadronic matrix elements of the twist four operators there is little predictive power in the operator formalism.

The experimental information on the size of higher twist terms in Deep Inelastic

Scattering is also quite meagre. At this conference Bodek has reported results on the ratio $R = \sigma_L/\sigma_T$ from experiment E-140 [9]. Fig. 1 shows R as measured by the experiments E140[9], CDHS[10] and BCDMS[11]. The solid line shows the prediction from $O(\alpha_S)$ perturbative QCD and the dashed line the prediction from $O(\alpha_S)$ QCD with target mass corrections[8]. These results demonstrate behaviour consistent with a $1/Q^2$ fall-off. Consistency is found with the ξ scaling formula, and the data can presumably be used to limit the size of a dynamical higher twist term.

Several groups have attempted combined fits to both the logarithmic and power scale violating effects in F_2. The particular functional form chosen was,

$$F_2(x,Q^2) = F_2^{\rm LT}(x,Q^2)\left[1+\frac{H_4(x)}{Q^2}\right] \tag{1.9}$$

The results of the fits to this form are shown in Fig. 2. There is an apparent discrepancy between the results obtained by combining the SLAC results with higher energy experiments[12] and the results obtained by WA59[13]. The higher twist fits to the combined data sets of SLAC/EMC[12] and SLAC/BCDMS[14] give qualitatively similar results.

2. Power suppressed effects at kinematic boundaries

In the previous section the general description of power suppressed effects in Deep Inelastic Scattering was reviewed. In this section I shall discuss a model of power suppressed effects in processes involving mesons due to Berger, Brodsky (BB) and co-workers[15]. As the kinematic limit in which one of the partons carries all the momentum of the incoming meson is approached, the approximation in which the active parton is treated as being on its mass shell becomes less and less accurate. The meson participates as a whole in the reaction, and the longitudinal momentum is transferred from one constituent to the other by the exchange of a single gluon. Since the exchanged gluon is also far off-shell the use of perturbation theory is presumably justified. In this model the pion is described by the quark-antiquark Fock state augmented with one gluon exchange.

The predictions of this model are most developed for the Drell-Yan process. In

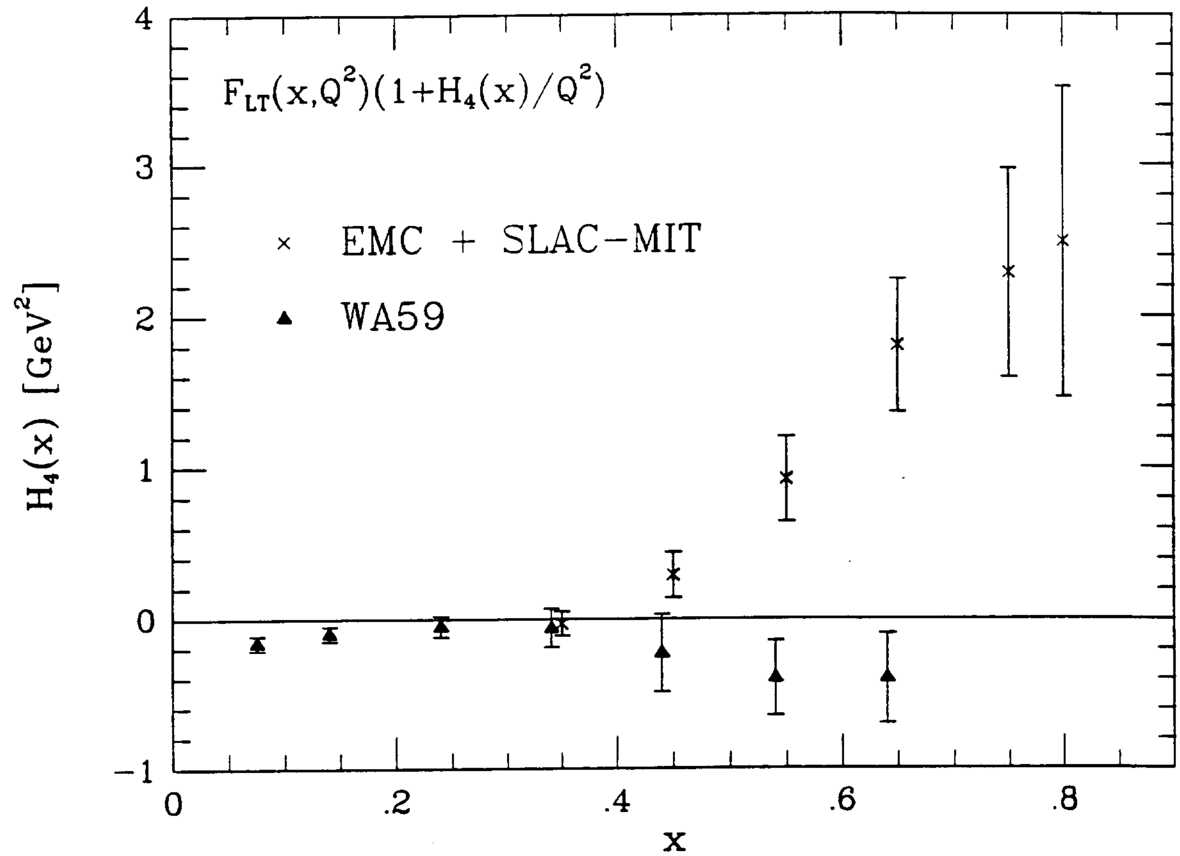

Figure 2: Extraction of higher twist terms from fits to F_2.

this case the pion structure function is predicted to behave asymptotically as,

$$q_\pi(x) \sim (1-x)^2 + \frac{2}{9}\frac{\langle k_T^2 \rangle x^2}{Q^2}. \tag{2.1}$$

although sub-asymptotic effects may be important[17]. The experimental results[18] indicate a behaviour,

$$q_\pi(x) \sim x^\alpha (1-x)^\beta + \frac{2}{9}\frac{\gamma}{Q^2} \tag{2.2}$$

with $\alpha \sim 0.4, \beta \sim 1.2$ and $\gamma \sim 0.5$ GeV^2.

Evidence for these effects have been reported in many contexts at this meeting.

1. $\pi N \to \mu^+\mu^- + X$, [18,19]

2. $\pi N \to \mathrm{Jet}_1 + \mathrm{Jet}_2 + X$, [20]

3. $\nu N \to \mu^+\pi + X$, [21]

4. $\mu N \to \mu\pi + X$, [22]

5. $\gamma p \to \pi + X$, [23]

6. $\pi N \to \rho + X$, [24]

Since these power suppressed effects which are present near kinematic boundaries have been reviewed by Berger[16], I shall limit myself to a few qualitative remarks. The great merit of the model of the Berger *et al.* is the fact that it makes concrete predictions. There can be no doubt some of the features expected in the BB model have been observed in the data. It is therefore important to take the model more seriously. From the data one would like to see whether the absolute normalisations of the observed effects are in agreement with the predictions of BB. It is also important to test the predicted $1/Q^2$ fall-off. On the theoretical side one should remark that for many of these processes only the asymptotic terms have been evaluated. Experience from the Drell-Yan process indicates that the asymptotic solutions may not be sufficient[17].

Let me conclude this section by remarking that such effects are extremely small cross-sections which occur near to the edge of phase space. It is also important to understand the bulk of the cross-section!

3. Heavy flavour production

The photoproduction and hadroproduction of hadrons containing heavy quarks has been extensively studied during the last two years. It is believed[25], (even though no all orders proof exists), that the total cross-section for the inclusive production of a heavy quark pair is described by a QCD improved parton model formula,

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \; \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) \; F_i^A(x_1, \mu) F_j^B(x_2, \mu) \tag{3.1}$$

where S is the square of the centre of mass energy of the colliding hadrons A and B, and F are the number distributions of partons. The sum on i and j runs over the light quarks and gluons, but not over the heavy quarks. Processes involving heavy quark constituents from the incoming hadrons are suppressed by powers of m, the heavy quark mass[26]. Interactions of the produced heavy quarks with spectator partons are also suppressed by powers of m and are therefore higher twist effects. A model of spectator interactions involving a limited class of graphs treated in a non-relativistic approximation has been presented in ref. [27]. When integrated outside the range of validity of the approximation the model suggests that these power corrections may be of order Λ/m, rather than Λ^2/m^2 as originally expected. In view of the potential phenomenological importance of these terms for charm production, it is important to check whether the result of ref. [27] survives in a complete analysis.

The short distance cross-section $\hat{\sigma}$ given in Eq. (3.1) is calculable as a perturbation series in the running coupling $\alpha_S(\mu)$ where μ is the renormalisation and factorisation scale. Early work on QCD corrections to heavy quark production is given in refs. [28, 29, 30, 31]. In ref. [32] the first radiative corrections to the heavy quark hadroproduction cross-section were presented, taking into account all sub-processes and including both real and virtual corrections. The resultant form of the short distance cross-section is,

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_S^2(\mu^2)}{m^2} \left\{ f_{ij}^{(0)}(\rho) + 4\pi\alpha_S(\mu^2) \left[f_{ij}^{(1)}(\rho) + \bar{f}_{ij}^{(1)}(\rho) \ln(\frac{\mu^2}{m^2}) \right] + O(\alpha_S^2). \right\} \tag{3.2}$$

with $\rho = 4m^2/s$, and s the square of the partonic centre of mass energy. In ref. [32] a complete description of the functions f_{ij} including the first non-leading correction was presented. These may be used to calculate heavy quark production at any energy and heavy quark mass. The functions f_{gg} have been also calculated in ref. [33] and the results are found to be in agreement with ref. [32].

The important properties of the functions f are as follows. The functions $f_{gg}^{(1)}, f_{gq}^{(1)}, \overline{f}_{gg}^{(1)}$ and $\overline{f}_{gq}^{(1)}$ tend to calculated constants[34] at high energy, because the higher order corrections involve the exchange of a spin one gluon in the t-channel. The lowest order terms, $f_{q\bar{q}}^{(0)}$ and $f_{gg}^{(0)}$ involve at most t-channel quark exchange and therefore fall off at large s. Near threshold the higher order terms $f_{q\bar{q}}^{(1)}$ and $f_{gg}^{(1)}$ display a very rapid $\ln^2(\beta^2)$ growth, $\beta = \sqrt{1 - 4m^2/s}$. The origin of these correction terms which are numerically important is explained in ref. [32]. For attempts to resum terms of this form in Drell-Yan processes we refer the reader to ref. [35].

Complete theoretical results for the case of photoproduction of heavy quarks in order $\alpha_S^2\alpha$ have been presented in ref. [36]. The real $O(\alpha_S^2\alpha)$ matrix elements for photoproduction are given in ref. [37].

3.1 Photoproduction of heavy flavours

New results have been presented on the photoproduction of charmed particles by E691[38]. This experiment has a large number of fully reconstructed charmed particles ($\sim 10^4$) and good acceptance in the forward region where the bulk of the photoproduction cross-section is expected to be produced. Since the experiment is performed on a light nucleus the extraction of the cross-section per nucleon does not introduce a major ambiguity. Fig. 3 shows a comparison of the new data with the lowest acceptable theoretical predictions[36] taking into account the uncertainties associated with the choice of input parameters. The major uncertainty associated with the value of the heavy quark mass is shown explicitly. Values of the heavy quark mass $m_c < 1.5$ GeV are excluded. Substantial agreement is found between the data points in Fig. 3 and earlier values, for example those of the EMC collaboration[39].

3.2 Hadroproduction of heavy flavours

The experimental situation for the hadroproduction of heavy quarks has been reviewed in refs. [40,41]. Before comparison can be made with total cross-section predictions per nucleon, one must extrapolate the measured rate to the whole of phase

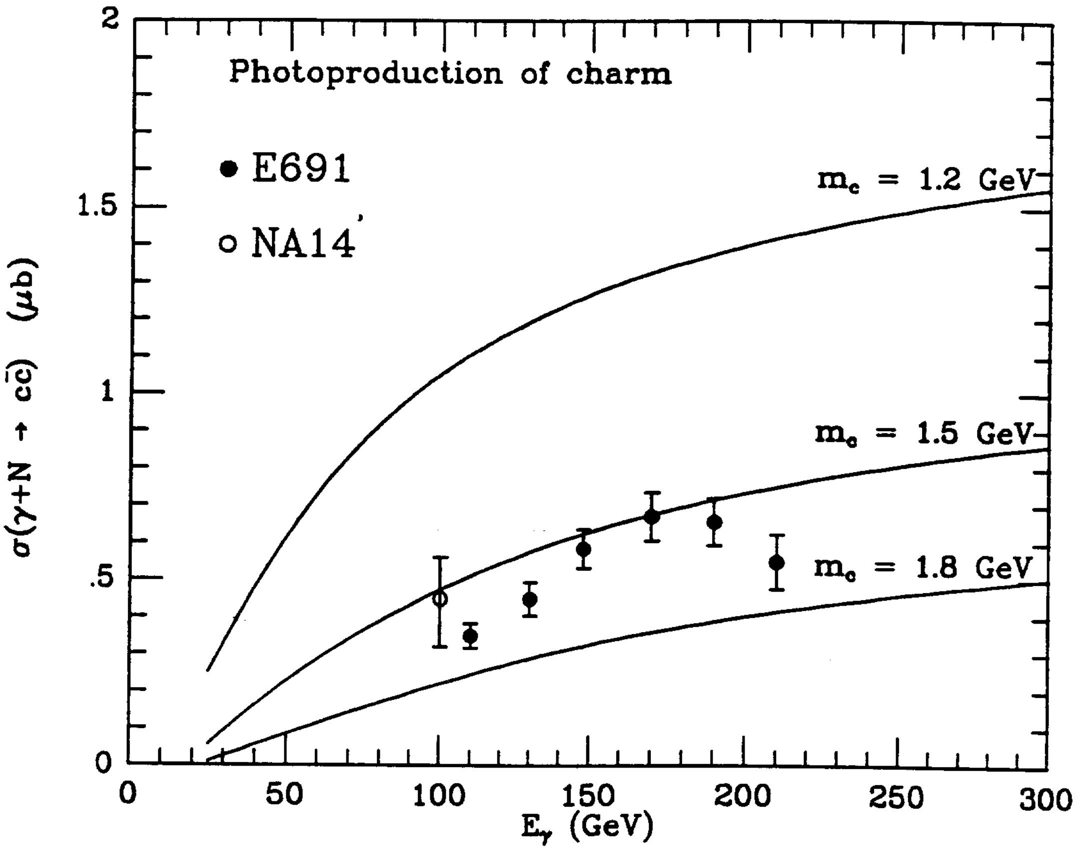

Figure 3: The measured $\gamma N \to c\bar{c} + X$ cross-section compared with theoretical lower bounds for various values of the mass.

space, correct for the branching ratio to the observed decay mode and if necessary reduce the nuclear cross-section to a cross-section per nucleon.

The new experimental results for charm production can be summarised as follows. There is now considerable evidence that the nuclear dependence of the cross-sections is given by A^{α} with $\alpha \neq 1$[42]. There are a large number of experiments for which agreement can be found with the QCD parton model predictions[43] including the higher order corrections, with a charm quark mass $m_c \sim 1.5$ GeV. However there are still some experiments which apparently after extrapolation lead to much larger total cross-sections than predicted by the leading terms in the QCD parton model.

Experimental rates for bottom production have been given in refs. [44,45]. Because of the heavier quark mass the predictions for bottom production are expected to be more reliable. This is true at fixed target energies but at collider energies they become less certain because of the presence of two scales $S \gg m^2$. The agreement of the measurements with theoretical predictions is fair[40].

The expected rates for top quark production are shown in Fig. 4 based on the calculations of ref. [32] and using the structure functions of Diemoz *et al.*[46]. As a result of the α_S^3 calculations the UA1 limit[47] has been revised[43] and is now $m_t > 41$ GeV. From this limit and Fig. 4 one can estimate that about 1000 t or $\bar{t}$ events need to be produced in order to set a limit. Based on this number one can extrapolate to the likely discovery limit on the top quark in the upcoming runs at CERN and FNAL. With 1 pb^{-1} at $\sqrt{S} = 1.8$ TeV or 10 pb^{-1} at $\sqrt{S} = 0.63$ TeV one should be able to discover a top quark with a mass less than 80 GeV.

3.3 The production of J/ψ

Because of its relatively clean experimental signature the production of J/ψ is an attractive channel in which to investigate heavy flavour production. The theory of J/ψ production has been reviewed by Rückl[48]. There are two main reasons for the interest in the production of J/ψ. Firstly, it has been suggested that quarkonium production is a good channel in which to measure the gluon distribution function[49,50]. In addition the secondary production of a J/ψ from the decay of a bottom meson, which occurs with a branching ratio,

$$BR(B \to J/\psi + X) \sim 1.2 \pm 0.3\% \tag{3.3}$$

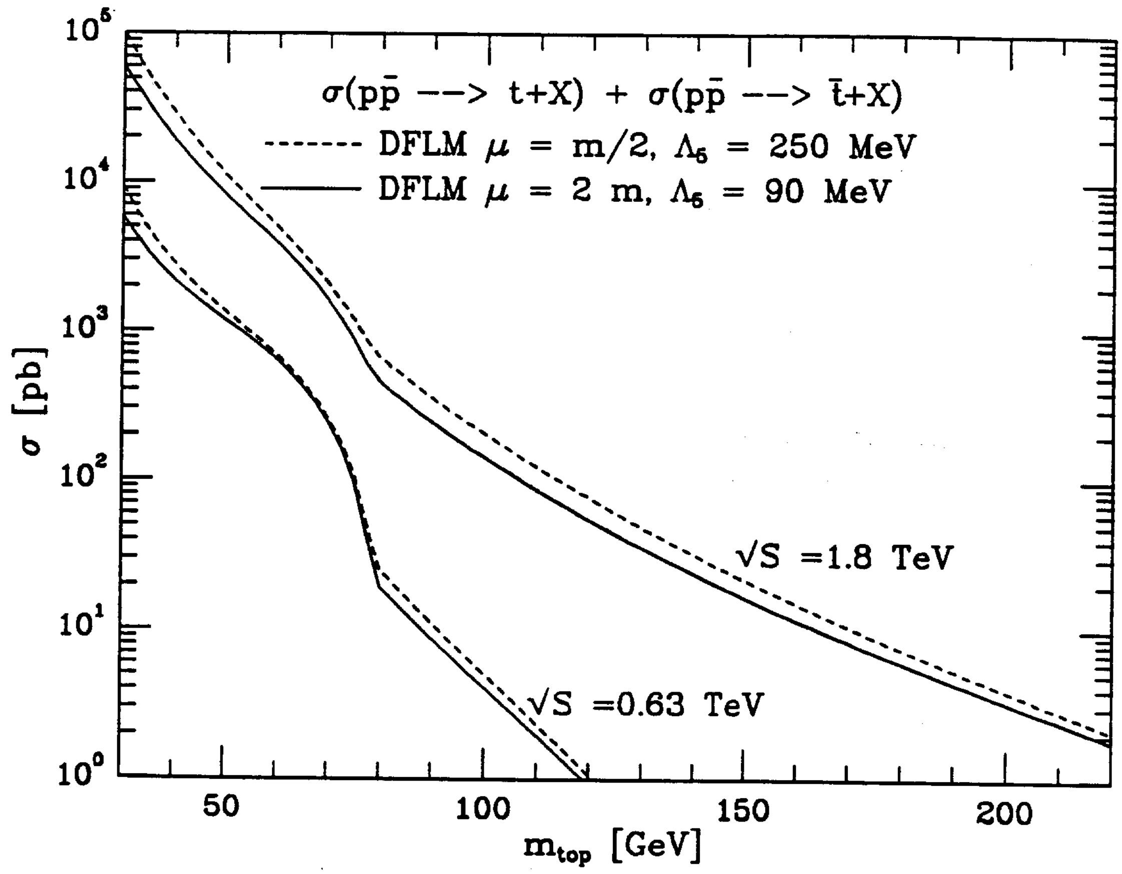

Figure 4: Top quark production as a function of the mass

provides a good way to tag bottom quarks. Both of these techniques obviously rely on an accurate understanding of the QCD production mechanism.

The theory of the photoproduction of J/ψ is described in the paper of Berger and Jones[51]. The older treatment[52] based on semi-local duality arguments is not predictive enough, since it gives no information on the relative rates of different channels. Perturbative QCD can only describe the inelastic part of the cross-section. In practice it is found[53] that for

$$z \equiv \frac{p_\psi . p_N}{Q . p_N} = \frac{E_\psi}{E} < 0.8, \quad \frac{p_T^2}{m_\psi^2} > 0.1 \tag{3.4}$$

the shape in z and p_T of the data is adequately described by the model of Berger and Jones. The results on the photoproduction of J/ψ presented at this conference by the EMC and NA14 collaboration confirm this conclusion[39,54]. Note however that with $m_c \sim 1.5$ GeV and $\Lambda \sim 0.2$ GeV the normalisation of the theoretical prediction lies well below the data.

The theoretical normalisation is uncertain because the higher order corrections are unknown. There is no information on the correct choice of scale μ in the running coupling constant except that it should be of order m_c. *A priori* such higher order corrections can also change the shape of the p_T and z distribution, but experience from other processes suggests that any such changes will be rather small. The treatment of the J/ψ wave function using the non-relativistic approximation introduces another source of normalisation uncertainty.

First results on the extraction of the gluon distribution function from J/ψ production have been presented by the EMC collaboration[39]. In view of the scarcity of reliable methods of obtaining information about the gluon distribution, the photoproduction of J/ψ merits further theoretical attention. However just as in the production of open charm the fact that m_c is not very much bigger than the scale of the strong interactions Λ means that these experiments will never give high precision tests of perturbative QCD.

4. Vector boson production

4.1 Direct photon production

Experiment	$\sqrt{S}$ (GeV)	x_T range
WA70[59]	22.9	$0.35 < x_T < 0.56$
UA6[60]	24.3	$0.25 < x_T < 0.58$
E706	31.5	$0.25 < x_T < 0.51$
UA2[61]	630	$0.04 < x_T < 0.22$
UA1[62]	630	$0.06 < x_T < 0.28$
CDF	1800	$0.02 < x_T < 0.03$

Table 1: Reported x_T ranges for experiments giving information on the gluon distribution function in the proton.

The production of direct photons at large p_T proceeds at lowest order via the two parton sub-processes,

$$q + \bar{q} \to \gamma + g, \quad g + q(\bar{q}) \to \gamma + q(\bar{q}). \tag{4.1}$$

The observation of the direct photon p_T spectrum therefore provides an alternative way to measure the gluon distribution function. The advantage of this method over deep inelastic scattering is that the gluon distribution function contributes directly in the Born approximation. In addition the produced photon is observed directly, allowing a relatively simple kinematic reconstruction of the event. In practice it is often necessary to introduce an isolation cut in order to remove the background from hadronic decays. This isolation cut must be taken into account when making the comparison with theory. A complete next to leading order calculation $O(\alpha\alpha_S^2)$ is available[55,56] so that in principle both the value of $\Lambda_{\overline{MS}}$ and the gluon distribution function can be determined from these experiments.

A large number of experiments have been analysed[56,57] using the structure functions of Duke and Owens[58], which exist in two versions, a soft gluon version (DO1) with $\Lambda = 0.2$ GeV and a hard gluon version with $\Lambda = 0.4$ GeV. The experiments favour the soft gluon fit (DO1) corresponding to the smaller value of Λ. First results have also been presented on a combined second order fit to deep inelastic scattering and direct photon data. They are found to provide complementary information on $\Lambda_{\overline{MS}}$ and the form of the gluon distribution[55].

The separation of the direct photon signal from the background is only possible for a limited range of $x_T = 2p_T/\sqrt{S}$. The values of x_T to which the the experiments are sensitive is shown in Table. 3. The range of x_T also determines the range in x in which the gluon distribution can be measured. The minimum value of x to which an experiment is sensitive is given by $x_{\min} = x_T/(2 - x_T)$. Therefore it is more appropriate to consider that these experiments determine the value of $\alpha_S(\mu^2)\, G(x, \mu^2)$ for $\mu \sim p_T, x \sim x_T$ rather than the shape of the whole gluon distribution function.

The observation of quasi-real photons which materialise into lepton pairs may allow the extension of direct photon studies down to lower values of p_T, yielding valuable information about the low x behaviour of the gluon distribution function[63]. Experimental results using this technique have been presented in ref. [64].

4.2 W and Z production

The production and decay properties of the W and Z bosons observed at hadron colliders test many features of the standard model. The branching ratio of the W into leptons depends on the mass of the top quark and/or the mass of a possible heavy lepton. The branching ratio of the Z also depends on the number of massless neutrino species. The production cross sections of the W and Z are calculable in perturbative QCD using the parton distribution functions measured in Deep Inelastic Scattering. The simplest quantity to analyse is the total cross-section. The total cross section for the production of a boson of mass M is determined by the convolution of the parton distribution functions F with the short distance cross-section Δ,

$$\sigma = \sum_{i,j} \int dx_1 dx_2 dz\; \delta(x_1 x_2 z - \tau)\; F_i(x_1, \mu) F_j(x_2, \mu) \Delta_{ij}(z) \tag{4.2}$$

where $\Delta_{ij}(z) = \Delta_{ij}^{(0)}(z) + \alpha_S \Delta_{ij}^{(1)}(z) + \alpha_S^2 \Delta_{ij}^{(2)}(z) + \ldots$ and $\tau = M^2/S$. In lowest order only quark antiquark annihilation contributes, $\Delta_{q\bar{q}}^{(0)}(z) \sim \delta(1-z)$, whereas in higher orders initial states containing gluons give non-zero contributions. The full result for $\Delta_{ij}^{(1)}(z)$ is known[65]. In the case of W production at $\sqrt{S} = 0.63$ TeV, the inclusion of the term $\Delta^{(1)}$ increases the estimate based on $\Delta^{(0)}$ alone by about 30%.

A partial result has been presented in ref. [66] for $\Delta_{q\bar{q}}^{(2)}(z)$. All terms associated with soft and virtual gluons are included, but the effects of the emission of hard partons are neglected. Applying the same approximation to $\Delta^{(1)}(z)$, which is fully known, one can test the validity of the soft and virtual approximation. It is found

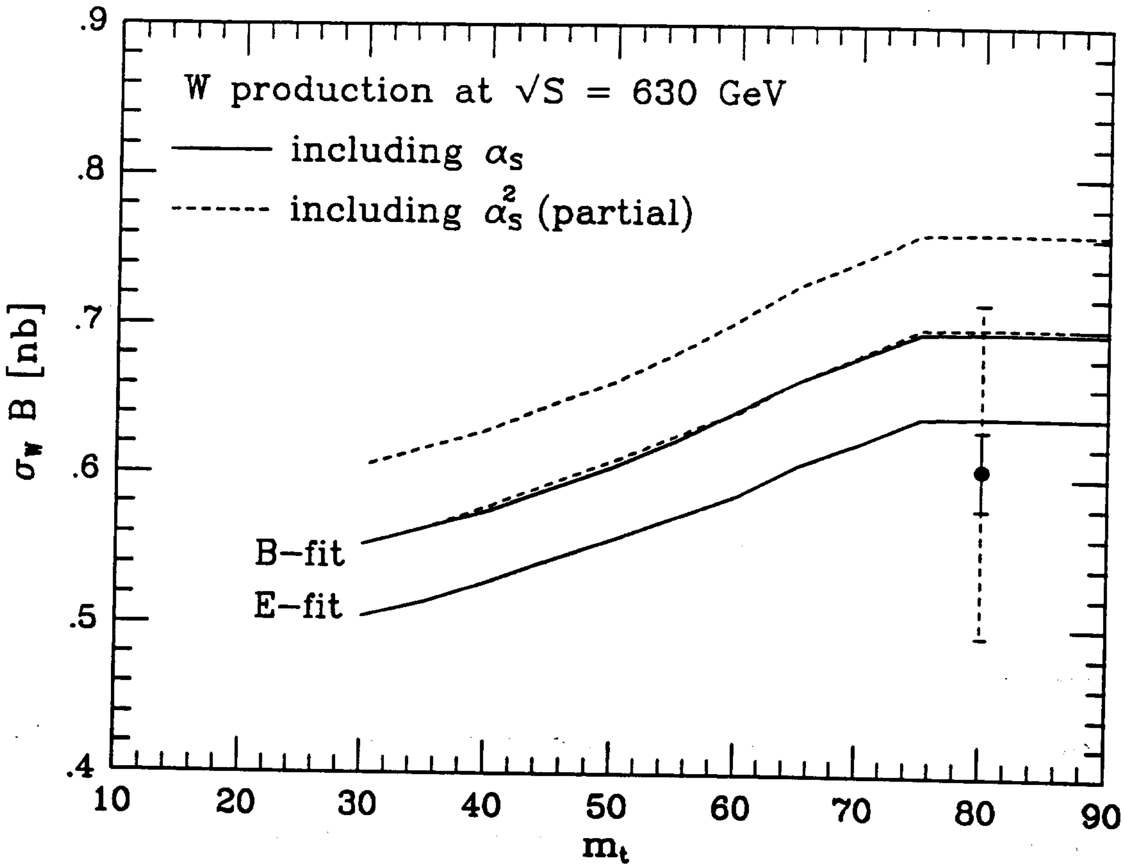

Figure 5: $\sigma_W B$ as a function of the top quark mass compared with data.

that approximate form deviates from the full result for $\Delta^{(1)}(z)$ by about 20% in the calculation of W production at CERN energies[67].

In Fig. 5 a comparison of the theoretical prediction for $\sigma_W B(W \to e\nu)$ with the experimental results of UA1[68] and UA2[69] is shown. The statistical errors of the two experiments have been combined in quadrature and the resultant error added linearly to the average of systematic errors of the two experiments. The theoretical curves correspond to the BCDMS like fit and the EMC like fit of ref. [70]. They are shown as a function of the top quark mass before and after the inclusion of the partial $O(\alpha_S^2)$ result. With the present errors no information on the top quark mass can be obtained from σB. Note however that agreement with the experimental results relies on the inclusion of the $O(\alpha_S)$ contribution. The quoted result for the W cross section at $\sqrt{S} = 1.8$ TeV[71] is $\sigma B(W \to e\nu) = 2.6 \pm 0.6 \pm 0.5$ nb which is consistent with the QCD prediction for all values of the top quark mass.

5. The running of the coupling constant

Much of this conference has been dedicated to the study of effects which fall off like powers of Q, where Q is a large scale which characterises the hardness of the interaction. In order to put this research into perspective it is useful to summarise the progress which has been made in the description of effects which fall off like logarithms of the large scale. Logarithmic effects are formally always more important than power suppressed effects. In this section I shall discuss the the status of the search for the running of the strong coupling α_S.

5.1 The e^+e^- total cross-section

One of the theoretically cleanest predictions of QCD is $R^{e^+e^-}$, the ratio of the total e^+e^- hadronic cross-section to the muon pair production cross-section. An interesting new result for this quantity has been obtained by Gorishny *et al.*[72] who have calculated the third order coefficient in the perturbative expansion of $R^{e^+e^-}$. Ignoring for the moment weak interaction effects, the expansion for $R^{e^+e^-}$ is found to be,

$$\begin{aligned} R^{e^+e^-} &= 3\sum_f Q_f^2\left\{1+\left(\frac{\alpha_S}{\pi}\right)+\left(1.986-0.115f\right)\left(\frac{\alpha_S}{\pi}\right)^2\right. \\ &+ \left.\left(70.986-1.200f-0.005f^2\right)\left(\frac{\alpha_S}{\pi}\right)^3\right\}-\left(\sum_f Q_f\right)^2 1.679\left(\frac{\alpha_S}{\pi}\right)^3 \quad (5.1) \end{aligned}$$

The scale for the strong coupling has been chosen to be $\mu = \sqrt{S}$ and the renormalisation is performed in the $\overline{MS}$ scheme. Specialising to the case of $f = 5$, Eq. (5.1) becomes,

$$R^{e^+e^-} = 3\sum_f Q_f^2\left\{1+\left(\frac{\alpha_S}{\pi}\right)+1.411\left(\frac{\alpha_S}{\pi}\right)^2+64.810\left(\frac{\alpha_S}{\pi}\right)^3\right\}. \quad (5.2)$$

The third order term is very large, and perhaps larger than one would have expected based on the second order term alone. Two groups[73,74] have performed

Expression for R	$\alpha_S(\mu^2 = 1000 \text{ GeV}^2)$	$\sin^2\theta_W$	$\Lambda_{\overline{MS}}$ (MeV)
2 loops, $7 < \sqrt{S} < 56$	0.158 ± 0.020	0.23	420^{+300}_{-230}
3 loops, $7 < \sqrt{S} < 56$	0.143 ± 0.016	0.23	250^{+160}_{-130}

Table 2: Results of global fit to e^+e^- data on total cross-section

global fits to all data in e^+e^- annihilation using this result. Of course at the higher energies the effects of Z boson exchange cannot be ignored. Leaving $\sin^2\theta_W$ free, both groups find values consistent with the world average value[75]. The values of α_S obtained in ref. [73] fixing $\sin^2\theta_W$ at the world average are given in Table 1. The inclusion of the third order leads to a 10% change in the value of α_S at $\mu^2 = 1000$ GeV^2. Note that within the experimental errors the values of α_S derived using second and third order perturbation theory agree.

5.2 The QCD perturbation series

Given a value of α_S, one can extract a value of the QCD parameter Λ. The relationship between the measured coupling constant and Λ is unfortunately not unambiguous. In order to make it completely clear one must also specify, a) which renormalisation scheme was used, b) how the heavy flavour thresholds were treated, c) which renormalisation scale was used, d) which functional relation between α_S and Λ was used. Some of these ambiguities can be eliminated simply by specifying α_S at a particular scale. The running coupling is a solution of the renormalisation group equation, $d\alpha_S(\mu)/d\mu^2 = -b_0\alpha_S^2(1 + b_1\alpha_S + b_2\alpha_S^2 \ldots)$. By convention Λ is defined by,

$$\alpha_S(\mu) = \frac{1}{b_0 \ln(\mu^2/\Lambda^2)}\left[1 - \frac{b_1}{b_0}\frac{\ln\ln(\mu^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)} + \ldots\right]. \tag{5.3}$$

Λ depends on the number of active flavours. Values of Λ for different numbers of flavours are defined by imposing the continuity of α_S at the scale $\mu = m$, where m is the mass of the heavy quark. In table 2 the values of the coefficients b_0 and b_1 for 3, 4 and 5 active light flavours are shown.

The coefficients in the perturbation expansion, Eq. (5.1) are given for the special choice of the renormalisation scale $\mu = \sqrt{S}$. In general the coefficients of all QCD

f	b_0	b_1
3	.7162	.5659
4	.6631	.4902
5	.6101	.4013

Table 3: Corresponding values of b_0 and b_1 with f active flavours.

perturbative expansions depend on the choice made for the renormalisation scale μ. Thus, for example, for an arbitrary choice of the scale μ, Eq. (5.1) can be written,

$$R^{(j)} = R^{QPM}\left\{1+\sum_{i=1}^{j} r_i(\mu)\left(\frac{\alpha_S(\mu)}{\pi}\right)^i\right\}, \quad r_1 = 1. \tag{5.4}$$

The dependence on the scale μ retaining only the first, second or third correction terms is shown in Fig. 6. In the literature, it has often been advocated that one should make specific choices for the scale μ such that,

$$\begin{aligned} R^{(1)}(\mu) &= R^{(2)}(\mu), && \text{FAC} \\ \mu\frac{d}{d\mu}R^{(2)}(\mu) &= 0, && \text{PMS.} \end{aligned} \tag{5.5}$$

After the inclusion of the recently calculated third order term we see that neither of these guesses do much better than any other choice for the scale μ (see also ref. [76]). Note that the third order correction cannot be defined away. The value of the renormalisation scheme invariant quantity $r_3 - r_2^2 - \pi b_1 r_2 + \pi^2 b_2$ is also large. Proponents of schemes in Eq. (5.5) are of course free to make these choices for μ. The error on a physical prediction which has been calculated to $O(\alpha_S^n)$ remains $O(\alpha_S^{n+1})$. Another question which is raised by the large coefficient of the third order term is whether the perturbation series has begun to manifest the behaviour expected of a divergent, asymptotic series. Without further terms in the series this question cannot be definitively answered.

5.3 Other determinations of α_S

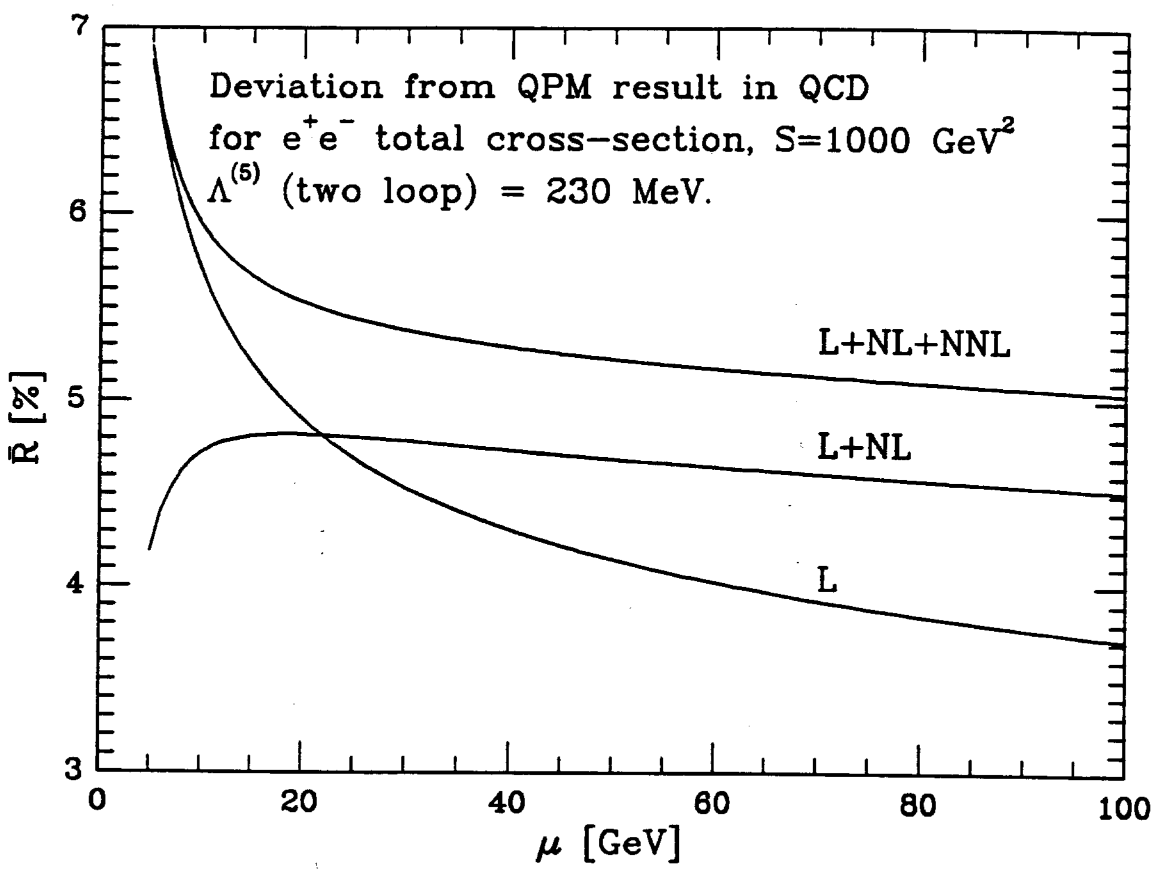

Figure 6: The quantity $\overline{R} = [R^{(j)}/R^{\mathrm{QPM}} - 1]$ as a function of the scale μ.

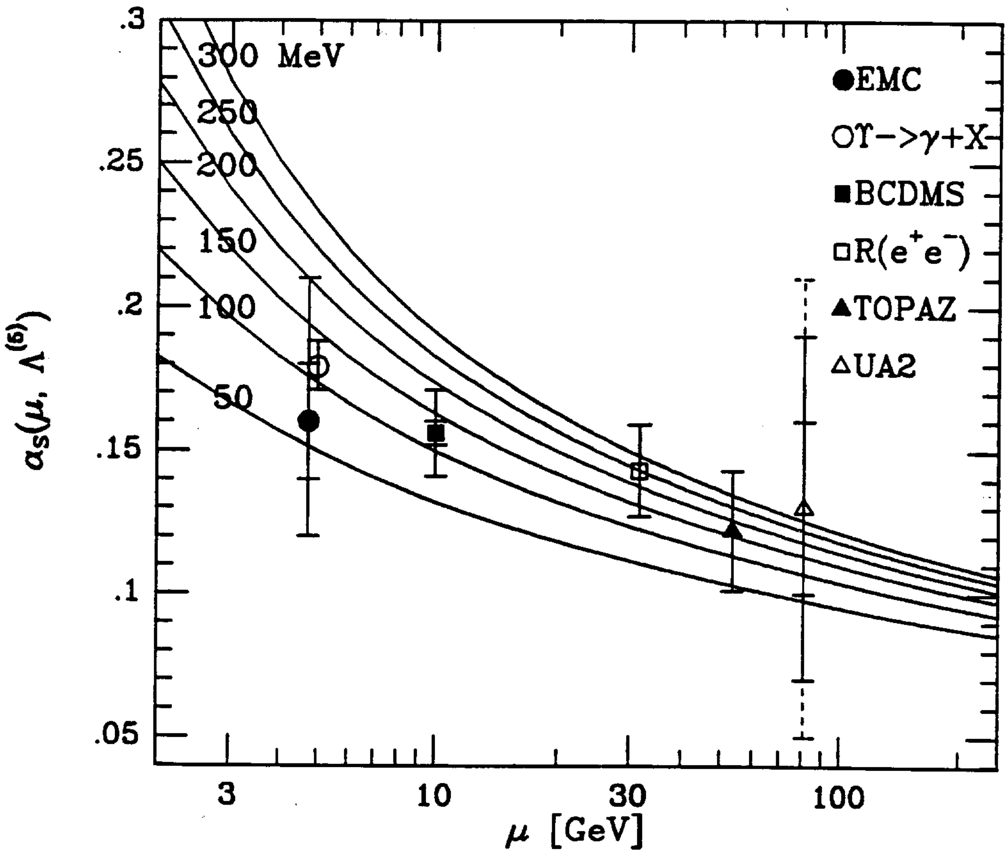

Figure 7: α_S measurements compared with theory for various $\Lambda^{(5)}_{\overline{MS}}$.

In Fig. 7 a partial compilation of α_S measurements is given. The value ascribed in Fig. 7 to the decay $\Upsilon \rightarrow \gamma + X$ is obtained from measurement of the branching ratio, $\Gamma(\Upsilon \rightarrow \gamma gg)/\Gamma(\Upsilon \rightarrow ggg)$. The value [77] shown in Fig. 7 includes only the statistical error which is very small. The extraction of the branching ratio from the data depends on a fit to a model of the photon spectrum which is based on a Monte Carlo program[78] rather than on an exact perturbative QCD evaluation. It is therefore subject to an additional theoretical uncertainty.

The values denoted by EMC[12] and BCDMS[79] are obtained from fits to deep inelastic scattering off Hydrogen. Since the data for these two experiments are in disagreement, at least one of these determinations of α_S must be suspect.

The UA2 value[80] is determined from the $W+$ jet events. The dashed line indicates an estimate of the theoretical error due to the fact that the calculation performed in ref. [81] is only partial. Although the errors are still large it is the highest energy measurement of $\alpha_S^{\overline{MS}}$. The measurement from TOPAZ[82] is obtained from energy-energy correlations.

Analysis of multijet final states in e^+e^- annihilation[83] yields a 3-jet rate consistent with a logarithmically decreasing coupling constant, but a value of α_S fixed with energy cannot be ruled out because of the limited statistics and energy range of the data. From Fig. 7 we can conclude that there is still not any convincing evidence for the running of $\alpha_S^{\overline{MS}}$ and that the value of α_S is still subject to a considerable uncertainty. This uncertainty in the value of α_S is reflected directly in the uncertainty in QCD production cross-sections.

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