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Effect of the Sun's Gravity on the Distribution and Detection of Dark Matter Near the Earth

KIM GRIEST

Astronomy and Astrophysics Center, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637

and

NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510

ABSTRACT

The effect of the Sun's gravity on the distribution of Dark Matter (DM) particles in the vicinity of the Earth is considered. The event rate in a cryogenic detector is found and the annual modulation of the signal due to this effect is compared to the annual modulation due to the relative velocities of the Sun and Earth. The effect is order 1% and probably too small to be seen experimentally. The effect of this distribution function on the capture rate of DM particles into the Earth is also considered. The rate could be suppressed for DM particles not well matched in mass to common elements in the Earth. Finally, the density of DM particles in bound orbits around the Sun is estimated and, contrary to earlier work, no way of generating an enhancement over "equilibrium" density is found. The inclusion of an "equilibrium" density of bound particles has negligible effect on direct detection, but removes the capture rate suppression.

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I. INTRODUCTION

It is possible that the dark matter (DM) which comprises the bulk of the material in our galaxy consists of some, as yet undiscovered, elementary particle. If so, there should be a substantial density of these particles in the neighborhood of the Earth and it may be possible to detect their presence either directly, by detecting the small, $O(keV)$, energy deposited by an elastic scatter of a DM particle with a nucleus in a cryogenic laboratory device[1,2] or indirectly, though detection of DM particle/anti-particle annihilation products. The most promising schemes of the latter type use the density enhancement caused by capture, over the age of the solar system, of DM particles into the body of the Earth[3,4] or Sun.[5,6]

The actual signal for both direct and indirect detection schemes depends crucially on the phase space density f of DM particles near the Earth. This function depends upon the average density of DM particles in the universe and on details of galaxy evolution, both of which are poorly known. Nonetheless, with the use of several assumptions, interesting limits have already been placed on several particle DM candidates, and many new and more sensitive experiments are being planned. In calculating event signal rates, previous authors[7,8] have taken for f a Maxwellian distribution of velocities parameterized by a velocity dispersion $\langle v^2 \rangle$ times a constant average halo DM density $\rho_0 = O(.4GeV/cm^3)$. The Earth and Sun move through this cloud of particles with velocities $v_e \approx 30$ km/sec, and $v_s \approx 235$ km/sec, and the distribution should have a cutoff at a galactic escape velocity v_{gal}. What has not been previously considered is the fact that to get to the Earth, the cloud of particles must fall through the gravitational field of the Sun. Since the escape velocity from the Sun at the Earth's orbit is $v_{es} = 42$ km/sec, apart from the Earth's motion around the Sun, no DM particle from this galactic cloud will be seen on Earth moving slower than this.[9] There may, in addition, be particles which have in some manner been captured into bound orbits around the Sun and which are slowly moving in the vicinity of the Earth, but the phase space density of these particles must be considered separately. Particles from the unbound cloud will also be focused by the Sun before reaching the Earth and the density of DM particles enhanced. Finally, the density and velocity distribution of particles from the unbound cloud will vary at different times of the year and in different directions in the sky. For example, when the Earth is in the "wake" of the Sun, the density will be higher than when the Earth is in front of the Sun. In this paper we consider the effect the Sun's gravity has on the distribution function, and the consequent effects this has on the direct and indirect detection of dark matter particles on Earth. Since $v_{es}^2/\langle v^2 \rangle$ is small, we expect the effect to be small in most cases.

In Sec. II we discuss the phase space density in the Earth frame for particles in a Maxwellian distribution under the influence of the solar potential. We also discuss the event rate in a cryogenic detector from this distribution. In Sec. III we apply the formulas of Sec. II to example detectors at various times of year and find an $O(1-2\%)$ focusing modulation of the event rate orthogonal to the already known $O(10\%)$ relative velocity modulation. In Sec. IV we consider the effect of our distribution function on the capture rate of DM particles into the body of the Earth and find that for particles with masses not closely matched to abundant elements in the Earth no capture is possible. For DM particles with masses between 8 and 90 GeV substantial mismatch does not occur, however, and in addition, a new paper by Gould shows that particles in bound orbits compensate this capture suppression. In Sec. V we consider the trickier problem of the density of DM particles in bound orbits. The "equilibrium density" is discussed and several capture scenarios considered, but in all cases no great enhancement over the equilibrium density is found. We conclude that the bound particle density is at or near equilibrium density. The effects of an example bound particle density on direct and indirect detection are touched upon. Sec. VI sums up the paper.

II. PHASE SPACE DENSITY

Fig. 1 shows schematically the relationship between the plane of the ecliptic, the velocity of the Sun around the galactic center and various times of year. The Sun's and Earth's velocities are most closely aligned in June and most closely anti-aligned in December. The Earth is most closely in the wake of the Sun in March and forward of the Sun in September. The angle between the Sun's velocity around the galactic center and the plane of the ecliptic is $\alpha = 59.6^0$, but if we include the Sun's motion toward the solar apex of 16.5 km/sec[10] and take 220 km/sec as the velocity of the local group of stars around the galactic center then the total solar velocity is 230 km/sec and the angle is $\alpha = 60.2^0$. In referring to March, June, etc. we are actually referring to the one day each year where the above is true. Leaving out motion towards the solar apex gives dates approximately 14 days before the the solstices and equinoxes, while including the motion gives dates approximately 19 days before the solstices and equinoxes; that is 2 June, etc.. Uncertainty in the Sun's velocity with respect to the halo translates into an uncertainty in these dates of $\simeq \pm 1.3$ days.

In what follows, we use spherical coordinates with the Sun at the origin and the positive z axis on the line from the Sun to the Earth (see Fig. 1). The Sun's motion is in direction (θ, ϕ), the Earth's motion is in direction (λ', ν') and we evaluate the distribution function in direction (λ, ν), and at velocity v. For the four times of year mentioned the values of (θ,ϕ), (λ,ν) are: June $(\frac{\pi}{2},\frac{3\pi}{2})$,$(\frac{\pi}{2}, \frac{3\pi}{2} - \alpha)$; December $(\frac{\pi}{2},\frac{3\pi}{2})$, $(\frac{\pi}{2}, \frac{\pi}{2} + \alpha)$; March $(\pi - \alpha,\frac{3\pi}{2})$, $(\frac{\pi}{2},\pi)$ and September $(\alpha,\frac{3\pi}{2})$,$(\frac{\pi}{2},\pi)$. The formula for f we will be using requires $\phi = \frac{3\pi}{2}$ always. We consider a cloud of DM particles which, far from the Sun, has a uniform density $\rho_0 \approx 0.4 GeV/cm^3$[11] and an isotropic Maxwellian distribution of velocities parameterized by $h = \sqrt{3/\langle v^2 \rangle} \approx (156 km/sec)^{-1}$.[11] In actuality, the halo of the Galaxy is probably neither Maxwellian, nor isotropic[12,13] but it is not clear what approximation is better. The cutoff velocity $v_{gal} \approx 640$ km/sec is not well known since it depends upon the unknown extent of the galactic halo and we are using here an estimate by Caldwell and Ostriker.[14] We denote the Earth's velocity with respect to the Sun as v_e and the Sun's velocity with respect to the cloud as v_s. The velocity distribution at a distance $a_\oplus$ from the Sun, in the Sun's rest frame, was found by Danby and Camm[15] in 1957, and applied by Danby and Bray[16] in 1967.

$$f(v,\theta,\phi,\lambda,\nu) = (2\pi)^{-3/2}\rho_0 h^3 \exp(\frac{-h^2}{2}F)\theta(J^2) \tag{1}$$

where

$$F = v_s^2 + J^2 + 2v_s \frac{J^2 Z + J(GM_\odot/a_\oplus)\cos\theta - JvZ\cos\lambda}{J^2 + (GM_\odot/a_\oplus) - Jv\cos\theta} \tag{2}$$

and $J^2 = v^2 - 2GM_\odot/a_\oplus$, $Z = v(\cos\lambda\cos\theta - \sin\lambda\sin\theta\sin\nu)$, $M_\odot$ is the mass of the Sun and $a_\oplus$ is the distance from the Sun to the Earth. In getting Eq. (2) from Ref. (15) we followed the method of Ref. (16), but corrected some errors, so our Eq. (2) differs from their Eq. (2). Eqs. (1) and (2)were derived by finding all four integrals of the motion, writing the distribution function in terms of them, and finding the values of the integrals which reduce f to the naive distribution function in the limit $GM_\odot/a_\oplus \to 0$. The $\theta(J^2)$ is necessary since particles which come from infinity must have energy greater than 0. That is, it incorporates our assumption that there are no bound orbits.

To include the motion of the Earth we shift the velocity $\vec{v} \to \vec{v} + \vec{v}_e$. Denoting the angle of the Earth's velocity (λ', ν') we then substitute

$$
\begin{aligned}
v^2 \to v''^2 &= v^2 + v_e^2 + 2vv_e \cos\theta_{wt} \\
\cos\lambda \to \cos\lambda'' &= \frac{1}{v''}(v\cos\lambda + v_e\cos\lambda') \\
\sin\lambda \to \sin\lambda'' &= \frac{1}{v''}[v^2\sin^2\lambda + v_e^2\sin^2\lambda' + \\
&\qquad 2vv_e\sin\lambda\sin\lambda'(\cos\nu\cos\nu' + \sin\nu\sin\nu')]^{1/2} \\
\cos\nu \to \cos\nu'' &= \frac{1}{v''\sin\lambda''}(v\sin\lambda\cos\nu + v_e\sin\lambda'\sin\nu') \\
\sin\nu \to \sin\nu'' &= \frac{1}{v''\sin\lambda''}(v\sin\lambda\sin\nu + v_e\sin\lambda'\sin\nu')
\end{aligned}
\tag{3}
$$

where $\cos\theta_{wt} = \cos\lambda\cos\lambda' + \sin\lambda\sin\lambda'(\cos\nu\cos\nu' + \sin\nu\sin\nu')$. We also introduce dimensionless parameters $t = v_e h$ for the Earth's velocity, $p = v_s h$ for the Sun's velocity, $w = vh$ for the DM velocity, and $q^2 = 2GM_\odot h^2/a_\oplus$ for the escape velocity. Using these we find

$$
f(v,\theta,\phi,\lambda,\nu) = (2\pi)^{-3/2}\rho_0 h^3 e^{-\frac{1}{2}F''}\theta(u'^2) \tag{4}
$$

where

$$
\begin{aligned}
F'' &= p^2 + u'^2 + 2p\left(\frac{u'^2 Z' + u'(q^2/2)\cos\theta - u'Z'(w\cos\lambda + t\cos\lambda')}{u'^2 + q^2/2 - u'(w\cos\lambda + t\cos\lambda')}\right), \\
Z' &= w(\cos\theta\cos\lambda - \sin\theta\sin\lambda\sin\nu) + t(\cos\theta\cos\lambda' - \sin\theta\sin\lambda'\sin\nu'), \\
w' &= (w^2 + t^2 + 2wt\cos\theta_{wt})^{1/2}, \\
u' &= (w'^2 - q^2)^{1/2},
\end{aligned}
\tag{5}
$$

and where λ and ν are as before, only now we are in the rest frame of the Earth. Eqs. (4) and (5) reduce to a Maxwellian shifted by $\vec{v}_e + \vec{v}_s$ in the limit $q^2 \to 0$. Note that since $q^2/2 \approx .03$ we expect the generic effect of the Sun's focusing to be about 3 percent.

Using the distribution function f we can evaluate the interaction rate in a detector on Earth as a function of v, λ, and ν.

$$dR = \frac{M_{det}}{m_N} d\sigma v_{rel} \frac{f(v,\lambda,\nu)}{m_X} d^3v \tag{6}$$

where M_{det}/m_N is the number of nuclei of mass m_N in the detector, v_{rel} is the relative velocity of the DM particle and the nucleus, $d\sigma$ is the elastic scattering cross section in the lab frame, and f/m_X is the number density of DM particles of mass m_X. Assuming the cross section is isotropic in the center of mass, $d\sigma = (\sigma d\cos\theta^*)/2$, where θ^* is the CMS scattering angle. The energy transferred (energy deposited in the detector) Q is determined by θ^*

$$Q = \frac{m_N m_X^2 v_{rel}^2 (1-\cos\theta^*)}{(m_n + m_X)^2}, \tag{7}$$

so with $v \approx v_{rel}$, $d\sigma = (m_X + m_N)^2 \sigma dQ/(2 m_N m_X^2 v^2)$ giving the interaction rate as a function of the energy deposited

$$\frac{dR}{dQ} = \frac{M_{det}}{m_N} \frac{\sigma(m_X+m_N)^2}{2m_N m_X^3} \frac{h^3 \rho_0}{(2\pi)^{3/2}} \int \frac{d^3v}{v} e^{-\frac{1}{2}F''}. \tag{8}$$

To find the limits of integration, we solve Eq. (7) for v and find $c\sqrt{m_N Q/2}(m_X + m_N)/(m_X m_N) < v < \infty$. However, the no bound orbits constraint implies that $v > v_{min} = -v_e \cos\theta_{wt} + \sqrt{v_e^2(\cos\theta_{wt}^2 - 1) + 2GM_\odot/a_\oplus}$, so $w > w_{min}$ where

$$w_{min} = \max\left(-t\cos\theta_{wt} + \sqrt{t^2(\cos^2\theta_{wt} - 1) + q^2}, \frac{hc(m_X+m_N)}{m_X m_N}\sqrt{\frac{m_N Q}{2}}\right). \tag{9}$$

In the case of a finite size galaxy halo there is also a maximum velocity $v_\infty < v_{gal} \approx 640$km/sec. The exact limit as a function of direction, after falling through the Sun's potential, is quite complicated, but with good accuracy we can approximate it as follows. At infinity, in the Sun's frame, the maximum velocity in direction (λ,ν) is $\vec{v}_{gal}$ plus $-\vec{v}_s$ (the cloud moves with velocity $-v_s$ in the Sun's frame) so the maximum velocity at infinity is $v_{m\infty} = |-v_s \cos\theta_{wp} + \sqrt{v_{gal}^2 - v_s^2 \sin^2\theta_{wp}}|$, where θ_{wp} is the angle between $\vec{v}_s$ and $\vec{v}$. If the angle of the DM particle does not change much in falling to the Earth, (This is a good approximation since $v_{gal} \gg \sqrt{2GM_\odot/a_\oplus}$). then the maximum velocity at Earth in the Sun's frame is $V_{max} = \sqrt{v_{m\infty}^2 + 2GM_\odot/a_\oplus}$. Finally, in the Earth's rest frame the maximum velocity is $\vec{V}_{max}$ plus $-\vec{v}_e$, and we have $v_{max} = -v_e \cos\theta_{wt} + \sqrt{V_{max}^2 - v_e^2 \sin^2\theta_{wt}}$, where we have neglected the angular change due to subtracting the Earth's velocity. So now, to find the interaction rate in a detector, one picks the time of year, which determines θ, ϕ, λ', and ν', and integrates Eq. (8) with $w_{min} < w < w_{max}$, $0 < \lambda \leq \pi$, and $0 < \nu \leq 2\pi$.

III. DIRECT DETECTION

Two challenges of direct detection schemes are the very low energy thresholds needed to detect light DM particles and the discrimination of the very small signal from a comparatively large background. One proof that the signal found is due to DM particles and not to some background may be the annual modulation of the signal due to motion of the Earth relative to the Sun as discussed by Drukier, Freese, and Spergel.[7] This may even aid in the discrimination of signal from background.[17] In June, the velocities of the Sun and Earth are most closely aligned and the cloud impacts the Earth with higher velocity, so a larger signal is seen at high Q and a smaller signal at low Q. In December, the velocities are most closely anti-aligned, and the reverse is seen. There are, of course, many environmental effects which vary on an annual basis (temperature, humidity, hours of daylight, power usage, drifting electronics, etc.) and care will have to be taken to prevent some subtle effect from mocking the modulation.

To find the effect of focusing by the Sun on the detection rate and the annual modulation, we evaluated the integral Eq. (8) for several values of deposited energy Q at the four different times of year mentioned previously. Table I. shows the event rate per unit energy for a 20 GeV DM particle and a Silicon ($m_N = 26.1$ GeV) detector for the four times of year. The rates with the focusing effect ($q^2 \neq 0$), and without the focusing effect ($q^2 = 0$) are shown, as is the percentage difference between them. The signal rates in Table I. are dimensionless and to get the rates in events/day/keV/(kg of detector) one multiplies by $(M_{det}/m_N)(\sigma/2)(m_N + m_X)^2 \rho_0 h/(m_N m_X^3) = 5.6 \frac{(m_N+m_X)^2}{m_N m_X} \frac{\sigma/10^{-36} cm^2}{(m_X/GeV)^2}$ events/day/keV/kg. Table I. can be used to find the rates for different nuclei mass m_N and different DM particle mass m_X by scaling Q by $[m_N m_X^2 (m_{Si} + 20GeV)^2]/[m_{Si}(20GeV)^2(m_N + m_X)^2]$, so for example the $Q = 10$ keV rate in Table I. corresponds to a $Q = .65$ keV rate for a 5 GeV DM particle and a Germanium ($m_N = 67.6$ GeV) detector.

In comparing the focusing case with the no focusing case we see in most cases a 0-3% difference in the event rate as expected. An increase this small will be completely buried, of course, in the astrophysical and particle physics uncertainties. More interesting is the modulation in signal rate that occurs between the different times of year. Table II. shows the modulation effects both with and without focusing. The relative velocity modulation discussed by Drukier, *et al.* is seen as the rather large ($O(10\%)$) increase in rate at high energy in both the $q^2 = 0$ and $q^2 \neq 0$ columns between December and June. Note the decrease in rate between these same months at low energy and the existence of one energy where no velocity modulation is seen. Also note that between March and September there is no relative velocity modulation at any Q since at these times the Sun's and Earth's velocities are perpendicular. When focusing is included, one then does see a roughly 2% increase in signal between September and March at low energies. This focusing modulation is caused mainly by the variation in DM density as the Earth moves in and out of the Sun's wake. In March, when the Earth is most nearly in the wake the density is 2.2% higher than for the $q^2 = 0$ case, while in September it is only .7% higher. The density is 1.1% higher in both December and June, explaining why there is no focusing modulation between these months. At higher Q the focusing modulation disappears even between March and September. Thus we see that the focusing modulation is much smaller and orthogonal to the relative velocity modulation.

To judge the significance of these modulation effects detailed statistical analysis is required, but for illustrative purposes one can consider a low threshold device and a heavy DM particle (for example, a 1 keV threshold Silicon device and a 20 GeV DM particle) and divide the events into two bins; one containing the 50% of the events with the lowest energies, and the other the 50% with the highest energies. There is then an overall 1.3% focusing modulation between March and September in the low energy bin and 0.5% modulation in the high energy bin. With the same crude binning the relative velocity modulation is 0 between March and September in both bins, -.6% between June and December in the low energy bin and 13% in the high energy bin. More details are shown in table II..

Given the smallness of the focusing modulation effect, one sees that it would require a very large signal rate to be able to see it. This is especially true since the effect is largest at low energies and the first generation of detectors will have comparatively large thresholds. So, given experimental realities, it is probably fair to conclude that this effect is unobservable in the near future, and the effect of the Sun's gravity on direct detection of DM particles can safely be ignored. However, as noted by other authors, the relative velocity modulation may be significant.

IV. INDIRECT DETECTION

The flux of light neutrinos in a proton decay detector from DM particle annihilation in the center of the Earth is proportional to the capture rate, so here we consider the consequences of Eqs. (4) and (5) on the capture of DM particles into the Earth. In the treatments of Krauss, Srednicki, and Wilczek,[3] and of Freese,[4] an approximation due to Press and Spergel[18] was used which counts as capturable only those DM particles which, at infinity, have velocities less than about $1.5v_{es\oplus}$, where $v_{es\oplus} = 11.2$ km/sec is the escape velocity from the surface of the Earth. They noted that particles on average lose *less* energy than this in a single collision, and given the small cross section, if a particle is to be captured, it must get below escape velocity in one collision. However, all particles from the unbound cloud which reach the Earth's orbit have velocities greater than 42 km/sec. There are therefore no particles which meet the above criterion, and in this approximation, the capture rate from the cloud is zero! (Actually, including the velocity of the Earth means particles coming from behind are traveling slower than 42 km/sec, but still faster than 11.2 km/sec.) However, as shown by Gould[19] , most captures are not made during low velocity collisions, but during nearly head on collisions between DM particles and nuclei of nearly equal masses, in which case much larger amounts of energy can be transferred. Following Gould, the capture rate from the unbound cloud can be written

$$C' = \int_0^{R_\oplus} 4\pi r^2 dr \frac{dC'}{dV} = \int_0^{R_\oplus} 4\pi r^2 dr \int_0^\infty \frac{dv\, f(v)}{v} w \Omega^-_{v_{es\oplus}}(w) \tag{10}$$

where dC'/dV is the capture rate per unit volume, $w = (v^2 + v_{es}^2 + v_{es\oplus}^2)^{\frac{1}{2}}$ is the velocity of a particle at Earth which had velocity v at infinity, $R_\oplus$ is the radius of the Earth, $f(v)$ is the distribution function, Eq. (4), integrated over angles (λ,ν), and $\Omega^-_{v_{es\oplus}}(w)$ is the probability per unit time that a particle with velocity w will scatter to a velocity less than $v_{es\oplus}$ (and thereby be captured). Besides the change in f, the form of $\Omega^-_{v_{es\oplus}}(w)$ changes when the Sun's gravity is included.

Using the notation of Ref. (19) we find

$$w\Omega^-_{v_{es\oplus}} = \frac{\sigma n v_{es\oplus}^2}{A^2}(A^2 - B^2 - x^2)\theta(A^2 - B^2 - x^2) \tag{11}$$

where $A^2 = h^2 \mu v_{es\oplus}^2/(2\mu_-^2)$, $B^2 = h^2 v_{es}^2/2$, $x^2 = h^2 v^2/2$, σ is the (assumed isotropic) scattering cross section, n is the number density of DM particles, $\mu =$

m_X/m_N, and $\mu_- = (\mu-1)/2$. This reduces to Gould's result when $B=0$. To find the capture rate one substitutes Eq. (11), integrates Eq. (10) and averages over the year. The integrals must be done numerically due to the complicated form of f, but from earlier discussion we expect the effect of changing f to be small and the big effect to be the theta function in Eq. (11). To get a feel for the effect we can compare with Gould's analytic result for a stationary Sun and Earth and no gravity ($v_s = v_e = 0$). For this purpose we take $f(v)dv = (4/\sqrt{\pi})x^2\exp(-x^2)dx$, integrate Eq. (10), and divide by Gould's result dC/dV to find

$$\frac{dC'/dV}{dC/dV} = \theta(A^2-B^2)\left(\frac{A^2-B^2}{A^2}\right)\left(\frac{1+[e^{-(A^2-B^2)}-1]/[A^2-B^2]}{1+(e^{-A^2}-1)/A^2}\right). \quad (12)$$

The biggest effect comes from the theta function which cuts off capture when $A^2 < B^2$. Using Gould's estimate of $A^2 \approx \mu/(400\mu_-^2)$ for the Earth's mantle and $A^2 \approx \mu/(290\mu_-^2)$ for the Earth's core and evaluating[20] $B^2 \approx .03$ we can solve the equation $\mu/\mu_-^2 = N$ to find $\mu = 1 + \frac{2}{N}(1 \pm \sqrt{1+N})$. So capture can take place in the mantle only when $.56 < m_X/m_N < 1.78$, and in the core only when $.5 < m_X/m_N < 2$. DM particles with masses outside these ranges are "off resonance" and the capture rate for them is zero in this limit. From Eq. (12) one sees that exactly "on resonance" when $\mu = 1$ ($m_X = m_N$), $dC'/dC = 1$ so the height of the resonance is not changed by the Sun. Also, even at $\mu = .8$ when the total capture rate is down by more than a factor of ten, dC'/dC is only .73.

In the usual picture of detection of DM through its annihilation in the Earth or Sun, one finds that the abundances of DM particles and anti- particles build up until the annihilation rate equals the capture rate of the minority species (see Ref. (6)), so, apart from capture from bound DM particles, the implication of the off resonance cutoff for annihilation product detection is that for particles with masses not well matched to any common element in the Earth the signal will be much suppressed. For most of the mass range 0-100 GeV there are common elements which can do the capture, as shown by Gould. However, the lightest common element is Oxygen,[21] and for DM particles below $.56m_{O^{16}} \simeq 8.3$ GeV any signal will be heavily suppressed. At the other end of the mass scale, the heaviest common element is Iron (perhaps Nickel also should be considered common) and so any signal will be heavily suppressed for DM particles heavier than around 93 GeV. It turns out, however, that DM particles with masses less than $O(12GeV)$ do not participate in the annihilations anyway because they evaporate from the Earth too quickly, thus making the cutoff of 8.5 GeV uninteresting. These are, of course, only rough estimates. The distribution function Eq. (4) which includes the velocities of the Sun and Earth must be properly integrated. As pointed out by Gould the effect of the Sun's motion will be to decrease the capture rate since the average kinetic energy of particles striking the Earth will be increased. However, off resonance, the motion of the Earth will have the opposite effect, since now particles with velocities down to $\vec{v}_{es\oplus} - \vec{v}_e$ will exist in the backward direction. At this point in this work we received a new preprint by Gould[22] which does the more complete calculation (using an approximation to Eqs. (4)and (5)) and we refer the reader to this paper for more details. Our conclusions above for the unbound particles are basically correct and the corrections to the event rate in a proton decay detector due to the Sun's gravity are quite small for matched DM and nuclei masses.

V. DENSITY OF DM BOUND TO THE SUN

Finally, we make some remarks concerning bound orbits. The phase space density Eq. (4) was derived by matching the general solution of the Vlasov equation to boundary conditions at infinity. Since bound orbits do not reach to infinity, Eq. (4) says nothing about them and their density must be considered separately. There are several sources of captured particles which can contribute to the bound orbit phase space density. Depending on the details of star formation a certain number of DM particles must have been captured as the Sun formed and may still be in orbits around the Sun. Also, unbound particles can interact gravitationally with the Earth, Jupiter and the other planets and be scattered into bound orbits. Of course, already bound particles can escape in the same way. Finally, DM particles of interest here can scatter off nuclei in the Sun, Earth, and planets and lose enough energy to become bound.

Now, while in general the phase space density of bound particles is unrestricted, since capture into bound orbits is presumably from the unbound cloud of particles described by Eq. (4), we can estimate the density of bound particles by considering the processes listed above. First note that the existence of 3-body gravitational interactions which both capture and free DM particles implies that an equilibrium value f_{eq} is possible (where the rate at which free particles are captured into bound orbits is equal to rate at which bound particles are freed). For isotropic scattering we expect f_{eq} to be nearly equal to the low energy limit of f (Eq. (4)). This can be seen by following a line of argument due to Gould.[23] Consider the gravitational scattering of a DM particle by the Earth. As discussed in Sec. II unbound particles are those which satisfy

$$u'^2 = w^2 + t^2 - q^2 + 2wt\cos\theta_{wt} \geq 0, \tag{13}$$

and bound particles satisfy $u'^2 < 0$, where w, t, and q are respectively the dimensionless DM, Earth, and escape from Earth orbit velocities and $\cos\theta_{wt}$ is the angle between $\vec{w}$ and $\vec{t}$. For a given DM velocity w, we see that in some directions there are only bound particles and in others only unbound. In the limit $u'^2 \to 0$, $f \to f_{eq} \equiv (2\pi)^{-3/2}h^3\rho_0\exp(-p^2/2)$, is independent of velocity and angles and so all directions which represent unbound particles are equally populated for a given w. Scattering by the Earth will not change the magnitude of the velocity, but the angle will change and the particle may afterward be bound. Thus, if the scattering rate is high enough, the directions which represent bound particles will be increasingly populated, up to the point where $f_{\rm bound} \approx f_{eq}$, when scattering out of bound angles will equal scattering into bound angles. Likewise, if $f_{\rm bound}$ for some w is initially larger than f_{eq}, we expect the bound directions to depopulate until $f_{\rm bound} \approx f_{eq}$ again.

With this in mind we also define an equilibrium density

$$\rho_{eq} = \int_0^{v_{min}} d^3v f_{eq} = \frac{\rho_0}{3}(1-2^{-3/2})\sqrt{\frac{2}{\pi}}\left(\frac{2GM_\odot h^2}{a_\oplus}\right)^{3/2} e^{-p^2/2} \approx 1.1\times 10^{-3}\rho_0, \quad (14)$$

where the v_{min} defined just before Eq. (9) is now the maximum velocity and all other symbols were defined in Sec. II. This is the density of bound DM particles at the Earth orbit which we would expect if an equilibrium phase space density existed. Note that the above equilibrating process does not apply for particles with w such that $w + t < q$, since in this case, no change of angle can change the sign of u'^2. So particles with $v < 12.4$ km/sec are not expected to be in equilibrium via this process. In addition, for these particles the effect of the Earth's gravity ($v_{es} = 11.2$ km/sec) is important; however, they do contribute less than 3% of the total to ρ_{eq}.

Before considering whether f_{eq} is reached in the age of the Earth and how much other capture processes contribute, we want to see how big an effect bound particles can have on direct and indirect DM detection. For direct detection we consider a bound particle phase space density $f_{\rm bound} = \epsilon f_{eq}$ where ϵ is some enhancement (or depletion) factor. The event rate in a cryogenic detector can be evaluated using Eq. (6)

$$\frac{dR_b}{dQ} = \frac{M_{det}}{m_N}\frac{\sigma(m_X+m_N)^2}{2m_N m_X^3}\epsilon f_{eq}(2\pi)(v_{es}^2 - v_e^2/3 - v_{Qmin}^2), \quad (15)$$

where v_{Qmin} is defined just after Eq. (8). For $m_X = 20$ GeV and a Silicon detector we can compare this bound particle event rate to the rate from unbound particles. As expected, the rate from bound particles is appreciable only at very small Q. For example, for an enhancement of $\epsilon = 1000$, the ratio of bound to unbound event rates is 15 at $Q = 10$ eV, 10 at $Q = 50$ eV, 6 at $Q = 100$ eV, 1 at $Q = 150$ eV, and zero at Q above 250 eV. We see that it would take a very substantial initial enhancement for the bound DM particle event rate to be significant. If such an enhancement did exist, however, it could be very interesting. With an enhancement of 10^6, for example, detectors would see a huge jump in signal at energies less than 100 eV and a smaller detector with lower energy threshold would be worth pursuing.

For indirect detection we need to know the capture rate into the body of the Earth from the cloud of bound DM particles. We first note that if a substantial density of bound particles exists, the sharp cutoff in capture rate found above for unbound particles with mismatched masses will be compensated for by capture from the bound cloud, since now particles with velocities all the way down to $v = 0$ will exist in the backward direction. In his new paper Gould[22] calculated the distribution of bound particles arising from gravitational scattering and weak interaction scattering and evaluated the resulting capture rate. He finds the largest contribution comes from bound particles in the range $0 < v < 12.4$ km/sec and, except right on resonance, is roughly equal to the capture rate from the unbound cloud. The compensating effect mentioned above is clearly seen in his Fig. 2. Thus, the density of bound particles is important for the indirect detection of DM particles and we refer the reader to Ref. 22 for more details.

The problem of whether f_{eq} is reached by gravitational scattering in the age of the Earth is also studied in some detail by Gould in his new paper.[22] His investigations are continuing[23] but I can oversimplify some of his results as follows. After a gravitational scatter, a newly bound particle leaves the Earth orbit with a speed w and with spherical angles with respect to the Earth's velocity (θ, ϕ). Under several assumptions, Gould showed these same values of w, θ, and ϕ will apply the next time this particle comes near the Earth. The new scatter will now change θ and ϕ but not w. The time scale to populate the directions which represent bound orbits is roughly the time scale to "random walk" into all parts of the bound "cone". Gould estimates the angular "random walk" of a bound particle in half the Earth lifetime to be

$$\Delta(w) \approx 1.5 \left(\frac{t}{w}\right)^{5/2} \text{radians}, \tag{16}$$

where, as before, t is the dimensionless Earth velocity. The half angle Φ of the cone inside which particles are unbound is found from Eq. (13)

$$\Phi = \cos^{-1}\left(\frac{t^2 - w^2}{2tw}\right), \tag{17}$$

where we have used $q^2 = 2t^2$. We see from Eq. (16) that DM particles with low w can sample the whole sphere and reach equilibrium in the age of the Earth. For particles with $w \simeq t$, $\Delta \simeq 1.5$ radians and $\Phi \simeq \pi/2$ and so here we just barely reach all of the sphere. For $v = wh \simeq 42$ km/sec, we have $\Delta \simeq .65$ and $\Phi \simeq 1.9$ and so much of the sphere is unreachable and we expect only partial progress toward equilibrium. As the velocity increases, the random walk angle decreases, but so does the size of the bound cone.

To be slightly more quantitative in these order of magnitude estimates we turn the problem around and ask how much of a large initial enhancement ϵ can survive over the age of the Earth. In the velocity range $0 < v < 12.4$ km/sec, $\Phi = 0$, there is no unbound cone, and given the assumptions in Ref. 22, an enhancement can survive until today undiminished. For $v > 12.4$ km/sec a certain fraction s of the particles will escape, and we define the angle $\Theta = \pi - \Phi - \Delta$ as the half angle of the "survival cone"; that is, bound particles which start within this cone will not, for the most part, under our assumptions, random walk into the unbound cone over the age of the Earth. The fraction of bound particles which survive can now be very roughly estimated as the ratio of the solid angle of the survival cone to the solid angle of the bound cone

$$s \simeq \frac{1+\cos(\Phi+\Delta)}{1+\cos\Phi}. \tag{18}$$

We find $s \simeq 2\times10^{-3}$ for $v = 30$ km/sec, $s \simeq .1$ for 35 km/sec, $s \simeq .3$ for 45 km/sec, and s peaks at around .4 for $v = 55$ to 65 km/sec. For higher velocities s decreases, dropping to .17 at $v = 70$ km/sec as the size of the bound orbit cone shrinks to zero at 72 km/sec. From these numbers we see that in the range $12.4 < v < 30$ km/sec enhancements do not survive, and $f \approx f_{eq}$. In the range $30 < v < 40$ km/sec the initial density drops by one to two orders of magnitude, but in the range $40 < v < 70$ km/sec any initial enhancement can be preserved. Our analysis is only order of magnitude and the interested reader is referred to an upcoming paper by Gould for a more exact treatment.

In the range 67 to 72 km/sec there is, however, one more effect which should be considered.[23] DM particles with these velocities at Earth orbit can reach to Jupiter's orbit, and since Jupiter is a more efficient scatterer than the Earth, one can guess that all such particles will be in equilibrium. Saturn will have a similar effect, but since it is further out will not change the above result. Summarizing the results so far, we see a possible enhancement in the velocity ranges $0 < v < 12.4$ km/sec and $40 < v < 67$ km/sec, and $f \approx f_{eq}$ in the rest of the range from 0 to 72 km/sec.

We now turn to an investigation of the initial phase space density; that is, the density that existed just after the Sun formed. As the proto-solar cloud of baryons collapsed, DM particles passing through the cloud could interact weakly with the nuclei, or gravitationally with the changing potential and lose enough energy to be captured. Also, if there were DM particles bound to the proto-solar cloud, then the density of these particles could be increased during the collapse. For weak interactions with the baryonic cloud very few particles could be captured, as can be seen from the capture rate formula in Ref. 19. The relatively quick time for solar formation ($\lesssim 10^7$ years), and the fact that the capture rate is (at best) proportional to the escape velocity from the cloud $(2GM_\odot h^2/R_i)$, coupled to the low initial density of DM particles means that nowhere near ρ_{eq} in DM particles could be captured in this way.

Gravitational capture is at first sight more promising. In 1978 Steigman, Sarazin, Quintana, and Faulkner[24] calculated the number of weakly interacting particles captured inside a sphere of final radius R_f during the slow collapse of a uniform spherical cloud of baryons with initial radius R_i. They defined a free fall time $t_f = R^3/\sqrt{GM_\odot}$ (approximately the time for a particle at rest to fall from $r = R$ to $r = 0$) and assumed that the ratio $\eta = t_f/t_c$ was constant in time and space, where t_c is the collapse time (the time for the cloud to collapse from $r = R$ to $r = R_f$). Under the assumption $\eta \ll 1$ they claimed

$$\frac{N_f}{N_i} \approx .05 \left(\frac{GM_\odot h^2}{R_i}\right)^{3/2} \eta^{3/2}, \tag{19}$$

where $N_f \approx \frac{4}{3}\pi n_f R_f^3$ is the number of DM particles inside radius R_f at the end, $N_i = \frac{4}{3}\pi n_0 R_i^3$ is the number of DM particles inside the precollapse sphere, and $n = \rho/m_X$. This would mean a density enhancement today at the Earth orbit $(R_f = a_\oplus)$ of

$$\rho_f \approx .05\rho_0 \left(\frac{GM_\odot h^2}{a_\oplus}\right)^{3/2} \eta^{3/2} \left(\frac{R_i}{a_\oplus}\right)^{3/2} \approx .3\rho_{eq}\eta^{3/2} \left(\frac{R_i}{a_\oplus}\right)^{3/2} . \tag{20}$$

Taking $a_\oplus \approx 1.5 \times 10^{13}$ cm, $R_i \approx 10^{17}$ cm, and $\eta \approx \frac{1}{5}$ this would be an enhancement of about $\epsilon \approx 15000$! However, we believe that Eq. (19) is in error. We first use a general argument due to Stebbins[25] to show that in any star formation process we expect $\rho_{\text{bound}} \lesssim \rho_{eq} e^{p^2/2}$, where $\frac{1}{2}p^2 \approx 1$, and then consider in detail the solar formation model of Steigman, *et al.*.

Let the bound particle phase space density be $f_{\text{bound}}(\vec{r}, \vec{v})$ and consider the particles in any small phase space volume. Follow these particles back in time to before the sun formed when they were unbound. Since along orbits $df/dt = 0$ by Liouville's Theorem, we have $f_{\text{bound}} = f_{\text{unbound}}$. Now, f_{unbound} is just a Maxwellian shifted by the solar velocity and has $(2\pi)^{-3/2}h^3\rho_0$ as its maximum, so $f_{\text{bound}} \leq (2\pi)^{-3/2}h^3\rho_0 = f_{eq}e^{p^2/2}$. Integrating f_{bound} over all bound velocities we see that $\rho_{\text{bound}} \leq \rho_{eq}e^{p^2/2}$. We can say more if we assume that the collapse was slow. Then only particles which were moving slowly with respect to the Sun (u'^2 small) could have been captured. This is a reasonable assumption since the dispersion velocity of the halo is ~ 220 km/sec and the escape velocity is ~ 42 km/sec. In this limit $f_{\text{unbound}} \approx f_{eq}$ and we predict $f_{\text{bound}} \approx f_{eq}$ and $\rho_{\text{bound}} \approx \rho_{eq}$ in disagreement with Eq. (20). To find the source of the disagreement we review the treatment of Ref. 24.

Steigman, *et al.* begin by finding the energy lost by a DM particle of energy E and angular momentum L transversing the collapsing cloud

$$\Delta E \approx -\frac{GM_{\odot}\eta}{R}g(\lambda^2), \tag{21}$$

where R is the radius of the cloud, g is a monotonically decreasing function of λ^2 given in Ref. 24 ($g(0) = 1.7$), and where $\lambda = L/L_{max} = L/\sqrt{2R^2E + 2GM_{\odot}R} \approx L/\sqrt{2GM_{\odot}R}$ is the dimensionless angular momentum. The $\eta \ll 1$ limit is necessary since the orbits were assumed to be those in a steady potential. We can drop terms of order $ER/GM_{\odot}$ since only DM particles with energies much less than $GM_{\odot}/R$ are captured. Also, since only particles which pass through the collapsing sphere can be captured, only particles with $\lambda \leq 1$ are of interest. The capture condition is $|\Delta E| > |E|$, which by taking the inverse of $g(\lambda^2)$ can be written $\lambda^2 < \lambda_*^2$, where λ_*^2 is the maximum value of λ for which the capture condition holds. Using Fig. 1 of Ref. 24, $\lambda_*^2(y)$ can be well approximated by $\lambda_*^2 \approx 1 - \frac{y}{1.7}$, where $y = ER/(\eta GM_{\odot})$ and $0 \leq y \leq 1.7$. The condition on λ_*^2 translates to a condition on the impact parameter b of DM particles at infinity $b < b_*$ where $\lambda^2 = Eb^2/(GM_{\odot}R)$. Using the cross section for capture πb_*^2 and integrating the velocity times this cross section over the phase space density at infinity, one derives the capture rate (Eq. 12 of Ref. 24).

$$\frac{dN_T}{dt} \approx .85(8\pi)^{1/2} n_0 (GM_\odot)^2 h^3 \eta, \tag{22}$$

where our h is the inverse of the Δv used in Ref. 24 and $n_0 = \rho_0/m_X$. The total number of DM particles captured is found by integrating Eq. (22) over the total collapse time $t_T = (R_i^{3/2} - R_f^{3/2})/(\eta\sqrt{GM_\odot}) \approx R_i^{3/2}/(\eta\sqrt{GM_\odot})$, which since dN_T/dt is independent of R (and therefore t) is easily found

$$N_T \approx N_i \left(\frac{GM_\odot h^2}{R_i}\right)^{3/2}, \tag{23}$$

which agrees with Eq. 13 of Ref. 24, apart from a factor of 2.

Steigman, *et al.* then point out that most of these captured particles are weakly bound and spend most of their time outside of the region of interest ($r \leq a_\oplus$). The maximum $|E|$ possible for a newly captured DM particle is $|\Delta E|$ which is smaller than $GM_\odot/R$ since $\eta \ll 1$ and $g < 1.7$. The time for a captured particle to orbit and pass through the contracting region again is $t_{\text{return}} \approx \pi R^{3/2}/(\eta^{3/2} g^{3/2} \sqrt{GM_\odot})$ which is greater than the collapse time $t_c = t_f/\eta$, so by the time the particle returns, the Sun has collapsed and the particle will therefore be unable to lose any more energy. Now, we can overestimate the fraction of particles inside $a_\oplus$ by saying that all captured particles have energy $|\Delta E|$ and that they are all on radial orbits. Then the fraction is the ratio of the time spent inside $a_\oplus$ to the orbit time. For $R > a_\oplus$ this ratio is $I \approx .3\,(a_\oplus \eta g/R)^{3/2}$ while for $R < \eta g a_\oplus$ it is $I \approx 1$. (In the latter case the particles stay inside $a_\oplus$ all the time.) The total number of particles inside $a_\oplus$ can be found by integration $N_f = \frac{N_T}{t_T}\int_0^{t_T} I dt$. Now using $R^{3/2} = R_i^{3/2} - \eta t \sqrt{GM_\odot}$

we find

$$\frac{N_f}{N_i} \lesssim .4 \left(\frac{GM_\odot h^2}{R_i}\right)^{3/2} \eta^{3/2} \left(\frac{a_\oplus}{R_i}\right)^{3/2} \ln \frac{R_i}{a_\oplus} \tag{24}$$

and $\rho_f \lesssim 2.5\rho_{eq}\eta^{3/2}\ln(R_i/a_\oplus)$, where we put g equal to its average value for simplicity. This is smaller than Eq. (20) by a factor of ~ 7500 and in rough agreement with the results of the general argument.

We must also consider the final collapse from $a_\oplus$ to $R_\odot \approx 7\times 10^{10}$ cm, where we can again apply Eq. (23) to find the number of particles captured in the final collapse $N_c \approx N_i(GM_\odot h^2/a_\oplus)^{3/2}$, where now $N_i \approx \frac{4}{3}\pi n_0 a_\oplus^3$. The fraction within $a_\oplus$ is $I \approx (\eta a_\oplus/R)^{3/2}$ for $\eta a_\oplus \le R \le a_\oplus$ and $I \approx 1$ for $R_\odot \le R \le \eta a_\oplus$. Integrating over the collapse time we find the density due to final collapse $\rho_c \approx 2\eta^{3/2}\rho_{eq}(.7 - \ln\eta)$, again in rough agreement with the general argument. In the above calculation, particles starting with negative energy were left out, which may explain the $\eta^{3/2}$ factor.

To this end, we lastly consider the fate of those DM particles already bound to the precollapse sphere of radius R_i. To make an estimate we can consider DM particles which orbit completely inside the sphere and which are on circular orbits. Then, for slow collapse, the adiabatic invariant $rM(r)$ can be used to calculate the density enhancement caused by the collapse of the sphere,[12] where here $M(r)$ is the total mass inside radius r. Defining the initial ratio of DM to total matter as f_X and considering a DM particle with initial radius r_i and final radius r, we have

$$r_i M(r_i) = r(M_X(r) + M_B(r))$$

where $M_X(r)$ and $M_B(r)$ are respectively the DM and baryonic masses inside r. Now $M_B(r_i) = M_\odot r_i^3/R_i^3$, $M_B(r) \approx M_\odot$, since by the end all baryons are within the very small radius of the Sun, and $M_X(r) = M_X(r_i)$, since DM orbits do not cross and we find

$$r = \frac{r_i^4}{f_X r_i^3 + (1-f_X)R_i^3} \approx \frac{r_i^4}{(1-f_X)R_i^3}.$$

So the density will be enhanced

$$\rho_{Xf} = \rho_{Xi}\frac{r_i^2 dr_i}{r^2 dr} \approx \frac{\rho_{Xi}(1-f_X)^{3/4}}{4}\left(\frac{R_i}{r}\right)^{9/4} \approx 10^8 \rho_{Xi}, \tag{25}$$

where in the last step we took $R_i \simeq 10^{17}$ cm and $r \simeq a_\oplus$. Eq. (25) is in agreement with the treatment of Ref. 24. But what is the density of initially bound DM particles ρ_{Xi}? If it was the equilibrium density for the precollapse sphere, $\rho_{Xi} \approx \rho_{eq}(a_\oplus/R_i)^{3/2}$ then we would have

$$\rho_{Xf} \approx \rho_{eq}\frac{(1-f_X)^{3/4}}{4}\left(\frac{R_i}{a_\oplus}\right)^{3/4} \approx 200\rho_{eq}, \tag{26}$$

a moderate enhancement which still must be multiplied by a factor less than unity to take into account the fact that not all orbits are circular and inside R_i. Here there is no factor of $\eta^{3/2}$, however, and given the crudeness of the approximations, we consider the above results to be in agreement with the general argument result that $\rho_{\rm bound} \approx \rho_{eq}$.

VI. CONCLUSIONS

In this paper we considered the effect the Sun's gravitational potential has on the distribution of cold dark matter particles near Earth. The distributions for particles with positive energy (unbound) and negative energy (bound) were considered separately. The phase space density for unbound particles is given in Eq. (4); the density for bound particles was considered only semi-quantitatively. Using the unbound phase space density we then estimated the effect on direct and indirect detection of DM. For direct detection we found a new $O(1-2\%)$

annual modulation of the signal rate in a cryogenic detector orthogonal to the larger relative velocity modulation. This effect is probably unobservable in the near future due to its small size and the low energy threshold needed to see it. For indirect detection we found that positive energy DM particles cannot be captured by the Earth if their mass is not well matched to a common nuclei in the Earth. This effect is probably not important either since for most of the mass range 12-90 GeV matching does occur and it is, in addition, compensated by the capture of DM particles from bound orbits.

The phase space density of negative energy particles is more difficult to estimate, but we reviewed some recent work by Gould and showed that for DM velocities in the range 12.4 to 40 km/sec we expect to reach an equilibrium density equal to the low energy limit of the unbound phase space density. This is due to 3-body gravitational scattering by the Earth. Including Jupiter equilibrates the range 67-72 km/sec as well. Under several assumptions, the phase space density in the ranges $0 < v < 12.4$ km/sec and $40 < v < 67$ km/sec does not equilibrate and we presumably have today roughly whatever density existed just after the Sun formed. In any case, using both a general argument and a crude model of solar formation we estimated the initial density due to solar collapse as being around the equilibrium density. This is in conflict with earlier estimates which suggested that a large enhancement was possible. A very large enhancement over equilibrium density would be interesting since it would mean large signal rates in cryogenic detectors at very low thresholds and would significantly alter the number of DM particles now in the body of the Earth. For equilibrium density, however, the effect in cryogenic detectors is negligible, and the capture rate into the Earth is of the same order of magnitude as other orbit capture mechanisms.[22] Overall we expect the bound DM particle distribution to be at or near the equilibrium value. Perhaps future generations of DM detectors will measure the energy spectrum of DM particles, give us real information about star formation (bound orbits) and galaxy formation (unbound orbits), and allow us to test these predictions.

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Table Caption

Event rates per unit energy in a Silicon detector for a 20 GeV DM particle at various times of year and at various values of energy deposited Q. The $q^2 \neq 0$ column includes the focusing effect of the Sun, while the $q^2 = 0$ column does not. See the text for factors to convert these rates into events/day/keV/kg and to scale the Q values for other nuclei and DM particle masses. The last column is the percentage difference between the rate columns.

TABLE I

	Q(keV)	Rate($q^2 \neq 0$)	Rate($q^2 = 0$)	% difference
September	.1	.566	.569	-.5
	1	.530	.527	.4
	5	.370	.366	1.1
	10	.225	.222	1.5
	20	.074	.073	1.9
March	.1	.579	.569	1.8
	1	.540	.527	2.4
	5	.374	.366	2.2
	10	.226	.222	2.2
	20	.074	.073	2.3
December	.1	.589	.588	.2
	1	.547	.541	1.1
	5	.369	.364	1.4
	10	.216	.212	1.7
	20	.067	.066	2.0
June	.1	.553	.551	.4
	1	.519	.514	1.0
	5	.373	.368	1.3
	10	.234	.230	1.6
	20	.081	.080	1.9

Table Caption

Modulation of signal rates in a Silicon detector for a 20 GeV DM particle at various times of year and at various values of deposited energy Q. The percentage change in event rate between the labeled months is shown. The $q^2 \neq 0$ columns include the focusing effect of the Sun, while the $q^2 = 0$ columns do not.

TABLE II

Q(keV)	Sep/Mar ($q^2 \neq 0$)	Dec/Jun ($q^2 \neq 0$)	Sep/Mar ($q^2 = 0$)	Dec/Jun ($q^2 = 0$)
.1	2.2	-6.6	0.0	-6.8
1	1.9	-5.3	0.0	-5.2
2	1.6	-3.6	0.0	-3.5
3	1.4	-2.0	0.0	-1.9
4	1.2	-0.4	0.0	-0.3
5	1.1	1.0	0.0	1.1
7.5	0.9	4.4	0.0	4.5
10	0.7	7.5	0.0	7.6
15	0.5	12.9	0.0	13.0
20	0.3	17.6	0.0	17.6
30	0.2	25.3	0.0	25.4

Figure Caption

The position and velocity of the Earth ($\oplus$) with respect to the Sun ($\odot$) at various times of year. The Earth's and Sun's velocities are most closely aligned in June; the Earth is most closely in the "wake" of the Sun in March. α is the angle between the solar velocity and the plane of the ecliptic. Also shown is the coordinate system used. The dark matter velocity $\vec{v}$ at Earth, in the Earth rest frame, is given in spherical coordinates by (v, λ, ν), where λ is the angle between $\vec{v}$ and the z axis, and ν is the azimuthal angle.

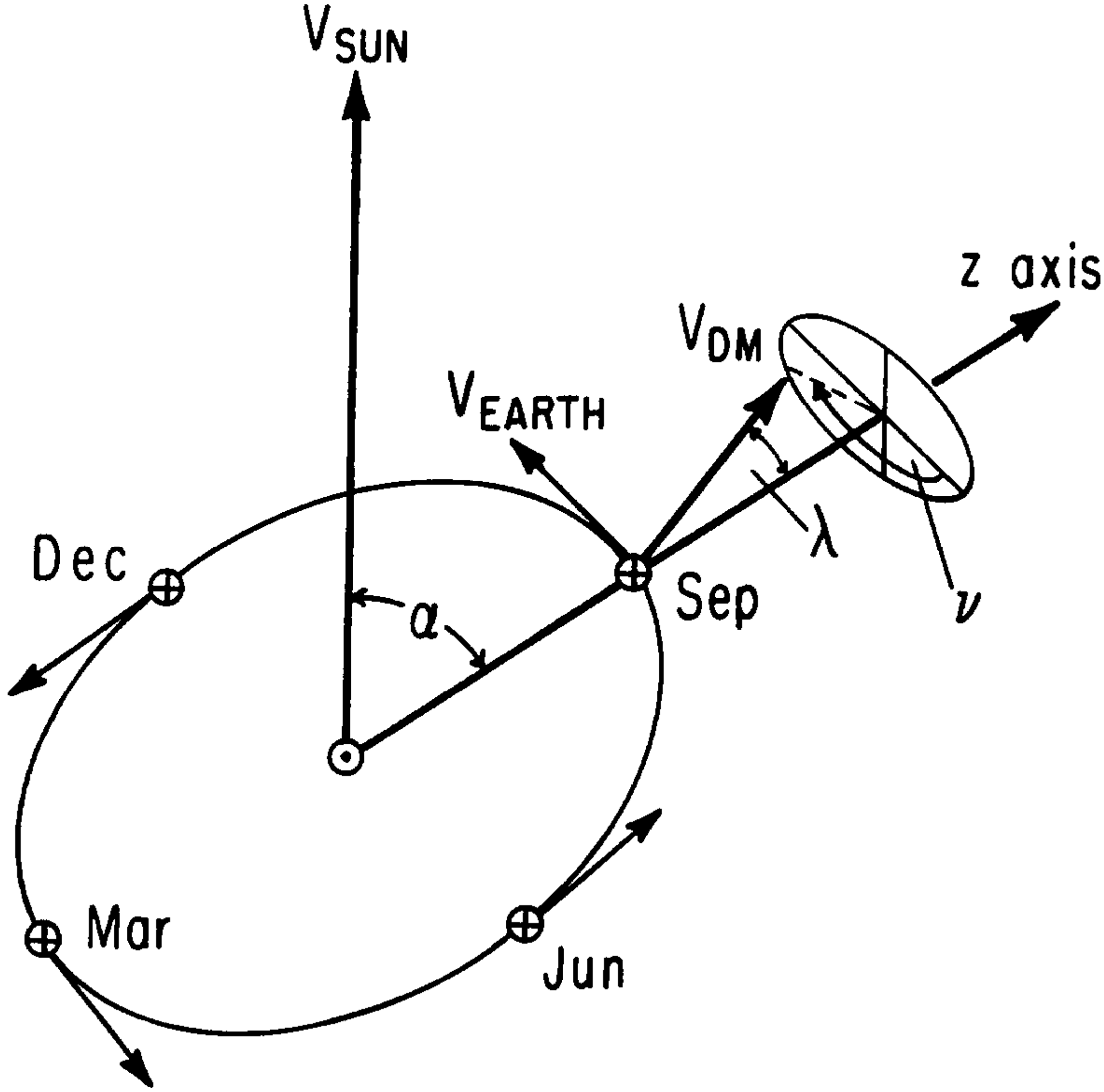

Figure 1