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On the dynamics of the power law inflation due to an exponential potential

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ABSTRACT

Power law inflationary universe model induced by a scalar field with an exponential potential is studied. A dissipation term due to particle creation is introduced in the inflaton's classical equation of motion. It is shown that the power index of the inflation increases prominently with an adequate viscosity. Consequently, even in theories with a rather steep exponential potential such as some supergravity or superstring models, it turns out that a "realistic" power law inflation (the power index: $p > 10$) is possible.

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The solution of the long-standing cosmological problems such as horizon and flatness problems is now generally attributed to inflation, or an exponential expansion of the cosmic scale in the early universe[1]. Such exponential inflation may realize if a scalar field with a sufficiently flat potential exists and its energy contributes dominantly to the energy density of the early universe[2,3]. Unfortunately, however, no scalar field with such a potential is yet known to exist naturally in high energy physics theories such as superstring theory which are expected to describe the early history of the universe[4].

The exponential inflation is, however, by no means a necessary condition in order to resolve above-mentioned cosmological problems. Models which may solve these problems without resorting to the exponential expansion are also known as generalized inflation models[5,6,7]. They are based on the fact that any type of accelerated expansion may solve horizon and flatness problems in principle[5].

Among various generalized inflationary models one with particular interest of us is the power law inflation model[5,6], in which the Friedmann-Robertson-Walker(FRW) scale factor $a(t)$ grows as

$$a(t) = a_0 t^p, \qquad p > 1. \tag{1}$$

Such power law expansion may realize if a scalar field ϕ with an exponential potential $V[\phi] = V_0 e^{-\lambda\phi}$ dominates the energy density of the universe. Indeed there is a set of exact solution in the spatially flat FRW space time in which $a(t)$ shows power law behavior as seen below. The Einstein equations and the classical equation of motion of the scalar field ϕ are written as

$$(\frac{\dot{a}}{a})^2 \equiv H^2 = \frac{1}{3}(\frac{1}{2}\dot{\phi}^2 + V_0 e^{-\lambda\phi}) - \frac{k}{a^2}, \tag{2}$$

$$\ddot{\phi} + 3H\dot{\phi} - \lambda V_0 e^{-\lambda\phi} = 0, \tag{3}$$

where V_0 and λ are constants and $8\pi G$ is taken unity. Then these equations posses the following exact solution if $k = 0$[8,9].

$$\phi_c = \phi_{c0} + \frac{2}{\lambda}\ln(\frac{t}{t_0}), \tag{4}$$

$$a(t) = a_0 t^p \qquad p \equiv \frac{2}{\lambda^2}, \tag{5}$$

where $\phi_c(t)$ denotes a classical solution and a_0, t_0 and ϕ_{c0} are integration constants. Thus power law inflation may take place if $\lambda < \sqrt{2}$. In fact it has been shown by Halliwell[8] that the above solution is an attractor for all the $k \le 0$ and some $k > 0$ initial conditions in the FRW universes.

Scalar fields with an effectively exponential potential appear naturally in supergravity theories and superstring models[10,11,12]. However, their potential is usually so steep that the power index of the scale factor cannot be much larger than unity, which makes it difficult to construct an acceptable inflationary universe model. For example, we find $\lambda = \sqrt{2}$ and $\sqrt{6}$ for two scalar fields in $N = 2$, 6-dimensional supergravity model with S^2-compactification [11], and $\lambda = \sqrt{2}$ and 2 for two scalar fields in $N = 1$, 10-dimensional supergravity model with gaugino condensation [12]. What we intend to do here is to re-consider the dynamics of the classical field

$\phi_c(t)$ in the power law inflationary stage to show the power index p increases sufficiently for inflation to occur with an adequate viscosity.

In the exponential inflation, time variation of the inflaton field φ is generally negligibly small during the inflationary stage so that the dynamics of the field is well described by an equation of motion of the form[3]

$$3H\dot{\varphi} + V'[\varphi] = 0. \tag{6}$$

While in the power law inflation, the inflaton ϕ_c rolls down the exponential potential rather rapidly even in the inflationary stage. Indeed under the exact solution of (4) the ratio of the potential energy to kinetic energy takes a finite value determined by the power p, as,

$$\frac{V[\phi_c]}{\dot{\phi}_c^2/2} = 3p - 1. \tag{7}$$

Thus in the power law inflation the time variation of the classical field ϕ_c is not negligible.

This implies that the couplings of some fields with ϕ become time dependent through $\phi_c(t)$ and those particles are produced due to ϕ_c's variation[13]. Then the classical field $\phi_c(t)$ loses its energy through these dissipation processes. In order to take into account this energy dissipation, we have to introduce a viscosity term in the inflaton's equation of motion[14,15,16]. That is, eq. (3) should be modified to

$$\ddot{\phi} + 3H\dot{\phi} - \lambda V_0 e^{-\lambda\phi} + C_v\dot{\phi} = 0, \tag{8}$$

where C_v is phenomenologically-introduced viscosity coefficient which generally depends on ϕ_c, $\dot{\phi}_c$, and coupling strength between ϕ and other fields.

Here let us investigate the effect of introducing a viscosity term as in (8) on the dynamics of the inflation. To do this, since we, unfortunately, have yet no method to calculate C_v properly, we assume that time scale of energy dissipation is proportional to the inflaton's "mass". That is, we assume C_v takes the following form,

$$C_v \equiv f M_\phi \equiv f V''[\phi_c]^{1/2}, \tag{9}$$

where a constant f is treated as a free parameter. We further assume, for simplicity, that the inflaton energy released through the viscosity term is converted into that of radiation. Then energy density of ϕ and that of radiation, which we write as ρ_{ϕ_c} and ρ_{rad}, respectively, evolve in terms of the following equations,

$$\frac{d\rho_{\phi_c}}{dt} = -3H\dot{\phi}_c^2 - C_v\dot{\phi}_c^2, \tag{10}$$

$$\frac{d\rho_{rad}}{dt} = -4H\rho_{rad} + C_v\dot{\phi}_c^2. \tag{11}$$

Then dynamical equations and a Hamiltonian constraint equation of the above system are given as follows.

$$\ddot{\phi}_c + 3H\dot{\phi}_c + f\lambda\sqrt{V}\dot{\phi}_c - \lambda V = 0, \tag{12}$$

$$\dot{H} = -(\frac{1}{2}\dot{\phi}_c^2 + \frac{2}{3}\rho_{rad}) + \frac{k}{a^2} \quad , \tag{13}$$

$$H^2 = \frac{1}{3}(\rho_{\phi_c} + \rho_{rad}) - \frac{k}{a^2} \quad . \tag{14}$$

Defining a new time variable by

$$\tau \equiv \int V[\phi_c(t)]^{1/2} dt \tag{15}$$

and

$$\alpha(\tau) \equiv \ln a(\tau), \qquad \delta(\tau) \equiv \frac{\rho_{rad}}{V[\phi_c]}, \tag{16}$$

we obtain from (12~14),

$$\phi_c'' = \frac{1}{2}\lambda\phi_c'^{\,2} - f\lambda\phi_c' - 3\phi_c'\alpha' + \lambda \ , \tag{17}$$

$$\alpha'' = -\frac{1}{6}\phi_c'^{\,2} + \frac{\lambda}{2}\phi_c'\alpha' - 2\alpha'^{\,2} + \frac{2}{3}, \tag{18}$$

$$\alpha'^{\,2} = \frac{1}{6}\phi_c'^{\,2} + \frac{1}{3} + \frac{\delta}{3}, \tag{19}$$

where we have set $k = 0$ for simplicity and prime denotes differentiation with respect to τ.

In order to examine the dynamics of the above system, we draw a diagram of (ϕ_c', α'). To do this, let us first draw lines of $\phi_c'' = 0$ and $\alpha'' = 0$, which are given by

$$\phi_c'' = 0 = \tfrac{\lambda}{2}[\phi_c' - \tfrac{3}{\lambda}(\alpha' + \tfrac{f\lambda}{3})]^2 - \tfrac{9}{2\lambda}(\alpha' + \tfrac{f\lambda}{3})^2 + \lambda, \tag{20}$$

$$\alpha'' = 0 = \tfrac{1}{6}(\phi_c' - \tfrac{3}{2}\lambda\alpha')^2 + (2 - \tfrac{3}{8}\lambda^2)\alpha'^{\,2} - \tfrac{2}{3}. \tag{21}$$

As is seen above the inclusion of the viscosity term of the form of (9) implies that α' is transported just to $\alpha' + f\lambda/3$ in eq. (20). Since δ is non-negative, only the region satisfying

$$\alpha'^{\,2} \geq \frac{1}{6}\phi_c'^{\,2} + \frac{1}{3} \tag{22}$$

has physical significance. The resultant diagram is shown in Fig. 1.

First let us consider the case $f = 0$(no viscosity). As is seen in Fig. 1, there are two stationary points with $\alpha' > 0$. Solving (20) and (21) with

$f = 0$, we find them

$$(\phi'_c, \alpha') = A_0(\frac{\sqrt{2}\lambda}{\sqrt{6-\lambda^2}}, \frac{\sqrt{2}}{\sqrt{6-\lambda^2}}), \quad B(\ 2,\ \frac{\lambda}{2}). \tag{23}$$

The point B appears outside the physical region as long as $\lambda < 2$. On the other hand the point A_0 is just on the $\delta = 0$ line and furthermore it is an attractor for all the initial conditions (ϕ'_c, α'), provided $\lambda < 2$. At $A_0(\phi'_{c0}, \alpha'_0)$ the scale factor behaves as

$$a(t) = a_0 t^p, \qquad p = \frac{2}{\lambda}(\frac{\alpha'_0}{\phi'_{c0}}) = \frac{2}{\lambda^2} \tag{24}$$

and $\delta = 0$. That is, power law expansion with $p = 2/\lambda^2$ realizes even in the initially radiation dominated case. Note that $p = 2/\lambda^2$ is larger than $1/2$ (the power index in the radiation dominated era) for $\lambda < 2$. Radiation energy redshifts to 0 owing to this rapid expansion caused by the exponential potential.

Next we consider the case with nonzero viscosity. The stationary point in the physical region, denoted by $A_f(\phi'_{cf}, \alpha'_f)$, is also an attractor for all the physical initial conditions (ϕ'_c, α') in this case. At A_f the scale factor shows asymptotically power law behavior with its power index given by

$$p_f = \frac{2}{\lambda}(\frac{\alpha'_f}{\phi'_{cf}}) \tag{25}$$

which is obviously larger than $2/\lambda^2$ as is seen in Fig. 1. At this point δ takes a finite value determined by (19). That is, if we take into account the effect of viscosity due to particle production, the power of inflation increases and finite radiation energy density remains. Figure 2, in which

$\lambda = 1$, shows an example of the dependence of p_f and δ on the viscosity strength f. As is seen in Fig. 2 the power index increases prominently if f is larger than ~ 3. This feature is common to all the cases with $\lambda < 2$, that is, the power becomes greater than ~ 10, if $f > O(3-10)$.

There are several merits in the power law inflation with larger power index. First the duration time of inflation which is necessary to explain the homogeneity and flatness of the present universe gets shorter. Second the spectrum of the density fluctuation after the inflation becomes closer to Zel'dovich type, since the spectrum depends on p as

$$\frac{\delta\rho}{\rho})_{t_H(k)} \propto k^{-1/(1+p)} \tag{26}$$

at the time $t_H(k)$ when fluctuation with wave number k gets into the horizon[6]. It is desirable to have a power law inflation model of a large power with the help of the effect of viscosity.

Hence it is important to examine if there is an adequate viscosity. In what follows, we investigate it briefly. As we mentioned earlier, we have no method to calculate C_v properly. However, it has been evaluated by perturbative expansion under the assumption that $\phi_c(t)$ varies adiabatically and shown to be related to the decay rate of ϕ particle through the imaginary part of its propagator[16]. We have performed similar calculation in the present case with an exponential potential and yielded at the lowest order

$$C_v \approx \frac{1}{2}\Gamma, \tag{27}$$

where Γ stands for the decay rate of ϕ particle. In order to evaluate Γ, the coupling between ϕ and external fields must be specified.

For the typical coupling of a scalar field which appears in supergravity theories with an exponential potential, we may consider Yukawa type coupling $\phi\overline{\chi}_i\chi_i$ or exponential coupling $e^{-m\lambda\phi}\overline{\chi}_i\chi_i$ with m a parameter where χ_i represents a fermion field such as gaugino and coupling strength may be of order of unity. Then the decay rate Γ for each case is written by

$$\Gamma \approx \frac{n}{4\pi}M_\phi, \qquad \Gamma \approx \frac{n}{4\pi}m^2\lambda^2 e^{-2m\lambda\phi_o}M_\phi, \tag{28}$$

respectively, for the lowest order of λ where n stands for the number of decay modes, which can be $\sim O(100)$. Putting (28) to (27) we find the dimensionless parameter f as

$$f \equiv \frac{C_v}{M_\phi} \approx \frac{n}{8\pi}, \qquad f \approx \frac{n}{8\pi}m^2\lambda^2 e^{-2m\lambda\phi_o}. \tag{29}$$

Since we are dealing with a strongly coupled case, the formula (27), which has been evaluated perturbatively, does not give quantitatively correct viscosity. However, from (29), we may know qualitatively the effect of viscosity on the dynamics of the universe.

For Yukawa coupling case, (29) possesses the same form as (9) so that the power index may increase prominently owing to viscosity. For the exponentially coupled case, the effective coupling strength decreases with time so that the initially large power index of inflation eventually decreases to $p = 2/\lambda^2$. We may need more analysis to see whether we can find a sufficient power law inflation for $\lambda \sim O(1)$.

Finally we mention the case $\lambda > 2$. In this case the stationary point B appears in the physical region as an attractor instead of A_0 or A_f. At B the universe is radiation dominated but δ remains constant. Hence this gives a particle physics model of "Cosmology with decaying vacuum energy"[17], if the potential keeps the exponential form until the low energy stage.

In summary we have introduced a viscosity term in the inflaton's classical equation of motion in order to take into account the energy dissipation due to particle creation caused by the time variation of $\phi_c(t)$. We have found that the power of the inflation increases prominently if there is an adequate viscosity. Hence even in the theories with a rather steep exponential potential such as some supergravity or superstring models, a sufficient power law inflation becomes possible, provided that it is coupled strongly with other fields.

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References

[1] A.H. Guth, Phys. Rev. **D23**(1981)347.
K. Sato, Mon. Not. R. astr. Soc. **195**(1981)467.
D. Kazanas, Astrophys. J. **241**(1980)L59.

[2] A.D. Linde, Phys. Lett. B **108**(1982)389.
A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**(1982)1220.

[3] A.D. Linde, Phys. Lett. B **129**(1983)177.

[4] K. Maeda and M.D. Pollock, Phys. Lett. B **173**(1986)251.
P. Binetruy and M.K. Gaillard, Phys. Rev. **D34**(1986)3069.

[5] L.F. Abbott and M.B. Wise, Nucl. Phys. **B244**(1987)541.

[6] F. Lucchin and S. Matarrese, Phys. Rev. **D32**(1985)1316.

[7] F. Lucchin and S. Matarrese, Phys. Lett. B **164**(1985)282.

[8] J.J. Halliwell, Phys. Lett. B **185**(1987)341.

[9] J.D. Barrow, Phys. Lett. B **187**(1987)12.

[10] E. Cremmer, S. Ferrara, C. Kounnas, and D.V. Nanopoulos, Phys. Lett. B **133**(1983)61.
J. Ellis, A.B. Lahanas, D.V. Nanopoulos, and K. Tamvakis, Phys. Lett. B **134**(1984)429.

[11] H. Nishino and E. Sezgin, Phys. Lett. B **144**(1984)187.
K. Maeda and H. Nishino, Phys. Lett. B **154**(1985)358; **158**(1985)381.

[12] E. Witten, Phys. Lett B **155**(1985)151.
J.P. Derendinger, L.E. Ibanez, and H.P. Nilles, Nucl. Phys. **B267** (1986)365.
M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. B **156**(1985)55.

[13] C. Pathinayake, and L.H. Ford, Phys. Rev. **D35**(1987)3709.

[14] A. Albrecht, P.J. Steinhardt, M.S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**(1982)1437.

[15] L.F. Abbott, E. Farhi, and M.B. Wise, Phys. Lett. B **117**(1982)29.

[16] M. Morikawa and M. Sasaki, Prog. Theor. Phys. **72**(1984)782.

[17] K. Freese, F.C. Adams, J.A. Frieman, and E. Mottola, Nucl. Phys. **B287** (1987)797.

Figure captions

Fig. 1 Dynamical behavior of the universe in the diagram of (ϕ'_c, α') for the case $\lambda < 2$. The power law solutions, $A_f(f \neq 0)$ and $A_0(f = 0)$ are attractors as shown by solid and broken arrows, respectively.

Fig. 2 Dependence of the power index p and δ on the viscosity strength f for the case $\lambda = 1$.

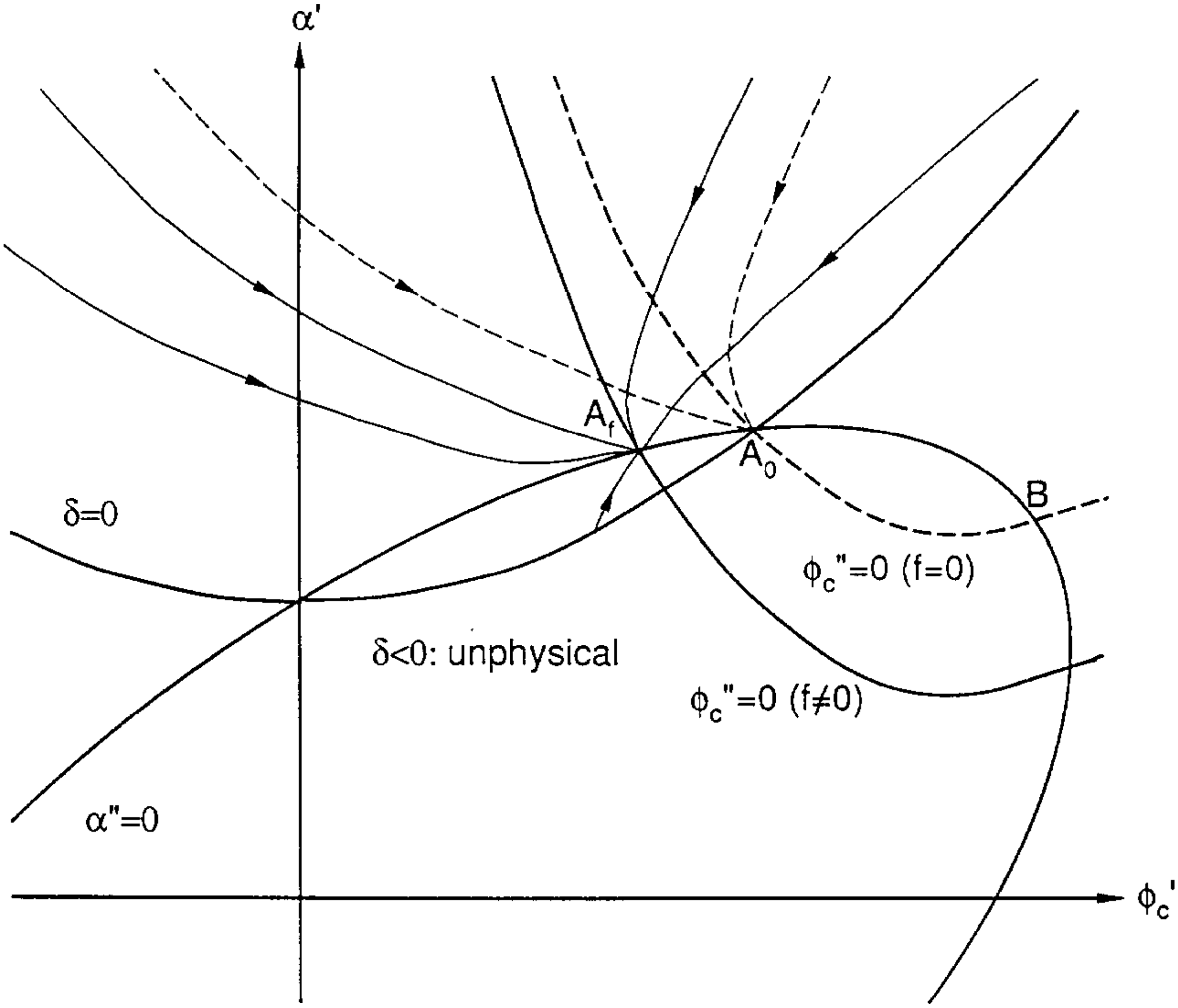

Figure 1

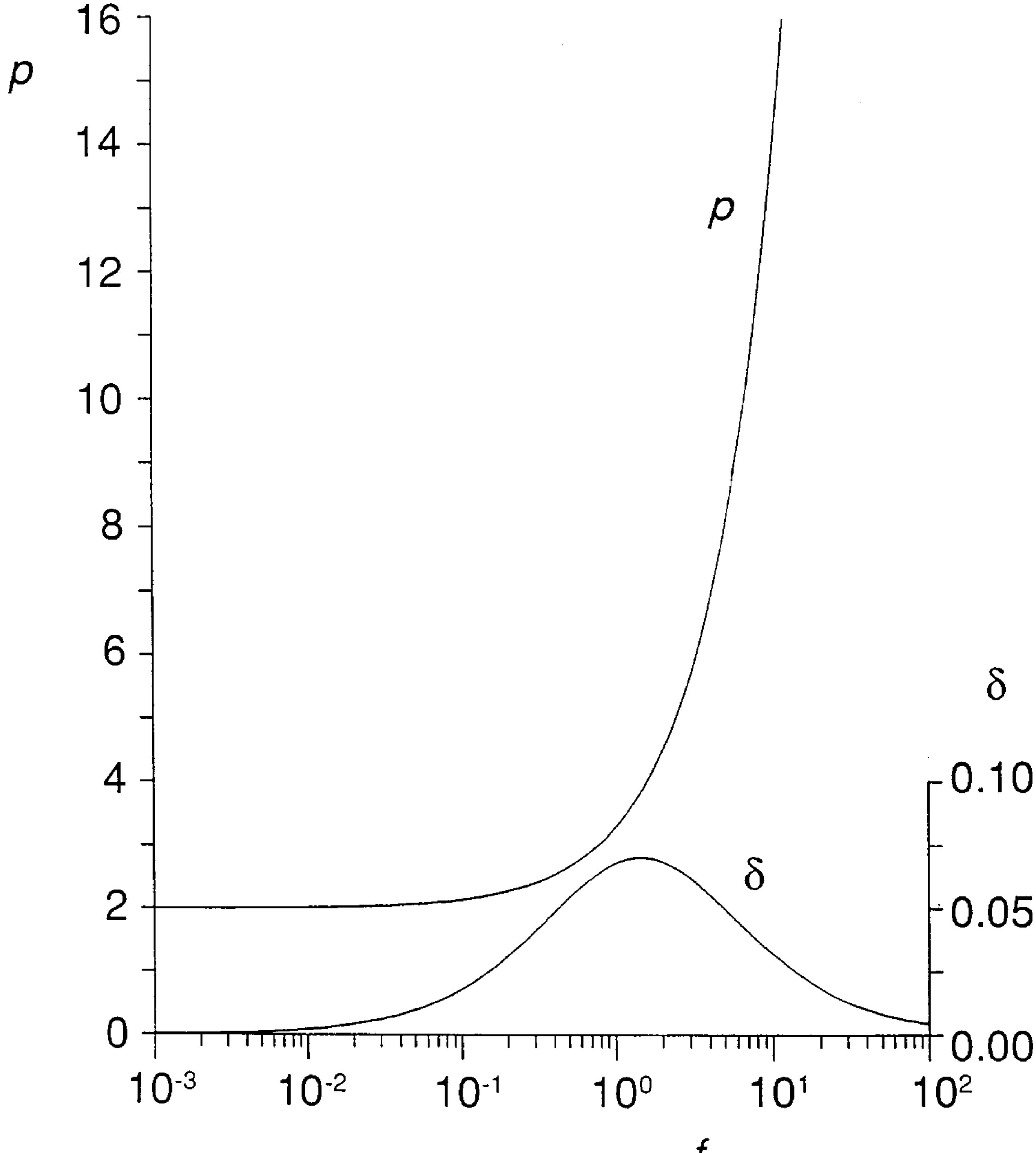

Figure 2