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Integrability, Conformal Symmetry, and Noncritical Virasoro Algebras

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We discuss the relation between integrability and conformal symmetry in the context of a unitary $c = 1$ lattice Virasoro algebra for the noncritical eight-vertex model. The lattice algebra is related to the critical conformal algebra by a modular transformation in spectral parameter (rapidity) space. Virasoro generators for massive free fermions are explicitly constructed in terms of integrals of local densities and a connection with higher conservation laws is exhibited. Comparing the Verma module states of the noncritical algebra with the Bethe ansatz states, we discuss a connection between the FQS discrete sequence and the bound state spectrum of the eight-vertex/massive- Thirring/sine-Gordon model.

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1 Introduction

The study of solvable two-dimensional models has long been a fertile ground for insight into statistical mechanics and field theory. In recent years, the work of Baxter on the eight-vertex model and the subsequent development of the quantum inverse scattering formalism has unified the once diverse fields of solvable statistical mechanics models and classical integrable systems and soliton theory, and focused attention in this field on the algebraic and geometrical structures associated with the Yang-Baxter relations. A parallel but largely separate development of the last few years has been the study of conformal field theories. These are massless theories (critical statistical systems) which possess an infinite dimensional symmetry associated with conformal distortions of Euclidean space-time. At first glance, the fundamental symmetries of conformal field theory would seem to be quite distinct from those which arise in integrable systems, since the latter appear in many theories which have finite correlation length (nonzero mass) and are therefore not invariant under space-time conformal transformations. However, there is growing evidence that these two kinds of symmetry are in fact intimately related, and that a full clarification of this relationship will yield a deeper understanding of both subjects. Here we will discuss an approach to this subject which focuses on the corner transfer matrix (CTM) technique introduced a decade ago by Baxter.[1] We'll show that the remarkable properties of the CTM in the eight-vertex (8V) model are related to the existence of a lattice Virasoro algebra in which the central element L_0 is essentially the log of the CTM.[3] (Recent results by Miwa, Jimbo, and coworkers [4] on the local height probabilities (LHP's) for a large class of solid-on-solid (SOS) models have revealed compelling evidence that infinite dimensional Lie algebras play a central role in the dynamics of these models even in the noncritical case. The corner transfer matrix is used in this work to obtain infinite product expressions for the LHP's.) The physical picture we'll present here is as follows: The CTM is interpreted as the exact lattice analog of a Lorentz boost (Euclidean rotation) operator which implements an overall real (imaginary) rapidity shift on eigenstates of the Hamiltonian or row-to-row transfer matrix. The lattice rapidity is Baxter's elliptic "spectral" parameter u which labels the infinite set of commuting transfer matrices $T(u)$. The crucial difference between the lattice Lorentz group and its continuum counterpart is that the complex parameter space of the lattice Lorentz group is compact in the real as well as the imaginary rapidity direction. As we will show below, the standard conformal algebra at the critical point can be formulated in terms of momentum space operators Fourier transformed around the imaginary rapidity direction of the spectral torus. The noncritical lattice Virasoro algebra [3] is isomorphic to the conformal algebra when written in terms of Fourier modes around the real rapidity direction. The two algebras are, however, physically distinct. In particular, the lattice algebra reflects a symmetry which is present in the noncritical theory and, as we'll see, is closely related to the existence of an infinite number of conserved densities.

A complete discussion of the CTM method is beyond the scope of this talk, but we will introduce the subject by discussing the physical significance of the corner transfer matrix and presenting an intuitive picture of the lattice Virasoro algebra for the eight-vertex (8V) model and its relation to the conformal algebra which appears at the critical point. We will then go on to discuss a specific example which illustrates most of the essential points, namely, the decoupling limit of the 8-V model where it is equivalent to two uncoupled Ising models and hence to a free massive Dirac field. (For a more complete discussion see [5].) We will conclude with some brief and incomplete remarks about the connection between the highest-weight modules of the lattice Virasoro algebra (i.e. eigenstates of the CTM) and the Bethe ansatz states which are the eigenstates of the row-to-row transfer matrix. This discussion suggests a remarkable connection between the FQS discrete sequence of central charges ($c = 1 - 6/(n+2)(n+3)$) and the sequence of n-soliton (breather) bound states of the sine-Gordon/massive Thirring model.[6]

2 The Corner Transfer Matrix

The corner transfer matrix may be thought of as one quadrant of a lattice (in the same sense that the ordinary transfer matrix is represented by a single row of vertices). It acts on a half-row of spins and turns it into a half-column of spins. With this definition, we might expect the spectral structure of the CTM to be at least as complicated as that of the row-to-row transfer matrix. However, contrary to this expectation, Baxter showed in his original work on the 8V CTM [1] that this operator has an amazingly simple eigenvalue spectrum. (The existence of infinite product representations for local order parameters is a manifestation of this simplicity.) There are three non-trivial parameters in the 8V model. In Baxter's elliptic function parametrization of the Boltzmann weights, these parameters include: (1) an elliptic modulus k which essentially measures the distance from criticality (and hence the mass of the Thirring fermion), (2) a parameter η associated with the Thirring four-fermion coupling constant, and (3) a "spectral" or "lattice rapidity" parameter u, which is related to the anisotropy between horizontal and vertical spin-spin couplings. After dividing the CTM by its largest eigenvalue and taking the infinite volume limit (from now on we will refer to this latter construct as the CTM), it turns out that its eigenvalues depend on only one of the three parameters, namely, the spectral parameter u. This is in itself quite remarkable, but there is more. The corner transfer matrix $A(u)$, expressed as an operator, has a very simple dependence on u, specifically $\log A(u) = -uL_0$ where L_0 is an operator which is independent of u, and is given (in arrow representation) by the first moment of the XYZ spin-chain operator,

$$L_0 = \sum_{j=1}^{\infty} j H_{XYZ}(j, j+1) \tag{2.1}$$

where

$$H_{XYZ}(j, j+1) = -\frac{1}{2}[J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y + J_z \sigma_j^z \sigma_{j+1}^z] \tag{2.2}$$

Furthermore, the eigenvalues of L_0 are discrete and, within an overall factor, are all equal to nonnegative integers. The XYZ spin chain operator (2.2) is the same one whose zeroth moment (the XYZ Hamiltonian) appears in the expansion of the row-to-row transfer matrix in powers of u.[7] Considering that the spectrum of the XYZ Hamiltonian is both continuous and rather complicated in general, [8] the fact that the first moment L_0 possesses such an exquisitely simple spectrum is quite remarkable. It has been pointed out [9] that the properties of the CTM reflect an exact lattice analog of Lorentz invariance and that the CTM itself (more precisely, $\mathcal{A} = A \otimes A$, the direct product of a lower-left and an upper-right CTM) is essentially a Lorentz boost or Euclidean rotation operator. The spectral parameter u is the lattice analog of a complex rapidity parameter, with real and imaginary values representing Minkowskian boost and Euclidean rotation angles respectively. In fact, it was pointed out long ago by Baxter in the context of a general inhomogeneous 8V model [10] that the elliptic parameter u for the Boltzmann weights at a vertex could be interpreted as the angle between the two lines forming the vertex. With this geometrical picture, the CTM may be seen as a "pie slice" with u measuring the (generally complex) angle subtended by the slice. The effect of applying the CTM product $\mathcal{A}(u)$ to a Bethe ansatz (Hamiltonian or row-to-row transfer matrix) eigenstate is to shift all the rapidities in the state by u. This follows from the fact that the CTM acts as a rapidity shift operator on the infinite volume monodromy matrix $\mathcal{T}(v)$, [9]

$$\mathcal{A}(u)\mathcal{T}(v)\mathcal{A}^{-1}(u) = \mathcal{T}(u+v) \tag{2.3}$$

The surprising fact that the 8V model exhibits a continuous Lorentz symmetry which is not broken by the lattice provides insight into some of the basic properties of the theory. The existence of a one-parameter set of commuting row-to-row transfer matrices $T(u)$ and the associated infinite number of conservation laws is easily understandable. In a Lorentz invariant theory two observers in different Lorentz frames will construct the same set of energy eigenstates, i.e. the Hamiltonians in the two frames are simultaneously diagonalizable and hence mutually commuting. In a continuum theory, the Poincaré algebra closes, and Lorentz invariance simply implies that energy eigenstates also conserve momentum. But in the lattice theory, repeated commutation of the XYZ Hamiltonian with the lattice boost generator produces the full infinite set of conserved operators. [9] Thus the lattice Lorentz invariance of the 8V model is a phenomenon uniquely associated with its integrability.

Now let us consider the topology of the lattice Lorentz group and introduce the "spectral torus" which will be fundamental to the remainder of the discussion. Consider first a continuum theory. Here the complexified parameter space of the Lorentz group forms a cylinder, with Euclidean rotations (imaginary rapidity) being defined modulo 2π but with Minkowski boosts of arbitrarily large real rapidity. Putting the theory on a lattice compactifies momentum space and thus turns the complex rapidity cylinder into a torus, with the period in the real rapidity direction associated with the Brillouin zone of the lattice dispersion relation. Baxter's elliptic function parametrization reflects the double periodicity of this spectral torus. Within this conceptual framework we can introduce the lattice Virasoro algebra and discuss its relation to the conformal algebra at the critical point. I'll consider specifically the Ising case of the 8V model, which corresponds to a free Dirac fermion.

3 Conformal Symmetry in Momentum Space

To begin, let's briefly recall the conformal field theory of a massless Dirac fermion which describes the critical behavior of the 8V/Ising model. From the massless equations of motion, it follows that the chiral components ψ_1 and ψ_2 are analytic functions of the complex Euclidean coordinate z and $\bar{z}$ respectively. The radially quantized conformal algebra, e.g. for ψ_1, may be constructed from the Fourier components of the field around the Euclidean unit circle,

$$b(l) = \frac{1}{2\pi i} \oint dz z^{l-\frac{1}{2}} \psi_1(z) \tag{3.1}$$

(Here we are taking the field $\psi_1(z)$ to be double valued in the z-plane.) The Virasoro operators are then

$$L_n = \sum_{l=-\infty}^{\infty} (n + \frac{l}{2}) : \tilde{b}(l) b(l+n) : \tag{3.2}$$

where b and $\tilde{b}$ are canonically conjugate and $\tilde{b} = b^\dagger$ when the integration contour in (3.1) is on the unit circle. The operators L_n form a unitary Virasoro algebra with central charge $c = 1$. To establish the connection with the lattice Virasoro algebra, we now want to rewrite the mode operators $b(l)$ in terms of the Fourier components of a momentum space operator around the imaginary rapidity direction of the critical spectral cylinder. Consider the double contour integral

$$\Psi_c(l) = \frac{i}{(2\pi)^2} \int_C dp dz (-ip)^{-l-\frac{1}{2}} e^{-ipz} \psi_1(z) \tag{3.3}$$

The contour here is a two-dimensional surface in the four-dimensional space of complex p and z. There is a square-root branch cut in both p and z, the latter coming from the fact that $\psi_1(z)$ is double valued. For any fixed value of one integration variable, the contour in the other variable is chosen to wrap around the cut in the counterclockwise direction. The phases of the p and z cuts must be chosen so that the factor of e^{-ipz} in (3.3) is exponentially decreasing. This requirement enforces a topological equivalence between paths in z-space and paths in p-space. If we carry out the p-integration in (3.3) first, we recover an expression proportional to $b(l)$, Eq.(3.1). For each fixed value of z we choose the square root branch cut in the p-plane to go to infinity in the direction for which ipz is positive real and take the p-contour to be wrapped around the cut. After changing variables to ipz and z, one of the integrals is just Hankel's representation of an inverse gamma function, and we find that

$$b(l) = \Gamma(l + \frac{1}{2})\Psi_c(l). \tag{3.4}$$

On the other hand, we may carry out the z-integration first and write $\Psi_c(l)$ in terms of the momentum space operator

$$\Psi_c(l) = \frac{i}{(2\pi)^2} \oint dp(-ip)^{-l-\frac{1}{2}} a(p) \tag{3.5}$$

where

$$a(p) = \int_C dz e^{-ipz} \psi_1(z). \tag{3.6}$$

Here the z contour goes around the branch cut of the double-valued field in the clockwise direction, with the branch cut being chosen so that the exponential is decreasing. The physical significance of $a(p)$ is that it is the eigenmode operator of L_{-1}. It is the analytic continuation of the usual momentum space operator in fixed time quantization. Since $p = e^{i\alpha}$ with $i\alpha =$ rapidity, we see that $\Psi_c(l)$ is the Fourier component of $a(p)$ integrated around the compact Euclidean (imaginary rapidity) direction of the spectral cylinder. We also introduce in this way the conjugate operator $\tilde{\Psi}_c(l) = \pi(-1)^l \tilde{b}(l)/\Gamma(\frac{1}{2} - l)$. Canonical anticommutation relations for Ψ_c and $\tilde{\Psi}_c$ follow from the reflection formula for Γ-functions. In terms of the new mode operators Ψ_c, the conformal Virasoro operators are

$$L_n = \sum_{l=-\infty}^{\infty} (l + \frac{n}{2}) \frac{\Gamma(l + n + \frac{1}{2})}{\Gamma(l + \frac{1}{2})} : \tilde{\Psi}_c(l)\Psi_c(l+n) : \tag{3.7}$$

At this point we have merely rewritten the standard conformal algebra in terms of the Fourier transforms of momentum operators around the (critical) spectral cylinder. The reason for doing this is that we can now construct the noncritical lattice algebra by simply rerouting the Fourier transform contours to go around the real rapidity direction of the spectral torus. The sense in which the latter algebra represents an exact symmetry of the general noncritical model and how this is related to the integrability of the system will be explored in the remainder of the talk. [It should be noted that there is a nonunitary (Feigen-Fuks) generalization of the algebra (3.7), obtained by adding a surface term of the form $\lambda \frac{\partial}{\partial z}(\psi_1^\dagger \psi_1)$ to the stress tensor. This changes the factor $(l+\frac{n}{2})$ to $(l+\frac{n}{2})+\lambda(n+1)$. There is a completely analogous generalization of the lattice Virasoro algebra. The algebra presented in [3] was the case $\lambda=\frac{1}{2}$ which is not unitary and has a central charge $c=-2$. Here we are considering the case $\lambda=0$ which is unitary and has $c=1$.)

4 The Ising Model

As is often the case in the 8V model, one can gain a lot of insight by studying the model in particular limits. In this section I'll use the Ising/XY model [1] to illustrate the construction of the $\Psi(l)$ operators for the lattice Virasoro algebra. I will only touch on some of the essential points. A more detailed discussion will be presented elsewhere. We want to construct eigenmode operators for the central element L_0 of the Virasoro algebra, which is obtained from the log of the CTM. We rely on Baxter's work to write this as the first moment of the XY spin chain density,

$$L_0=\sum_{j=1}^{\infty} j[\sigma_j^x\sigma_{j+1}^x+k\sigma_j^y\sigma_{j+1}^y] \tag{4.1}$$

As in the case of the XY Hamiltonian, we can diagonalize L_0 by introducing fermion variables via a Jordan-Wigner transformation,

$$c_j^{x,y}=\sigma_j^{x,y}(\prod_{l>j}\sigma_l^z) \tag{4.2}$$

Now construct the momentum space operators

$$a^{x,y}(\alpha)=\sum_j(-i\sqrt{k}\,\mathrm{sn}\alpha)^{j-1}c_j^{x,y} \tag{4.3}$$

Note that the momentum p is related to the lattice rapidity α by $e^{ip}=-i\sqrt{k}\,\mathrm{sn}\alpha$. After some analysis, we find that L_0 is diagonalized by the operators

$$\Psi(l)=N_l\int d\alpha e^{-i\alpha l\pi/2K}\chi(\alpha), \tag{4.4}$$

where

$$\chi(\alpha)=\mathrm{dn}\alpha\, a^x(\alpha)+i\sqrt{k}\,\mathrm{cn}\alpha\, a^y(\alpha) \tag{4.5}$$

Here N_l is a normalization factor that will not be of concern. The integration over α in (4.4) is over one complete real period of the elliptic functions from $-2K+$

$iK'/2$ to $2K + iK'/2$ i.e. it winds once around the real rapidity (Brillouin zone) direction of the spectral torus. The physical vacuum corresponds to filled modes over half the Brillouin zone, giving two Fermi surfaces . We can now see the precise connection between the lattice Virasoro algebra associated with the CTM and the critical conformal algebra. At the critical point, the elliptic modulus $k \to 1$ and the Brillouin period goes to infinity. In order to recover the conformal algebra, we take the zero mass limit of the operator $\chi(\alpha)$ and Fourier transform it around the imaginary rapidity direction. Two distinct sets of operators can be constructed in this way, corresponding to taking the constant real part of the rapidity variable at the right or left Fermi surface. This explains why there are two (left- and right-moving) conformal algebras while there is only one noncritical lattice algebra. In the critical limit, the lattice fermion operators (4.2) can be expressed in terms of Dirac field components, and we obtain the double integral representation (3.3) of the conformal operators Ψ_c.

Now we want to mention some results for the case of continuum massive free fermions, which can be recovered from the Ising/XY model by taking its scaling limit. Again expressing the lattice fermion operators in terms of Dirac field components, we find that the scaling limit of (4.5) is just the Bogoliubov rotated momentum space operator which diagonalizes the massive Dirac hamiltonian,

$$\chi(\beta) \to \sqrt{\frac{m}{2}} \int dx e^{-imx \sinh \beta}[e^{\beta/2}\psi_1(x) + e^{-\beta/2}\psi_2(x)] \tag{4.6}$$

where β is the continuum rapidity. The Virasoro operators L_n for $n \geq -1$ may be constructed from these momentum space operators directly, using

$$L_n = \int_C d\beta e^{\beta}\,\tilde{\chi}(\beta)[(\frac{\partial}{\partial e^{\beta}})^{n+1} - \frac{1}{2}e^{-\beta}(\frac{\partial}{\partial e^{\beta}})^{n}]\chi(\beta). \tag{4.7}$$

The β contour is over the two lines $\beta =$ real and $\beta = i\pi-$ real. Interestingly, it turns out that all of these operators can also be written as integrals of local densities in coordinate space,

$$L_n = \int dx J_0^{(n)}(x) \tag{4.8}$$

where J_0 is the zeroth component of a conserved but explicitly space-time dependent current. For example, $J_0^{(-1)}$ is the hamiltonian (with a mass term) plus the momentum operator,

$$J_0^{(-1)} = \frac{1}{m}[2i\psi_1^{\dagger}\overleftrightarrow{\partial_x}\psi_1 + m(\psi_1^{\dagger}\psi_2 + \psi_2^{\dagger}\psi 1)] \tag{4.9}$$

and $J_0^{(0)}$ at $t = 0$ is the first moment of the massive Dirac hamiltonian,

$$J_0^{(0)} = x[i\psi_1^\dagger \overleftrightarrow{\partial} \psi_1 - i\psi_2^\dagger \overleftrightarrow{\partial} \psi_2 + m(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1)]. \tag{4.10}$$

For $t \neq 0$ this will also include a term $-t\mathcal{P}$ where $\mathcal{P}$ is the momentum density. The local densities $J_0^{(n)}$ associated with the higher L_n's contain both higher powers of x and t and higher derivative operators. These operators play a role in the Virasoro algebra analogous to that of the higher moments of the stress tensor in the conformal theory. However, in this case they are related to the infinite sequence of conserved charges that arise from the integrability of the system. (Although free massive fermions constitute a somewhat trivial case of an integrable system, this connection between the Virasoro operators and higher conserved charges is probably more general.) Let us define the following infinite set of ordinary (i.e. not explicitly space-time dependent) charges. Let

$$Q_n = \int d\alpha e^{n\alpha} \chi^\dagger(\alpha)\chi(\alpha). \tag{4.11}$$

By inserting (4.5) into this expression, we see that all the $Q_n's$ can be written as integrals over local densities $j_0^{(n)}$. These higher conserved charges contain Dirac bilinears with up to n derivatives. Now consider the Virasoro charges. At $t = 0$, $J_0^{(n)}$ is a polynomial in x of order $n + 1$,

$$J_0^{(n)} = \sum_{l=0}^{n+1} x^l \mathcal{O}_l^{(n)} \tag{4.12}$$

where the coefficients $\mathcal{O}_l^{(n)}$ are local operators with no explicit space-time dependence. From the above discussion it is not difficult to show that the operator coefficient of the leading power of x in (4.12), i.e. $\mathcal{O}_{n+1}^{(n)}$ is an ordinary conserved density, and is equal to a linear combination of the higher conserved charges obtained from (4.11). Thus, L_{-1} and L_0 are expressed in terms of the energy and momentum density, but for $n > 0$ each new L_n introduces a new member of the sequence of higher conserved densities. This establishes a direct link between the existence of a noncritical Virasoro algebra and the integrability of the system.

5 Verma Modules, Solitons, and the FQS Discrete Series

We began by pointing out the remarkable contrast between the complicated eigenvalue spectrum of the XYZ hamiltonian (zeroth moment of eq.(2.2)) and the simple integer spectrum of L_0. However, there must be a close relationship between the eigenstates of these two operators. In fact the operator L_{-1} of the lattice Virasoro algebra is one of the infinite sequence of conserved charges, and therefore has the same set of eigenstates as the XYZ hamiltonian. In massive Thirring (sine-Gordon) language, these states are made up of fermions (solitons), antifermions (antisolitons), and fermion-antifermion bound states (quantized breathers or n-boson bound states), with the number of bound states being controlled by the choice of the coupling constant parameter η. On the other hand, the eigenstates of L_0 can be classified into highest weight states and their Verma modules. The connection between these Verma modules and the particle spectrum is being studied and will be discussed in detail elsewhere.[6] Here we want to point out an intriguing connection which is suggested by this study, namely, a relation between the n^{th} member of the FQS discrete series of Virasoro central charges, and the n-boson bound state of the sine-Gordon model. In the Bethe ansatz solution of the model the n-boson bound state is represented in the complex rapidity plane by a string of n modes (called "n-strings") with the same real part and separted from each other in the imaginary direction by a spacing of η, the 8V coupling constant parameter. Now consider two contrasting limits, the free fermion case which we have already discussed ($J_z = 0$ in Eq.(2.2)), and the strong ferromagnetic coupling limit $J_z \gg |J_x|, |J_y|$. In the free fermion case the eigenstates of L_0 are obtained from the hamiltonian eigenstates by simply Fourier transforming over the rapidity variable of each fermion in the state. This leads to the obvious free fermion interpretation of the Verma module constructed by applying L_{-n}'s to the vacuum. The ferromagnetic limit gives a rather different and less straightforward picture. Here the lowlying excitations are bosonic n-string solitons. Consider for example the XXZ chain with $J_x = J_y = 1, J_z = \Delta$. For $\Delta > 1$ it can be shown [8] that all states are n-string solitons with n arbitrarily large. In the limit $\Delta \to \infty$ an n-string state reduces to one obtained by turning over n adjacent spins in the ferromagnetic ground state. In this limit, we may easily associate eigenstates of the hamiltonian with those of L_0 and obtain an interpretation of the Verma module in terms of n-boson bound states. In the range $0 < \Delta < 1$ there is an infinite sequence of thresholds at values of coupling for which the elliptic period is an integer multiple of the parameter η (where η is the separation between adjacent modes in an n-string). These are the values of coupling constant at which the n-boson bound state is just becoming unbound. But these values of η are also the ones for which the 8V model is related to the n^{th} ABF solid-on-solid model.[11] The critical exponents of these models realize the FQS discrete sequence of central charges.[12] It is likely that there are noncritical Virasoro algebras for the ABF models with central charges $c < 1$. The emergence of these models from the full 8V model is associated with the level crossing between the n^{th} bound state and the unbound fermion-antifermion pair. This seems to point toward an appealing physical interpretation of the discrete sequence in terms of the spectrum of sine-Gordon solitons.

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