Fermi National Accelerator Laboratory

FERMILAB-Conf-88/101

Analytical and Numerical Evaluation of Landau Cavities in the Fermilab Booster*

Yu-Chiu Chao and King-Yuen Ng
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

June 9, 1988

*Presented at the European Particle Accelerator Conference, Rome, Italy, June 7-11, 1988

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

ANALYTICAL AND NUMERICAL EVALUATION OF LANDAU CAVITIES IN THE FERMILAB BOOSTER

Yu-Chiu Chao, King-Yuen Ng
*Fermi National Accelerator Laboratory,*Batavia, IL 60510*

Abstract

Longitudinal coupled bunch instability has been observed in the Fermilab Booster at high intensity. It is a cause for concern due to its effect on the Tevatron collider performance[1]. We study this phenomenon using initial value technique[2] to correctly account for the underlying transient nature. Analytic result is obtained for any mode and comparison is made between ordinary harmonic potential and higher harmonic (Landau) cavity potential. In the latter case we consider the mode coupling effect as in [3]. A computer program is developed to facilitate the calculation. The result shows that the merit of Landau cavity is best realized in cases where the resonance is of a broad band nature.

The major offending resonances are due to the parasitic modes in the rf cavities. Table 1 lists the parameters of these resonances in the Fermilab Booster[4]. The other relevant parameters are listed in Table 2.

Resonance Number	Frequency (MHz)	Shunt Impedance (MΩ)	Q
1	52.3	0.43	1307
2	85.8	1.56	3380
3	109.7	0.15	2258
4	167.2	0.07	1960
5	171.5	0.07	1190
6	225.4	0.33	2090
7	318.1	0.09	1570
8	342.6	0.50	530
9	391.0	0.11	460
10	448.8	0.48	3590
11	448.8	0.11	1206
12	559.7	0.07	430
13	685.9	0.71	2440

Table 1: Measured resonant frequency, shunt impedance, and Q of the Booster accelerating cavities.

I The harmonic cavity

In the case of harmonic rf potential, we establish the dispersion relation as follows: First the linearized Vlasov equation in the presence of instability induced voltage V is obtained

$$\frac{\partial\Psi_1}{\partial s}+\frac{\omega_s(r)}{\beta c}\frac{\partial\Psi_1}{\partial\varphi}-\frac{e\eta V}{T_0\beta^3cE\omega_s}\sin\varphi\frac{d\Psi_0}{dr}=0, \tag{I.1}$$

where $\Psi_0=\Psi_0(r,\varphi,s)$ and $\Psi_1=\Psi_1(r,\varphi,s)$ stand for the equilibrium and perturbed distributions respectively. $V(\tau)$ and T_0 are the voltage induced by the longitudinal impedance and the revolution period respectively. η is the frequency dispersion factor and ω_s is the synchrotron frequency. r and φ are the action-angle variables in the longitudinal phase space: $\tau=r\cos\varphi,(\eta\delta/\beta^2\omega_s)=r\sin\varphi$, where τ is the advance time of the particle with respect to the synchronous particle and $\delta=\Delta E/E$ is the fractional energy deviation from the synchronous particle. We relate the time and frequency domains via the Laplace transform

$$\tilde{R}_l(r,\omega)=\int_{-\pi}^{\pi}\frac{d\varphi}{2\pi}e^{-il\varphi}\int_0^\infty\frac{d(s/c)}{2\pi}e^{i\omega s/c}\Psi_1(r,\varphi,s) \tag{I.2}$$

The inverse Laplace transformation for $\tilde{R}_l$ is

$$R_l(r,s)=\int_W d\omega e^{-i\omega s/c}\tilde{R}_l(r,\omega), \tag{I.3}$$

with similar definitions for the perturbed voltage V_l. The integration path W is taken so that it traverses the complex plane above all poles of R_l or V_l and the real axis. The form of Eqs. (I.2) and (I.3) ensures the preservation of causality. Next we need to establish the relation between the induced voltage V and Ψ_1 through the impedance of the cavity $Z(\omega)$ [5]. This is

$$\tilde{V}_l(r,\omega)=\frac{2\pi e\beta^2\omega_sM}{\eta T_0}\int_0^\infty r'dr'\sum_{l'}\sum_p\tilde{R}_l(r',\omega)$$
$$\otimes\frac{Z(\omega_p)}{\omega_p}\frac{(-l)i^{l-l'}}{r}J_l(\omega_pr)J_{l'}(\omega_pr'). \tag{I.4}$$

where $\omega_p=\frac{\omega v}{c}+p\omega_0$, and

$$\sum_p=\sum_{j=-\infty,p=Mj+s}^{\infty}.$$

$J_l(x)$ is the Bessel function. Applying the Laplace transform (I.2) on (I.1), we obtain a second relation between $\tilde{R}_l$ and $\tilde{V}_l$ besides (I.4). These lead to the dispersion relation

$$\tilde{R}_l(r,\omega)=\frac{1}{\omega-\frac{l\omega_s}{\beta}}\left[iR_{l0}(r)+\frac{ie^2\omega_0M}{\beta T_0E}\frac{\Psi_0'(r)}{r}\sum_p\sum_{l'}(-li^{l-l'})\right.$$
$$\left.\otimes\int_0^\infty r'dr'\tilde{R}_{l'}(r',\omega)J_l(\omega_pr)J_{l'}(\omega_pr')\frac{Z(\omega_p)}{\omega_p}\right] \tag{I.5}$$

where $R_{l0}(r)$ is related to the perturbed distribution at $t=0$.

This is actually an infinite dimensional eigenvalue problem, We will ignore the usually small coupling between modes with different absolute values of l. We then use small bunch approximation so that only the lowest synchrotron modes contribute. These are the $l=1$ and $l=-1$ modes. We diagonalize the space spanned by these two modes and arrive at an expression for the eigenmode of the problem

$$R(s/c)=\int_W d\omega\tilde{R}(\omega)e^{-i\omega s/c}=\int_W d\omega\frac{\Phi(\omega)e^{-i\omega s/c}}{1+iA(S_--S_+)} \tag{I.6}$$

where R is the perturbed time domain amplitude corresponding to the eigenmode. $\Phi(\omega)$ is related to the R_{l0} in (I.5) and not of interest here. $A=(e^2\omega_0M/4T_0E)\sum_p\omega_pZ(\omega_p)$. the unperturbed distribution is

$$\Psi_0(r)=\frac{\eta N}{2\pi r_0^2\beta^2\omega_s}e^{-r^2/2r_0^2},\qquad r_0=\frac{\sigma_\varphi}{h\omega_0} \tag{I.7}$$

*Operated by the Universities Research Association under contract with the U. S. Department of Energy.

The denominator of (I.6) gives the dispersion relation. Finally we observe that when the two modes $l = 1$ and $l = -1$ are far enough apart, we can approximate one eigenmode with the mode $l = 1$. This leads to our final form of the dispersion relation

$$H(\omega) = 1 - i\frac{\overline{A}x_0^2}{s}I(z) = 0 \qquad \text{(I.8)}$$

where

$$z = \frac{y}{x_0^2} = \frac{\omega\beta - \omega_{s0}}{s\omega_{s0}x_0^2}, \quad and \quad \overline{A} = \frac{e^2N\eta\omega_0^2M}{16\pi^2E\sigma_\varphi^4\beta^2\omega_s^2}\sum_p \omega_p Z(\omega_p),$$

The $I(z)$ is given by

$$I(z) = \begin{cases} 1 - \frac{z}{2}e^{z/2}E_1(z/2) & \text{for } \mathrm{Im}(\omega) > 0 \text{ and} \\ & \mathrm{Re}(\omega) \geq 0,\ \mathrm{Im}(\omega) \leq 0 \\ 1 - \frac{z}{2}e^{z/2}E_1(z/2) + i\pi z e^{z/2} & \text{for } \mathrm{Im}(\omega) < 0,\ \mathrm{Re}(\omega) < 0. \end{cases}$$

The analytic continuation across the negative real axis is included to give analytic results for $Im(\omega) < 0$. $E_1(z)$ is defined by

$$E_1(z) = e^{-z/2}\int_0^\infty dt \frac{e^{-t}}{z/2 + t}$$

II The Landau cavity

We consider a 4th harmonic (Landau) cavity added to the main rf potential such that the first and second derivative of the combined voltage vanishes in the vicinity of the acceleration phase ϕ_s. the additional potential takes on the form $V_L(\phi) = kV_0 \sin(n\phi + n\phi_n)$

We proceed in the same manner as in the previous section. The major differences are that we use a different unperturbed distribution consistent with the quartic potential and that we can no longer separate the two initially degenerate modes $l = +1$ and $l = -1$.

The dispersion relation is

$$H(z) = \begin{cases} 1 - F\int_{-\infty}^{\infty} \frac{x^5 e^{-x^4}}{z - x}dx & \mathrm{Im}(z) > 0 \\ 1 - F\left[\mathcal{P}\int_{-\infty}^{\infty} \frac{x^5 e^{-x^4}}{z - x}dx - (\pi i z^5 e^{-z^4})\right] & \mathrm{Im}(z) = 0 \\ 1 - F\left[\int_{-\infty}^{\infty} \frac{x^5 e^{-x^4}}{z - x}dx - (2\pi i z^5 e^{-z^4})\right] & \mathrm{Im}(z) < 0 \end{cases}$$

with

$$F = i\sum_p \omega_p Z(\omega_p)\frac{I_b\sqrt{6}M}{\pi\Gamma(3/4)(E/e)\sigma_\delta}\frac{1}{h(n^2 - 1)^{1/2}\omega_{s0}} \qquad \text{(II.1)}$$

$\mathcal{P}$ denotes the principle value of the integral and I_b is the electric current per bunch. z is related to ω through $z = \omega\beta/(\delta\omega_s)$, and $\delta\omega_s = (\pi c\beta/K)(a\sigma_\delta^2)^{1/4}$. $\sigma_\delta = \sigma_E/E$ is the fractional energy spread.

III Numerical evaluation

A computer program is developed to solve the growth rates $Im(\omega)$ from (I.8) and (II.1). Both equations can be cast into the form

$$I(\omega) = A(E)Z_{\text{eff}}(E) \qquad \text{(III.1)}$$

Where $I(\omega)$ is a function of ω involving all the integrals and analytic continuations, but independent of energy. $A(E)$ is a proportional constant, and $Z_{\text{eff}}(E)$ is a quantity which characterizes the instability-inducing impedance for a given mode. For our purpose we use short bunch approximation

$$Z_{\text{eff}} \sim \sum_p e^{-(\nu_p\sigma_l/R)^2}\omega_p Z(\omega_p)$$

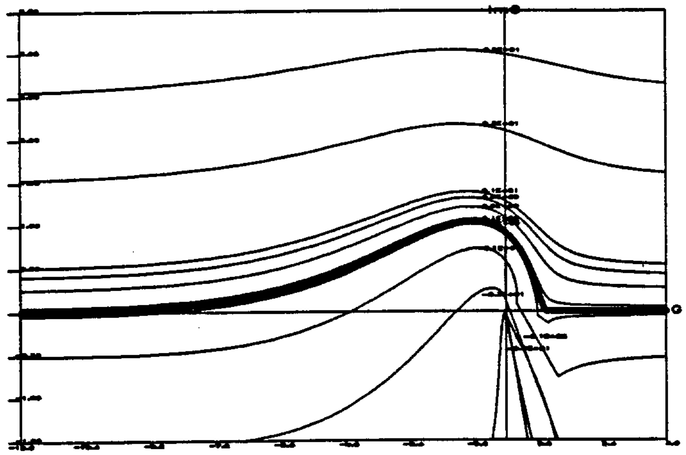

Figure 1: Stability diagram for the harmonic potential with one dipole mode only. The thick line marks $Im(\omega) = 0$, with increasing $Im(\omega)$ along the positive imaginary axis. The cut along the negative imaginary axis explains the erratic behavior there.

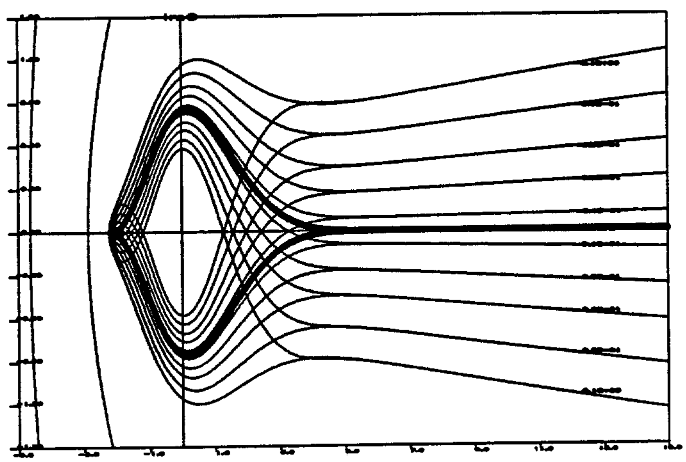

Figure 2: Stability diagram for the Landau cavity potential with both dipole modes. The thick line marks $Im(\omega) = 0$, with increasing $Im(\omega)$ going out of the closed contour.

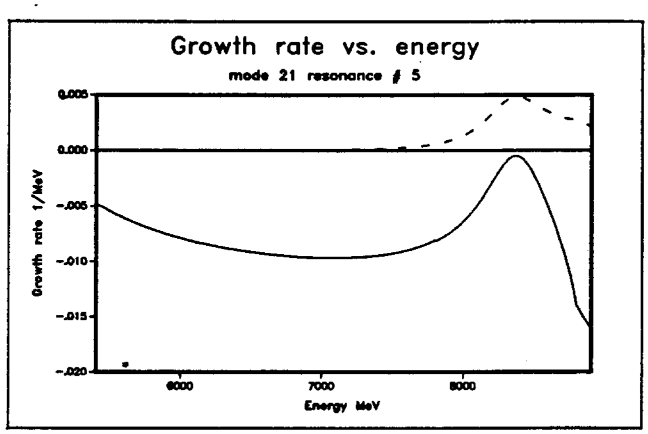

Figure 3: Mode 21 resonance number 5 as given in Table 1

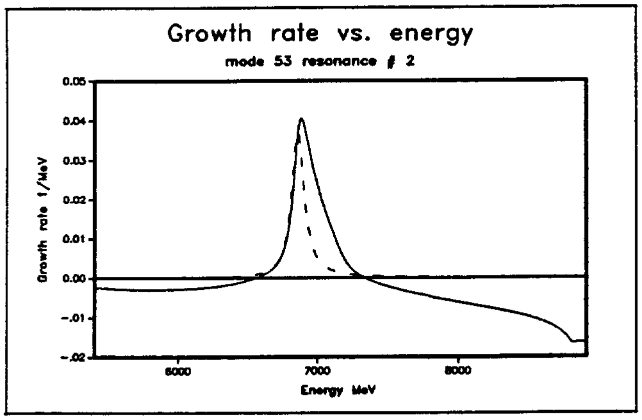

Figure 4: Mode 53 resonance number 2 as given in Table 1

where $\nu_p = p + \omega\beta/\omega_0$, (σ_l/R) is the bunch length in ring radian. Our problem is simply one of finding $Im(\omega)$ given E in (III.1). Figure 1 shows the "stability diagram",namely, the contours of constant $Im(\omega)$ in the complex plane of $A(E)Z_{\text{eff}}(E)$, for the harmonic potential. Figure 2 shows same plot for Landau cavity potential. The computer program solves $Im(\omega)$ by iteration for a given value of $A(E)Z_{\text{eff}}(E)$. The result of two of the resonances in Table 1 are displayed in Figures 3 and 4. The integrated growths for all modes in Table 1 are listed in Table 3.

IV Discussions

Figures 3 and 4 do not afford a coherent picture of the Landau cavity. To understand this, let us look at equation (I.8), If we assume that right at the peak the impedance is so large that the tune spread does not play any role, we can approximate the dispersion relation by taking z out of the denominator of the integrand:

$$Im(\Delta\omega) = \frac{\eta M I_b \omega_0^2}{4\pi\beta^2\omega_s(E/e)} Z_{\text{eff}} \tag{IV.1}$$

where $\Delta\omega$ is the complex frequency shift and I_b the current per bunch. In the case of the Landau cavity, both modes have to be included. Again neglecting the tune spread near the peak, we get the approximated dispersion relation:

$$Im(\Delta\omega) = \sqrt{\frac{\eta M I_b \omega_0^2}{4\pi\beta^2 E/e} Z_{\text{eff}}} \tag{IV.2}$$

Comparing Eq. (IV.1) with Eq. (IV.2), we can derive the following conclusions:

a. When Z_{eff} is very large and therefore dominant, Landau cavity would suppress the growth simply by power counting in Eqs. (IV.1) and (IV.2). However, when Z_{eff} is big enough for the difference between Eqs. (IV.1) and (IV.2) to be appreciable, Eq. (IV.2) itself is usually too big for the Landau cavity to look attractive.

b. When Z_{eff} is small enough that the tune spread has a dominant effect even near the peak, our approximations Eqs. (IV.1) and (IV.2) break down and don't teach us anything about the growth rates. In this event the dominant tune spread would act to discourage any coherent pattern accumulated within the bunch and we also expect the Landau cavity to reduce the growth rate significantly.

c. In the intermediate region where none of the above applies, it requires a detailed knowledge of all the factors which have effects on $\Delta\omega$ to reach a conclusion. This could be difficult.

Among the three possibilities discussed above, (b) is where a Landau cavity will be useful. When dealing with broad band impedances or resonances with weak enough peaks, we can in general apply the Landau cavity to suppress the growth.

Generally speaking, the effectiveness of Landau damping is determined by the competition between the growth rate $\Delta\omega$ in our calculation and the extent of the synchrotron tune spread $\delta\omega_s$. The tune spread is inversely proportional to the time scale during which the particles can remain coherent. Any meaningful growth has to take place in a time scale much shorter than this one in order not to be wiped out simply by decoherence of the beam. This leads to the general criterion for Landau damping:

$$\Delta\omega \ll \delta\omega_s \tag{IV.3}$$

This can rarely be satisfied at the peak in a general sense. Thus most of the time there will be some growth right at the peak even for a Landau cavity. It is also true however that this growth could have been bigger without the Landau cavity.

Injection Energy (Kinetic)	204 MeV
Extraction Energy (Kinetic)	8 GeV
Circumference	474.20 meters
Number of Bunches	84
Max. Beam Intensity	3×10^{12}
Transition Gamma γ_t	5.4
RF frequency(Inj.)	30.31 MHz
RF frequency(Ext.)	52.81 MHz
RF voltage (Maximum)	950 KV

Table 2: Booster parameters

Mode	Res.	Harmonic			Landau		
		Total Growth	Max. Growth 1/sec	Energy MeV	Total Growth	Max. Growth 1/sec	Energy MeV
14	4	2.91E+0	9.64E+2	8798	3.33E−1	3.14E+2	8788
16	4	3.35E−1	8.10E+2	5800	6.34E−2	5.06E+2	5804
21	5	3.37E+0	9.40E+2	8379	0.00E+0	0.00E+0	
23	6	8.16E+0	4.77E+3	8012	6.74E+0	5.06E+3	8054
43	8	1.26E+1	5.58E+3	6898	5.67E+0	3.29E+3	6944
45	8	8.65E+0	5.82E+3	5860	5.27E+0	4.12E+3	5916
45	11	9.24E−1	8.54E+2	6681	0.00E+0	0.00E+0	
53	2	6.20E+0	1.24E+4	6860	9.89E+0	1.35E+4	6884

Table 3: Total growth, maximum growth rate and minimum growth rate for various coupled bunch modes and driving resonances. The energy values at which these happen are also listed.

References

1. Y. Chao et al. ,Improving the Fermilab Booster Emittance, these proceedings.

Y. Chao and K.-Y. Ng, Analytical and Numerical Studies of the Landau Cavity, Fermilab Report FN-470 (1988). Readers interested in details of the current article should consult this work.

2. J.D. Jackson, Nuclear Energy Part C: Plasma Physics,1, p.171, (1960)

3. S. Krinsky and J.M. Wang, Particle Accelerators, 17, p109, (1985) J.M. Wang, BNL 51302 (1980)

4. J. Crisp, Private communications.

5. A. Chao, AIP Proceedings 105, p353, (1983)