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GRAVITATIONAL PRODUCTION OF SCALAR PARTICLES IN INFLATIONARY UNIVERSE MODELS

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Abstract

We study the production of massive scalar particles (with $m < H_o$) in inflationary Universe models. A given mode of the scalar field is quantum-mechanically excited when it is well within the Hubble radius ($\equiv H_o^{-1}$) during the de Sitter phase of inflation. It then crosses outside the Hubble radius after which it is treated classically. Ultimately, long after re-entering the Hubble radius, the fluctuations correspond to non-relativistic scalar particles. The energy density in these particles depends on the Hubble parameter during inflation, the mass of the particle species, and the coupling of the scalar field to the curvature scalar. The energy density may contribute significantly to the total energy density in the Universe; in addition, new constraints to the value of the Hubble constant during inflation follow. As an example we apply our results to axions.

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I. Introduction

For some time now, the study of quantum fields in cosmological models has been the focus of active and fruitful research. (For a comprehensive review of quantum fields in curved space, see ref. 1.) The suggestion by Guth [2], and later Linde [3] and Albrecht and Steinhardt [4], that an inflationary Universe might solve a number of the puzzles which plague the standard big bang cosmology, has lead to the belief that a de Sitter phase in the early Universe can arise quite naturally and may even be expected [5]. However, this yet to be an established fact! A period of inflation would have a profound effect on the quantum fields present and these quantum fields may be the progenitors of classical entities such as density inhomogeneities, particles, and perhaps primordial magnetic fields [6]. For example, it has been shown that fluctuations in the scalar field which drives inflation give rise to (almost) scale-free adiabatic density perturbations [7]. Fluctuations in other fields (e.g., the axion) can give rise to isothermal density perturbations [8]. Both adiabatic and isothermal density perturbations are candidates for the seed fluctuations necessary to initiate galaxy formation. Furthermore, a number of authors have looked at particle production for higher spin fields. In particular, graviton production leads to a definite spectrum of relic gravitational waves and to large angular scale (quadrupole) distortions in the microwave background temperature with $\delta T/T \simeq H_o/m_{pl}$ where H_o is the Hubble parameter during inflation [9]. The requirement that these distortions be consistent with the present limits to the isotropy of the microwave background provides a stringent constraint to the value of the Hubble parameter during inflation. In short, in an inflationary Universe model, quantum processes operating early on have profound implications for phenomena on large-scales today. The crucial aspect of inflation that makes this possible is the kinetic fact that sub Hubble radius sized fluctuations grow to enormous size ($\gg$ Hubble radius) during inflation.

Much of the work on quantum fields in curved space has focused on scalar fields. The effects of the gravitational field on a scalar field are often studied by solving the Klein-Gordon equation in a fixed background spacetime. One then determines whether the energy density in particles produced is cosmologically interesting. For example, a number of authors have studied scalar fields in anisotropic [10] and inhomogeneous spacetimes [11] and find that gravitational production of these scalar particles can efficiently isotropize the Universe. More recently, Ford [12] has studied the production of scalar particles due to the time dependence of the scalar curvature during the transition from a de Sitter phase to a radiation-dominated Robertson-Walker phase. He finds that particles are produced with an energy density corresponding to a thermal bath at the Gibbons-Hawking temperature, $T = H_o/2\pi$, provided that the particles are not conformally coupled and the transition from the de Sitter phase to the radiation-dominated phase is not too abrupt. He suggests that this particle production mechanism may be responsible for reheating the Universe after an inflationary epoch in contrast to the usual picture where reheating is due to the decay of the inflaton field (the scalar field responsible for inflation itself).

In this paper, we will be concerned with the gravitational production of massive scalar particles in inflationary Universe models. [Our analysis is restricted to masses < Hubble constant during inflation; i.e., massive, but light in the context of inflation.] A given Fourier mode of a scalar field is quantum-mechanically excited when its wavelength is well inside the horizon. The mode crosses outside the horizon where it is then treated classically, i.e., 'freezes in' as a classical fluctuation. The mode begins to behave as non-relativistic matter during either the reheating, radiation-dominated, or matter-dominated phases which follow inflation. We sum the contributions from all modes and compute the energy density today in these massive scalar particles. The final result depends on the mass of the particle, its coupling to gravity, and on the energy scales for inflation and reheating. We find that for a wide range of these parameters, the energy density in the particles produced may significantly contribute to the total energy density of the Universe.

The outline of the paper is as follows: In Section II, we give preliminary calculations relevant for the general case. In Sections III and IV we compute the total energy density in ϕ-particles for particular choices of ξ, the coupling of the field to the curvature scalar.

minimal coupling ($\xi = 0$), and arbitrary ξ. Finally, in Section V we apply our results to the axion, a particle whose mass is temperature and therefore time-dependent.

II. Preliminary Calculations

We consider a massive scalar field coupled to gravity in a spatially flat Friedmann-Robertson-Walker (FRW) cosmology. The line element can be written as

$$ds^2 = \begin{cases} -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \\ a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2) \end{cases} \tag{2.1}$$

where t (η) is the clock (conformal) time and $a(t)$ is the cosmic scale factor. In what follows, dot (prime) will denote the derivative with respect to clock (conformal) time. Comoving scales (those measured by $(x,\ y,\ z)$) are related to physical scales by: (physical scale) = $a(t)$ × (comoving scale). The cosmic scale factor $a(t)$ is normalized so that today, (physical scale) = (comoving scale). We use the system of units in which $k_B = c = \hbar = 1$ and $G = m_{pl}^{-2}$ where $m_{pl} = 1.2 \times 10^{19} GeV$ is the planck mass. Throughout we describe the stress energy of the Universe by a perfect fluid with an equation of state $p = \gamma\rho_{tot}$ where ρ_{tot} is the total energy density of the Universe; it follows that $\rho_{tot} \propto a^{-3(1+\gamma)}$. The Hubble radius or 'physics horizon', $H^{-1} \propto a^{3(1+\gamma)/2}$, determines the scale over which coherent microphysical processes can operate. Today, $H = 100\ h\ km\ sec^{-1}\ Mpc^{-1}$ and $H^{-1} = 3000h^{-1}Mpc$ ($h \approx 0.4 - 1.0$). [For brevity, we refer to the Hubble radius, H^{-1}, as the horizon, although this, of course, is not technically correct.]

We assume that the scalar field is weakly-interacting so that its couplings to itself and to other fields can be, for our purposes, ignored (and in this limit the ϕ-particle is stable). The Lagrangian for the field is

$$\mathcal{L} \ = \ -\frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi \ + \ m^2\phi^2 \ + \ \xi R\phi^2\right] \tag{2.2}$$

where $R = 6\left(\ddot{a}/a + (\dot{a}/a)^2\right) = 6a''/a^3$ is the scalar curvature and the parameter ξ specifies the coupling of the particle to gravity. In the present discussion, we take the mass, m, of the ϕ-field to be constant. The classical equation of motion for the field is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + m^2\phi + \xi R\phi = 0 \tag{2.3a}$$

or equivalently, in conformal coordinates,

$$\omega'' + \left(a^2m^2 - \nabla^2\right)\omega + (\xi - 1/6)\,a^2R\omega = 0 \tag{2.3b}$$

where $\omega = a\phi$. In conformal coordinates it is clear that for $\xi < 1/6$, the gravitational coupling results in a negative mass-squared proportional to R. This negative mass-squared is responsible for the instability which leads to the scalar particle production we will discuss. [The equation of motion for the graviton is identical and $\xi_{graviton} = 0$.] In a later section, we will consider a specific example of a particle whose mass is not constant: the axion.

Let us review some important aspects of inflation[5]. Inflation occurs when a scalar field, called the inflaton (and different from ϕ), is displaced from the zero-energy minimum of its potential and 'slowly' evolves to this minimum. During inflation ρ_{tot} is dominated by the potential energy of the inflaton and is approximately constant: $\rho_{tot} \simeq \rho_o \equiv M^4$ where M is the energy scale for the inflaton potential. It follows that during inflation, the Universe is in a nearly de Sitter phase (dS). In order for inflation to solve the usual horizon and flatness problems, a must grow by at least a factor of $O(e^{60})$. After inflation follows the epoch of reheating (RH) during which the energy density is dominated by the coherent oscillations of the inflaton field (equivalently, non-relativistic inflatons) and $\rho_{tot} \propto a^{-3}$. During RH, the temperature T ($\simeq \rho_{rad}^{1/4}$ where $\rho_{rad}^{1/4}$ is the energy density in relativistic particles produced by the decay of the coherent oscillations) rises quickly (in a Hubble time) to $(T_{RH}M)^{1/2}$ and then decreases to T_{RH}. The temperature reaches $T_{RH}(\simeq (\Gamma m_{pl})^{1/2})$, when the age of the Universe $\simeq \Gamma^{-1}$ (Γ = decay width of the inflaton). At this time $\rho_{tot} \simeq \rho_{rad}$ and the energy density in the coherent oscillations (inflaton particles) begins to decrease exponentially[13]. Following RH come the usual radiation-dominated (RD) and matter-dominated (MD) phases of the standard big bang model with $T_{eq} = 6h^2 eV$ being the temperature at the epoch of equal matter and radiation.

In constructing an acceptable inflationary scenario, it is important to keep in mind two basic constraints to M and T_{RH}. First, graviton production imposes the constraint that $\rho_o = M^4 < 10^{-8} m_{pl}$ or equivalently $H_o/m_{pl} < 10^{-4}$ where $H_o \simeq M^2/m_{pl}$ is the Hubble parameter during inflation[9]. Gravitons are produced during inflation and those modes just entering the horizon today lead to large angular scale (quadrupole) distortions in the microwave background. The above constraint follows from the requirement that these distortions be consistent with present limits to the microwave isotropy[14]. We must also ensure that the Universe is radiation dominated by the epoch of primordial nucleosynthesis so that the successful predictions of nucleosynthesis are not spoiled and we therefore require that M, $T_{RH} \gtrsim 1 GeV$. Baryogenesis almost certainly provides a much more stringent constraint to T_{RH}; however, the details of baryogenesis in inflationary models are far from being settled. Furthermore, the most stringent constraint to inflationary models is that from adiabatic density perturbations; however, this constraint does not easily translate into a simple constraint to M, T_{RH}.

In Table I, we give the scale factor and scalar curvature in terms of the conformal time η for each phase in the history of the Universe. Note that in using the expressions for $a = a(\eta)$ in Table I care must be taken in joining one phase to the next. One must require that a and its first derivative be continuous. However, it is not necessary to have η continuous (if care is taken) and it is simpler not to require that η be continuous.

In this paper, we calculate the energy density today in non-relativistic ϕ particles which have their origin as quantum fluctuations during an inflationary phase. Our starting point is to write ϕ as the sum over Fourier modes each labeled by its comoving wavelength λ and comoving wavenumber $k \equiv 2\pi/\lambda$: $\phi(\vec{x}, t) = \int d^3k e^{ik\cdot x} \phi_k(t)$. The equation of motion for ϕ_k is

$$\ddot{\phi}_k + 3\frac{\dot{a}}{a}\dot{\phi}_k + (k/a)^2\, \phi_k + m^2\phi_k + \xi R\phi_k = 0 \tag{2.4a}$$

or equivalently,

$$\omega_k'' + \left(k^2 + a^2m^2\right)\omega_k + (\xi - 1/6)\, a^2 R\omega_k = 0 \tag{2.4b}$$

where $\omega_k = a\phi_k$.

A given mode is initially excited when it is well inside the horizon ($a\lambda < H^{-1}$ or $k > aH$) during the de Sitter epoch of inflation. It is well known that in de Sitter space, there are fluctuations in a massless, minimally-coupled scalar field such that a comoving observer detects a thermal bath of ϕ-particles with the Gibbons-Hawking temperature, $H_o/2\pi$ [15]. This implies that at first horizon crossing, $\rho_\phi(k = aH)/\rho_{tot} \simeq (H/m_{pl})^2 \simeq (M/m_{pl})^4$ where $\rho_\phi(k) \simeq k d\rho_\phi/dk$ is the energy density in the kth mode. Since $\rho_\phi(k) \simeq k^5|\phi_k|^2/a^2$

it follows that at first horizon crossing $|\phi_k|^2 \simeq H_o^2/k^3$. Furthermore, Bunch and Davis[16] have shown that a massless, conformally-coupled scalar field will have fluctuations with a stress energy density that is de Sitter invariant (and hence non-thermal) and is of order H_o^4. We therefore make the seemingly reasonable assumption that at first horizon crossing, $\rho_\phi(k)/\rho_{tot} \simeq H^4$ and $|\phi_k|^2 \simeq H_o^2/k^3$ for *any* scalar field satisfying $m < H_o$, regardless of the value of ξ.

A fluctuation mode crosses outside the horizon (first horizon crossing) during inflation and crosses back inside the horizon (second horizon crossing) during either RH, RD, or MD. The scale factors at first and second horizon crossing are labeled a_1 and a_2 respectively. It is useful at this point to refer to Fig. 1. Shown are the horizon size, H^{-1}, the physical wavelength of a given mode, $a(t)\lambda \simeq a(t)/k$, and the Compton wavelength of the ϕ-particle, m^{-1}. One should keep in mind that $a_1(\equiv H(a_1)/k)$, $a_2(\equiv H(a_2)/k)$ and the value of the scale factor when (the Compton wavelength) $\simeq$ (wavelength of the fluctuations), $a_{NR} \simeq k/m$, are fixed by the choice of k and in fact each could equally well serve to label any given mode. Furthermore, there is one particular mode such that $a_2 = a_{NR}$, i.e., equality of the Compton wavelength, mode wavelength, and horizon at second horizon crossing. We label the scale factor at second horizon crossing for this mode by a_*. More precisely, a_* is defined by the relation $3H(a_*) = m$.

Once outside the horizon, we assume that the fluctuation behaves classically i.e., obeys its classical equations of motion, Eqns. (2.4). By comparing the three scales displayed in Fig. 1, one can determine which of the terms in Eqns.(2.4) are dominant. In particular, once the Compton wavelength has entered the horizon ($m > H$) *and* $m^{-1} < a/k$ [i.e., $a > a_m \equiv min(a_*,\ k/m)$] ϕ will behave as non-relativistic matter. This can be shown explicitly by studying the equations of motion. Substituting $\phi_k = A_k e^{imt}$ into Eqn.(2.4a) and neglecting the k^2 term, one finds

$$\ddot{A}_k + 3\frac{\dot{a}}{a}\dot{A}_k + im\left(2\dot{A}_k + 3\frac{\dot{a}}{a}A_k\right) = 0 \tag{2.5}$$

which, for $m \gg 3H \sim t^{-1}$, gives $A_k \propto a^{-3/2}$ and $d\rho_k \propto |A_k|^2 \propto a^{-3}$. [Note that for $m \ll 3H$, there are two solutions: $A_k \propto cons't$ and $A_k \propto a^{-3}$.] For $a > a_m$, the energy density (= mass × number density) per comoving volume is conserved and it is therefore convenient to compare the differential energy density in the kth mode $d\rho_k$ to the entropy density, s ($= 4\rho_{rad}/3T$ where ρ_{rad} is again the energy density in relativistic matter):

$$\left.\frac{d\rho_k}{s}\right|_{a=a_m} = \frac{3}{4}f^{-1}\ T_m\ \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_m} \tag{2.6}$$

where $T_m = T(a_m)$ and $f \equiv \rho_{rad}/\rho_{tot}$ is the fraction of the total energy density contained in radiation. T_m, the temperature at which any ϕ particles created behave as non-relativistic matter, can occur during RH, RD, or MD and each of these cases must be treated separately.

In RD, f is 1 (by definition) while in MD, $f = T_m/T_{eq}$. During RD and MD in the standard big bang cosmology, the expansion of the Universe is adiabatic and therefore $d\rho_k/s$ is constant so that $d\rho_k/s|_{a=a_m} = d\rho_k/s|_{today}$. If there is entropy production during either RD or MD then $d\rho_k/s|_{today} = d\rho_k/s|_{a=a_m} P^{-1}$, where P is the factor by which the entropy per comoving volume sa^3 has changed since $T = T_m$. When T_m occurs during RH, both the ϕ-field and the coherent oscillations of the inflaton behave as massive, non-relativistic matter ($d\rho_k$ and $\rho \simeq \rho_{osc}$ are both $\propto a^{-3}$). Therefore,

$$\left.\frac{d\rho_k}{s}\right|_{a=a_{RH}} \simeq T_{RH} \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_{RH}} = T_{RH} \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_m}. \tag{2.7}$$

The above results can be summarized as follows:

$$\left.\frac{d\rho_k}{\rho_c}\right|_{today} = \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_m} \frac{T_m}{\rho_c/s_o}\, G \tag{2.8}$$

where

$$G = \begin{cases} T_{RH}/T_m & \text{RH} \\ 1 & \text{RD} \\ T_{eq}/T_m & \text{MD} \end{cases} . \tag{2.9}$$

The critical density is $\rho_c = 1.05 \times 10^4\ h^2\ eV\ cm^{-3}$ and the entropy density today is $s_o = 2810\ T_{2.7}^3\ cm^{-3}$, where $T = 2.7^\circ K\ T_{2.7}$ is the present temperature of the microwave background. The three cases given in Eqn (2.9) correspond to T_m occurring during RH, RD, and MD as indicated.

To complete the calculation, we must determine $d\rho_k/\rho_{tot}|_{a=a_m}$ and then integrate over k. As we shall see, the quantity $d\rho_k/\rho_{tot}|_{a=a_m}$ is very sensitive to the value of ξ. Furthermore, we will show that the integral is dominated by fluctuations centered around the particular mode which enters the horizon just as the mass term begins to dominate the k^2 term, i.e., the mode such that $a_2 = a_m \equiv a_*$ is satisfied.

III. Conformally and Minimally Coupled Scalar Field

We now consider two particular choices for ξ: conformal coupling ($\xi = 1/6$) and minimal coupling ($\xi = 0$). For $\xi = 1/6$ and $m^2 \ll (k/a)^2$, the solutions to Eqn.(2.4b) are $\omega_k = a\phi_k \propto e^{\pm ik\eta}$ and it follows that $d\rho_k \sim |\phi_k|^2/a^2 \propto a^{-4}$. These are the expected results: First, for a conformally coupled field (in the present example, the $\xi = 1/6$ scalar field) in a conformally flat spacetime (here, an FRW spacetime), the solution to the wave equation is just the Minkowski space solution multiplied by a conformal weight. For the $\xi = 1/6$ scalar field the conformal weight is a^{-1}. Furthermore, the energy density for a conformal field in an FRW spacetime always scales as a^{-4}.

With the above result, we can readily calculate $d\rho_k/\rho_{tot}$ at $a = a_m$ for a_m occurring during either RH, RD, or MD:

$$\left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_m} = e^{-4N}\left(\frac{H_o}{m_{pl}}\right)^2\frac{dk}{k}\times\begin{cases}\left(T_m^2/T_{RH}M\right)^{4/3} & \text{RH}\\ \left(T_{RH}/M\right)^{4/3} & \text{RD}\\ \left(T_{RH}/M\right)^{4/3}\left(T_m/T_{eq}\right) & \text{MD}\end{cases} \tag{3.1}$$

where $N(= N(\lambda) = 45 + \ln(\lambda/Mpc) + 2/3\ln(M/10^{14}GeV) + 1/3\ln(T_{RH}/10^{10}GeV))$ is the number of e-folds the Universe expands between first horizon crossing and the end of inflation. In the above expression, $\lambda = \lambda_{Mpc}Mpc \simeq 1/k$ is the comoving wavelength of a given scale and is related to the temperature at second horizon crossing for that scale, T_2:

$$T_2(\lambda) = \begin{cases}73\ eV/\lambda_{Mpc} & \lambda \leq 12h^{-2}Mpc\\ 860\ h^{-2}eV/\lambda_{Mpc}^2 & \lambda \geq 12h^{-2}Mpc.\end{cases} \tag{3.2}$$

From Eqn. 3.1, it is clear that the contribution to the energy density of the Universe from these particles is negligible.

Next, consider the minimally coupled scalar field. First, we show that $d\rho_k/\rho_{tot}$ at second horizon crossing is the same as it is at first horizon crossing provided that the Compton wavelength of ϕ remains greater than the horizon until second horizon crossing. For $H \gg m$ and $H \gg k/a$, Eqn. (2.4b) becomes

$$\omega_k'' = \begin{cases}2\omega_k/\eta^2 & \text{in dS, RH, or MD}\\ -(k^2 + a^2m^2)\omega_k & \text{in RD}\end{cases}. \tag{3.3}$$

The general solutions are

$$\omega_k = \begin{cases}Sa + Ta^{-2} & \text{in dS}\\ Ua + Va^{-1/2} & \text{in RH and MD}\\ Wa + X & \text{in RD with } k > ma\\ Ya + Za^{-2} & \text{in RD with } k < ma\end{cases} \tag{3.4}$$

where S through Z are constants. [For the last of these results, see the comment which follows Eqn. (2.5).] At first horizon crossing, S and T are comparable. However, once a fluctuation crosses outside the horizon, $|k\eta| = k/aH_o \ll 1$, so that $\omega_k \simeq Sa$, $\phi_k \simeq cons't$ and therefore during dS (and after first horizon crossing), $\phi_k \simeq H_o/k^{3/2}$. Since the solutions in each of the subsequent phases has a part with $\omega_k \propto a$ or equivalently $\phi_k \propto cons't$ it easy to match solutions from one phase to the next. [The decaying mode solution decreases with time and thus can be neglected.] Therefore, as long as $a/k > H^{-1}$ and $m^{-1} > H^{-1}$, $\phi_k \simeq H_o/k^{3/2} \propto cons't$ and $d\rho_k \propto a^{-2}$ or equivalently

$$\left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_2} = \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_1}\left(\frac{a_1}{a_2}\right)^2\left(\frac{\rho_{tot}(a_1)}{\rho_{tot}(a_2)}\right). \tag{3.5}$$

From the relations $H(a_1) = k/a_1$ and $H(a_2) = k/a_2$ it follows that

$$\frac{H(a_2)}{H(a_1)} = \left(\frac{\rho_{tot}(a_2)}{\rho_{tot}(a_1)}\right)^{1/2} = \frac{a_1}{a_2} \tag{3.6}$$

and therefore

$$\left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_2} = \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_1} \simeq \left(\frac{H_o}{m_{pl}}\right)^2 \tag{3.7}$$

regardless of whether second horizon crossing occurs during RH, RD, or MD.

We note that the equation of motion for gravitons, the tensor perturbations of $g_{\mu\nu}$, is the same as that for a minimally-coupled scalar field. The fact that the energy density of a given mode for the graviton field, as well as for the minimally-coupled scalar field, decreases only as a^{-2} as compared with a^{-4} for a conformally-coupled field is known as 'superadiabatic amplification'.

[The graviton constraint discussed earlier is easily derived using the results of this section. Fluctuations in the microwave temperature, $\delta T/T$, are approximately equal to the amplitude of metric fluctuations. The metric fluctuations are $O(H_o/m_{pl})$ at first horizon crossing and, as in the case of the minimally-coupled scalar field, are $O(H_o/m_{pl})$ at second horizon crossing. Modes just entering the horizon today give rise to large scale (e.g., quadrupole) distortions in the microwave background temperature with $\delta T/T = O(H_o/m_{pl})$ and the requirement that these distortions be consistent with present limits on the microwave isotropy leads to the constraint : $H_o \simeq 10^{-4} m_{pl}$ [9,14].]

Let us rewrite Eqn.(2.8) in the following form:

$$\begin{aligned} \left.\frac{d\rho_k}{\rho_c}\right|_{today} &= \frac{T_*}{\rho_c/s_o}\,\frac{T_m}{T_*}\,F\,G\,\left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_1} \\ &= \frac{T_*}{\rho_c/s_o}\left(\frac{H_o}{m_{pl}}\right)^2\frac{T_m}{T_*}\,F\,G \end{aligned} \tag{3.8}$$

where $d\rho_k/\rho_{tot}|_{a=a_m} = F d\rho_k/\rho_{tot}|_{a=a_1}$ and G is given by Eqn.(2.9). We now discuss the quantity F. Consider, for example, modes which have both second horizon crossing and a_m during RD. For those modes where $a_2 < a_m$ (labeled λ_B in Fig. 1), we see that after a_2 but before a_m, $\phi \propto 1/a$ (Eqn. (2.4b) dominated by k^2 term) and $\rho_k \propto a^{-4}$. For these modes, $d\rho_k/\rho_{tot}$ is constant after second horizon crossing. Since $d\rho_k/\rho_{tot}$ is the same at second horizon crossing as it is at first horizon crossing, F is simply 1. For modes with $a_2 > a_m$ (labeled λ_A in Fig. 1), $d\rho_k/\rho_{tot}|_{a=a_2} = d\rho_k/\rho_{tot}|_{a=a_1}(a_*/a_2)^2$ and $F = (a_*/a_2)^2$. In Table II we give the values of F for the various possibilities.

From Eqn. (3.8), we calculate the energy density today in ϕ-particles, $\rho_\phi = \int d\rho_k$, as well as $\Omega_\phi \equiv \rho_\phi/\rho_c$. While it is rather difficult and cumbersome to display results for Ω_ϕ

in the most general case, specific examples are easy to work out. Consider the case where a_* occurs during RD. We first calculate the contribution to Ω_ϕ from modes such that *both* a_2 and a_m occur during RD. The integral is broken into two parts:

$$\Omega_\phi \simeq \left(\frac{H_o}{m_{pl}}\right)^2 \frac{T_*}{\rho_c/s_o}\left[\int_{k(a_m=a_*)}^{k_{min}} \frac{a_*}{a_m}\frac{dk}{k} + \int_{k(a_2=a_{eq})}^{k(a_2=a_*)} \left(\frac{a_*}{a_2}\right)^2 \frac{dk}{k}\right]. \tag{3.9}$$

where $k_{min} = min(k(a_m = a_{eq}),\ k(a_2 = a_{RH}))$. [$k_{min}$ is defined as such to insure that both second horizon crossing and a_m occur during RD for all modes included in the integral.] The first (second) integral gives the contribution from modes with $a_2 < a_m$ ($a_2 > a_m$). Using the relations $a_m \propto k$ and $a_2 \propto k^{-1}$ (for a_2 occurring during RD), we find that

$$\Omega_\phi = \left(\frac{H_o}{m_{pl}}\right)^2 \frac{T_*}{\rho_c/s_o}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{T_{eq}}{T_*}\right)^2 - max\left(\frac{T_*}{T_{RH}}, \frac{T_{eq}}{T_*}\right)\right]. \tag{3.10}$$

The contributions from modes which have $a = k/m$ during MD or have second horizon crossing during either RH or MD will introduce corrections of the order T_{eq}/T_* and T_*/T_{RH} and for the case at hand (i.e., $T_{RH} \geq T_* \geq T_{eq}$) these contributions are at most of order unity. We obtain then, as an estimate for Ω_ϕ:

$$\Omega_\phi \approx \left(\frac{H_o}{m_{pl}}\right)^2 \frac{T_*}{\rho_c/s_o} \simeq \frac{T_{2.7}^3}{h^2}\left(\frac{H_o}{m_{pl}}\right)^2 \frac{T_*}{4\ eV} \tag{3.11}$$

The dominant contribution to the energy density in this example comes from those fluctuation modes which have $a_2 \simeq a_*$. This is also true when a_* occurs during RH or MD as can be seen by inspection of Table II and Eqn.(3.7) which shows that the integrand needed to compute Ω_ϕ is maximal when $a_2 = a_m = a_*$. It is easy to check that the result of the integration in either of these two cases is well approximated by simply evaluating the integrand at $a_2 = a_m = a_*$.

To summarize the results for the three cases where T_* occurs during either RH, RD, or MD we write:

$$\Omega_\phi \simeq \left(\frac{H_o}{m_{pl}}\right)^2 \left(\frac{1}{4\ eV}\right)\begin{cases} T_{RH} & \text{RH} \\ T_* & \text{RD} \\ T_{eq} & \text{MD}\end{cases} \tag{3.12}$$

The expression $m = 3H(T_*) = (4\pi^3 g_*(T_*)/5)^{1/2} T_*^2/m_{pl}$ can be used to relate T_* to the mass of the ϕ-particle:

$$T_* \simeq 1.6 \times 10^9\ GeV\ g_*^{-1/4}(T_*)(m/GeV)^{1/2} \tag{3.13}$$

Eqn. (3.12) for the energy density in ϕ-particles today can then be rewritten as

$$\Omega_\phi \simeq \left(\frac{H_o}{m_{pl}}\right)^2 \begin{cases} 2.5\times 10^{18} T_{10} & m \geq 40\ GeV\ g_*^{1/2}(T_{RH})T_{10}^2 \\ 3.9\times 10^{17} g_*^{-1/4}(T_*)(m/GeV)^{1/2} & 40\ GeV\ g_*^{1/2}(T_{RH})T_{10}^2 \geq m \\ & \geq 3\times 10^{-35} \\ 1.5 & 3\times 10^{-35} GeV \geq m \end{cases} \tag{3.12a}$$

where $T_{RH} = T_{10}10^{10}\ GeV$. Note that for $m \leq 3\times 10^{-35}\ GeV$, T_* is less than $T_{eq}(\simeq GeV)$, and $\Omega_\phi \leq (H_o/m_{pl})^2 \leq 10^{-8}$ is probably uninterestingly small.

Now consider the possibility that ϕ particles created during inflation as deSitter space QM fluctuations might provide closure density today. For $m \geq 40GeV\ g_*^{1/2}(T_{RH})T_{10}^2$ (i.e., corresponding to $T_* \geq T_{RH}$), this requires that

$$\Omega_\phi \simeq 2.5\times 10^{18} T_{10}(H_o/m_{pl})^2 \simeq 1,$$

or

$$T_{RH} \simeq 0.4GeV[(m_{pl}/H_o)/10^4]^2. \tag{3.14a}$$

For scalar particle masses in the range: $3\times 10^{-35}GeV \leq m \leq 40GeV g_*^{1/2}(T_{RH})T_{10}^2$, (i.e., corresponding to: $T_{RH} \geq T_* \geq 6eV$), this requires that:

$$\Omega_\phi \simeq 3.9\times 10^{17} g_*^{-1/4}(T_*)(m/GeV)^{1/2}(H_o/m_{pl})^2 \simeq 1$$

, or

$$m/GeV \simeq 6.6\times 10^{-20} g_*^{1/2}(T_*)[(m_{pl}/H_o)/10^4]^4. \tag{3.14b}$$

Conversely, the existence of a minimally-coupled scalar particle places constraints on the value of the Hubble constant during inflation to ensure that $\Omega_\phi < 1$:

$$\frac{H_o}{m_{pl}} \leq \begin{cases} 6.3\times 10^{-10} T_{10}^{-1/2} & m \geq 40GeV g_*^{1/2}(T_{Rh})T_{10}^2 \\ 1.6\times 10^{-9} g_*^{1/8}(T_*)(m/GeV)^{-1/4} & m \leq 40GeV g_*^{1/2}(T_{RH})T_{10}^2 \end{cases} \tag{3.15}$$

In principle this constraint to H_o/m_{pl} can be more stringent than that from graviton production.

IV. Arbitrary Coupling to Gravity

In this section, we consider the case where ξ is arbitrary. As before, an estimate of Ω_ϕ is calculated by considering the contribution from the mode which has second horizon crossing when $a = a_*$. For modes outside the horizon and still massless, Eqn. (2.4b) reads

$$\omega_k'' = \begin{cases} 2(1-6\xi)\omega_k/\eta^2 & \text{in dS, RH, MD} \\ -(k^2 + a^2m^2)\omega_k & \text{in RD} \end{cases} \tag{4.1}$$

The solutions are easily found and the results for ϕ_k in terms of a are:

$$\phi_k = \begin{cases} Aa^{(\sigma-3)/2} + Ba^{-(\sigma+3)/2} & \text{in dS} \\ Ca^{(\sigma-3)/4} + Da^{-(\sigma+3)/4} & \text{in RH, MD} \\ E + Fa^{-1} & \text{in RD with } k > ma \\ G + Ha^{-3} & \text{in RD with } ma > k \end{cases} \tag{4.2}$$

where $\sigma \equiv \sqrt{1+8(1-6\xi)}$. As before, at first horizon crossing, the amplitude of ϕ_k is $\simeq H_o k^{-3/2}$. At the end of dS, $\phi_k \simeq H_o k^{-3/2} e^{N(\sigma-3)/2}$. By matching this to the solution in RH we find:

$$\phi_k \simeq \frac{H_o e^{N(\sigma-3)/2}}{k^{3/2}} \left(\frac{3(\sigma-1)}{2\sigma} \left(\frac{a}{a_e}\right)^{(\sigma-3)/4} + \frac{3-\sigma}{2\sigma} \left(\frac{a}{a_e}\right)^{-(\sigma+3)/4} \right) \tag{4.3}$$

where a_e is the scale factor at the end of inflation. Note that for $\xi \to 1/6$, $\sigma \to 1$ and the first term will vanish. In this limit, one recovers the result for the conformally-coupled field. Since this case leads to only negligible energy density in ϕ-particles, we will not consider it further. For $(a_e/a)^{\sigma/2} \gg \sigma - 1$ we can neglect the second term. During RD (but with $k > m/a$):

$$\phi_k \simeq \frac{H_o e^{N(\sigma-3)/2}}{k^{3/2}} \left(\frac{T_{RH}}{M}\right)^{(3-\sigma)/3} \frac{3(\sigma-1)}{8\sigma} \left(\sigma + 1 - (\sigma-3)\left(\frac{T}{T_{RH}}\right) \right) \tag{4.4}$$

Ignoring the second term, we find that

$$\phi_k \simeq \frac{3(\sigma^2-1)}{8\sigma} e^{N(\sigma-3)/2} \frac{H_o}{k^{3/2}} \left(\frac{T_{RH}}{M}\right)^{(3-\sigma)/3} \tag{4.5}$$

which is constant.

Consider the two cases where a_* occurs during RH or RD. One has that

$$\phi_k\Big|_{a=a_*} = \phi_k\Big|_{a=a_1} \frac{3(\sigma-1)}{8\sigma} e^{N(\sigma-3)/2} \begin{cases} 4\left(T_*^2/T_{RH}M\right)^{(3-\sigma)/3} & \text{RH} \\ (\sigma+1)\left(T_{RH}/M\right)^{(3-\sigma)/3} & \text{RD} \end{cases} \tag{4.6}$$

so that

$$\Omega_\phi(\xi) = \Omega_\phi(\xi=0) \times \frac{9(\sigma-1)^2}{64\sigma^2} e^{N(\sigma-3)} \begin{cases} 16\left(T_*^2/T_{RH}M\right)^{2(3-\sigma)/3} & \text{RH} \\ (\sigma+1)^2\left(T_{RH}/M\right)^{2(3-\sigma)/3} & \text{RD} \end{cases} \tag{4.7}$$

N must be evaluated for the particular mode which has $a_2 = a_*$ and we find that

$$e^N = \begin{cases} \left(MT_{RH}/T_*^2\right)^{2/3} & \text{RH} \\ \left(M/T_{RH}\right)^{2/3}\left(T_{RH}/T_*\right) & \text{RD} \end{cases}. \tag{4.8}$$

Using this, we can rewrite Eqn. (4.7):

$$\Omega_\phi(\xi) = \Omega_\phi(\xi=0) \times \frac{9(\sigma-1)^2}{64\sigma^2} \begin{cases} 16\left(MT_{RH}/T_*^2\right)^{4(\sigma-3)/3} & \text{RH} \\ (\sigma+1)^2\left(M/T_{RH}\right)^{4(\sigma-3)/3}\left(T_{RH}/T_*\right)^{(\sigma-3)} & \text{RD.} \end{cases} \tag{4.9}$$

Eqns. (4.6-4.9) clearly show that for $\xi < 0$ ($\sigma > 3$), ϕ_k grows while outside the horizon, i.e., $d\rho_k$ decreases more slowly than a^{-2}, and Ω_ϕ is enhanced over the $\xi = 0$ value. Given

M, T_{RH}, ξ, and m, one can compute Ω_ϕ. Alternatively, given three of these quantities, one can use Eqn. (4.9) to place constraints on the fourth by requiring that Ω_ϕ not be greater than 1.

Physically, the growth in ϕ_k while outside the horizon arises because, for $\xi < 1/6$, ϕ_k has a negative effective mass squared and this indicates an instability. One might expect that interaction terms (e.g., a ϕ^4 term) would halt the growth of ϕ_k but we have not considered these terms. Furthermore, for $\xi < 0$ there may be important backreaction effects during inflation. Ford [17], for example, finds that in a Universe with a cosmological constant and a scalar field with $\xi < 0$, the late time behavior of the system has $\phi \propto t$ and $a \propto t^\alpha$ where $\alpha \equiv (2|\xi| + 1)/4|\xi|$, i.e., the energy density in the scalar field cancels the cosmological constant with the residual energy density leading to power-law expansion. We will not discuss backreaction effects further except to note that as found by Ford[17], it is possible, under certain conditions, to have a scalar field with $\xi < 0$ without affecting inflation.

V. Axions

We now apply the results of the previous section to an interesting and very topical example: the axion. The axion is the (pseudo-) Nambu-Goldstone boson associated with spontaneous breaking of the Peccei-Quinn (PQ) global $U(1)_{PQ}$ symmetry. Symmetry breaking occurs at an energy scale f_A. At the QCD transition (energy scale $\Lambda = 200 MeV \Lambda_{200}$) instanton effects, which also break $U(1)_{PQ}$, become important and the axion becomes massive. The mass is temperature and therefore time-dependent. As will be shown below, we must modify the analysis of the previous section in order to take into account the special properties of the axion field. Before doing so, we review the general properties of axions relevant to the present discussion as well as the previous work on cosmological production of axions.

Let $\vec{\phi} = \phi e^{i\theta}$ be the complex scalar field responsible for breaking the $U(1)_{PQ}$ symmetry. Spontaneous symmetry breaking occurs when $\vec{\phi}$ acquires a vacuum expectation value: $< |\vec{\phi}| > = < \phi > = f_A$. The Nambu-Goldstone boson associated with the θ degree of freedom is labeled the axion, $A = f_A\theta$. Initially, the potential for θ is flat and the axion is massless. At a temperature of order Λ, QCD instanton effects induce M degenerate minima in the potential for θ and consequently, the axion develops a mass. M is a positive integer whose value depends on the $U(1)_{PQ}$ charges of the quarks (and any other particles which carry color) and, in the simplest models $M = 6$. The Lagrangian for the axion field is

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\left(\partial_\mu A \partial^\mu A + m_A^2(T)A^2 + \xi R A^2\right) \\ &= -\frac{f_A^2}{2}\left(\partial_\mu \theta \partial^\mu \theta + m_A^2(T)\theta^2 + \xi R \theta^2\right)\end{aligned} \tag{5.1}$$

where we have neglected the coupling of the axion to other fields (e.g., photons, fermions). [These couplings, though very important for axion detection and for understanding how axions may affect stellar evolution, can be for our purposes ignored.] The equation of motion for the kth Fourier component of A is

$$\ddot{A}_k + 3\frac{\dot{a}}{a}\dot{A}_k + \frac{k^2}{a^2}A_k + m_A^2(T)A_k = 0. \tag{5.2}$$

m_A is the temperature-dependent axion mass with $m_A(T) \to 0$ for $T \to \infty$ and $m_A(T) \to m_A = 3.7(M/6) \times 10^{-5} eV(10^{12}GeV/f_A)$ for $T \to 0$. We will need an explicit formula for the axion mass as a function of temperature and we use the results of Gross, Pisarski, and Yaffe [18]. They calculate the mass of the axion using the dilute-instanton gas approximation and find

$$\frac{m_A(T)}{m_A} = \begin{cases} B\,(\Lambda/T)^p & T < B^{1/p}\Lambda \equiv T_o \\ 1 & T > T_o \end{cases} \tag{5.3}$$

where $B = 7.7 \times 10^{-2\pm 0.6}$, $p = 3.7 \pm 0.1$, and T_o is the temperature at which the mass achieves its zero temperature value.

Let us briefly review the usual mechanism thought responsible for the cosmological production of axions[19,20]. At the QCD transition, $\theta \simeq \theta_1 \simeq cons't$ within a horizon volume, the higher momentum modes having been redshifted away. If inflation occurred beforehand, then θ will be constant in a region corresponding to the presently-observable Universe. However in general, θ_1 will not correspond to a minimum in the potential for θ. θ remains constant as long as the Compton wavelength of the axion is outside the horizon ($m_A < 3H$). Once $m_A > 3H$, θ will oscillate about a minimum of its potential and these coherent oscillations correspond to a condensate of non-relativistic axions. The energy density in axions due to this effect depends on θ_1, the finite temperature behavior of the axion mass, and the scale of the PQ symmetry breaking, f_A, or equivalently m_A. One finds that[20]

$$\Omega_a = 0.23\, \frac{T_{2.7}^3}{h^2 \Lambda_{200}^{0.7}}\, (\theta_1 M)^2 \left(\frac{f_A/M}{10^{12}GeV}\right)^{1.18} \tag{5.4}$$

where we have assumed that $f_A \lesssim 10^{18} GeV$.

Seckel and Turner [8] have studied de Sitter space fluctuations in the axion field. These fluctuations lead to fluctuations in the initial misalignment angle and therefore to fluctuations in the density of axions produced. They are in fact, fluctuations in the axion to photon ratio or isothermal density inhomogeneities. Seckel and Turner argue that this effect may be important in galaxy formation.

Along very different lines, Turner [21] has discussed the thermal production of axions in the early Universe via Primakoff and photoproduction processes. He finds that, for $f_A \leq 2 \times 10^8 GeV$ ($m_A \geq 3 \times 10^{-2} eV$), thermal production dominates over production due to coherent oscillations. The results for production due to both thermal effects and coherent oscillations are displayed in Fig. 2.

We now calculate the energy density in axions which originate as QM fluctuations during inflation. It is important to keep in mind that the axion field A, is the Nambu-Goldstone boson associated with the angular degree of freedom of $\vec{\phi}$ once $U(1)_{PQ}$ is broken and therefore A cannot have fluctuations greater than $O(f_A)$. In what follows, we will assume that $f_A \lesssim H_o$ so that $U(1)_{PQ}$ is broken during inflation and the axion can be considered a fundamental scalar field during this epoch.

Let us first consider the case where the axion is minimally coupled ($\xi = 0$). The scales H^{-1}, $m_A(T)^{-1}$, and $a\lambda \simeq a/k$ for the axion case are shown in Fig. 3. From the relation $m_A(T_*) \simeq 3H(T_*) = 5g_*^{1/2}(T_*)(T_*/\Lambda)^2\Lambda^2/m_{pl}$, it follows that

$$\begin{aligned}\frac{T_*}{\Lambda} &= \left[\frac{B}{5g_*^{1/2}(T_*)}\right]^{1/(2+p)} \left(\frac{m_A m_{pl}}{\Lambda^2}\right)^{1/(2+p)} \\ &= 4.7 \times (m_{-5}/\Lambda_{200}^2)^{0.18}\end{aligned} \tag{5.5}$$

where $m_A = 10^{-5} \times m_{-5} eV$. For $m_A(T) > 3H$ and $m_A(T) > k/a$ (i.e., $a > a_m \equiv min[a_*, k/m]$), $m_A^2(T)|A_k|^2 \propto d\rho_k \propto a^{-3}$ where $a_* \equiv a(T_*)$). This result, first obtained in the original work on the cosmological production of axions[19], can be seen directly from the equation of motion by making the substitution $A_k = \mathcal{A}_k \cos m_A T$ and using the fact that $\dot{m}_A/m_A \simeq p/t \simeq H < m_A$. One obtains $d(m_A\mathcal{A}_k^2)/dt + 3H(m_A\mathcal{A}_k^2) = 0$ and the above result follows immediately.

Let us first consider only those modes which have *both* second horizon crossing and T_m during RD. For these modes

$$\left.\frac{d\rho_k}{\rho_c}\right|_{today} \simeq \frac{T_m}{\rho_c/s_o} \left.\frac{d\rho_k}{\rho_{tot}}\right|_{a=a_m} \tag{5.5}$$

where, for simplicity, we assume that there is no entropy production during either RD or MD. In calculating $d\rho_k/\rho_{tot}|_{a=a_m}$ we note that there are three types of fluctuation modes to consider. First, there are modes with $a_2 > a_m$ (labeled λ_C in Fig. 3) so that $a_m = a_*$ and $d\rho_k/\rho_{tot}|_{a=a_m} = d\rho_k/\rho_{tot}|_{a=a_1}(a_*/a_2)^2$. Next, there are modes which become massive after second horizon crossing but before the mass reaches its zero temperature value at T_o (labeled λ_D in Fig. 3). For these modes, $a_m = k/m_A = (k/m_A)(T_m/\Lambda)^p$. Finally, there are modes which become massive after T_o (labeled λ_E in Fig. 3) and for these, $a_m = k/m_A$. For these last two cases, $d\rho_k/\rho_{tot}|_{a=a_m} = d\rho_k/\rho_{tot}|_{a=a_1}$. The contribution to Ω_A from the modes discussed above is given by

$$\Omega_A = \left(\frac{H_o}{m_{pl}}\right)^2 \frac{T_*}{\rho_c/s_o}\Big[\int_{k(a_2=a_{eq})}^{k(a_2=a_*)} \left(\frac{a_*}{a_2}\right)^2 \frac{dk}{k} + \int_{k(a_m=a_*)}^{k(a_m=a_o)} \frac{T_m}{T_*}\frac{dk}{k} + \int_{k(a_m=a_o)}^{k_{min}} \frac{T_m}{T_*}\frac{dk}{k}\Big] \tag{5.6}$$

where $a(T_o) \equiv a_o$ and again, $k_{min} = min[k(a_* = a_{eq}), k(a_2 = a_{RH})]$. The first and last integrals are just the ones encountered in the of minimal coupling case. For the second integral, we use the fact that $k \propto T_m^{-(1+p)}$ so that $dk/k = -(1+p)dT_m/T_m$. The final result is

$$\begin{aligned}\Omega_A &\simeq \left(\frac{H_o}{m_{pl}}\right)^2 \frac{T_*}{\rho_c/s_o}\left[\frac{3}{2} + p - p\frac{T_\alpha}{T_*} - \frac{1}{2}\left(\frac{T_{eq}}{T_*}\right)^2 - max\left(\frac{T_{eq}}{T_*}, \frac{T_*}{T_{RH}}\right)\right] \\ &\simeq 10^9 \frac{T_{2.7}^3}{h^2}\left(\frac{H_o}{m_{pl}}\right)^2 \left(\frac{m_{-5}}{\Lambda_{200}}\right)^{1/(p+2)}\end{aligned} \tag{5.7}$$

As before, the energy density is dominated by those modes just entering the horizon as the Compton wavelength enters the horizon and by including the contributions from the full spectrum of modes the above result will not change in any substantial way

It follows, from the assumption that $f_A \leq H_o$, and from the relation $m_{-5} = 4 \times 10^{12} GeV/f_A$ that

$$\Omega_A \leq 7 \times 10^7 \frac{T_{2.7}^3}{h^2 \Lambda_{200}^{.18}} \left(\frac{f_A}{m_{pl}}\right)^{1.82} \tag{5.8}$$

where the equality holds for $f_A = H_o$. In Fig. 2 this result (with $f_A = H_o$) together with the results for axion production due to the two aforementioned mechanisms are plotted. In choosing $f_A = H_o$ we are in fact plotting an upper limit to Ω_A for axions produced by this mechanism. Evidently, for the minimally-coupled axion, the energy density in axions arising from quantum fluctuations during inflation is always subdominant.

The situation is quite different for $\xi < 0$. For $\xi \neq 0$ we can use Eqn. (4.9) to determine how to modify Eqn. (5.8). For $\xi < 0$, $\sigma > 3$ and the density in axions is greatly enhanced. The results for Ω_A with $\xi = -0.033$ and $\xi = -0.068$ ($\sigma = 3.25$ and $\sigma = 3.5$ respectively) are shown in Fig. 2. [Again, we take $f_A = H_o$ and also, for simplicity, choose $M = T_{RH}$.] Clearly, the energy density in axions from deSitter-induced fluctuations can be important for $\xi \lesssim -0.033$.

VI. Conclusion

We have studied the production of scalar particles in inflationary Universe models. De Sitter space-induced fluctuations in a scalar field cross outside the horizon during inflation and then evolve classically. Ultimately, the fluctuations behave as non-relativistic particles and may significantly contribute to the energy density of the Universe.

The evolution of a scalar field while outside the horizon depends crucially on the coupling of the field to gravity. In particular, the coupling constant ξ determines the power-law dependence of the field on the scale factor so that small changes in ξ will change ρ_ϕ by many orders of magnitude. The dominant contribution to the energy density in ϕ-particles from this mechanism comes from modes which reenter the horizon just as the Compton wavelength of the ϕ-particles is entering the horizon. For a given inflationary Universe model, ξ and m determine ρ_ϕ.

While virtually all Grand Unified Theories predict the existence of fundamental scalar particles, no such particles have yet been observed. Furthermore, the currently popular quantum theories of gravity (e.g., supergravity, superstrings) have not (to the best of our knowledge) directly addressed the question of the coupling of scalar particles to gravity. We hope that this work will generate interest along these lines.

In summary, our results depend on many unknowns. First, one must determine the energy scales for inflation and reheating, M and T_{RH} (if indeed inflation did ever occur). Next, one must know the couplings to gravity as well as the masses of fundamental scalar particles (if such particles exist at all). In addition, one would have to address the question of whether or not the ϕ-particles once produced might thermalize (through their interaction with other particles), thereby ultimately reducing their final abundance. [The equilibrium number density of a massive particle is $n_{EQ} \simeq (mT/2\pi)^{3/2} exp^{-m/T}$.] We have shown here that for a large class of inflationary models and for a wide range of masses for nonconformally-coupled scalar fields, quantum fluctuations in the early Universe can produce a significant and potentially interesting number of scalar particles.

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Figure Captions

Fig. 1 - Summary of the evolution of two representative modes with comoving wavelengths λ_A and λ_B. The Universe is assumed to evolve through four phases: inflation, reheating (RH), radiation domination (RD), and matter domination (MD). The Hubble radius, H^{-1} is $\propto$ $cons't$ (inflation), $a^{3/2}$ (RH), a^2 (RD), and $a^{3/2}$ (MD). The Compton wavelength of the ϕ particle, m^{-1}, is constant throughout and a_* is defined by $3H(a_*) = m$. The physical wavelengths (either $a(t)\lambda_A$ or $a(t)\lambda_B$) begin subhorizon-sized, cross outside the horizon during inflation ($a = a_1$), and thereafter evolve as classical fluctuations. Second horizon crossing occurs at $a = a_2$. The fluctuation behaves as non-relativistic matter ($\rho_k \propto a^{-3}$) once $a > a_m$ where $a_m = min(a_*, k/m)$. For λ_A, $a_m < a_2$ and $a_m = a_*$ while for λ_B, $a_m > a_2$ and $a_m = k/m$. Gravitational production of ϕ particles is dominated by the modes for which $a_2 \simeq a_m$. This mode (not shown) lies between modes A and B.

Fig. 2 - Schematic summary of cosmological production mechanisms for axions. We plot $\Omega_A = \rho_A/\rho_c$ as a function of the PQ symmetry breaking scale f_A (or axion mass). Coherent production of axions resulting from an initial misalignment of the axion field is labeled 'coherent' (ref. 19, 20). Thermal production of axions (Primakoff and photoproduction processes) is labeled 'thermal'[21]. Gravitationally produced axions (considered in the present work) are shown for $\xi = (0,\ -0.033,\ -0.068)$. We have set $H_o = f_A$ so that these results should be considered upper limits for this production mechanism.

Fig. 3 - Summary of the evolution of three representative modes with wavelengths λ_C, λ_D, and λ_E for the axion, as in Fig. 1. Here, the axion mass and therefore the Compton wavelength for the axion, $m_A(T)^{-1}$, is temperature-dependent. As before, a_* is defined by $3H(a_*) = m_A(T_*)$ and a_o is the scale factor when the axion mass reaches its zero temperature value. For λ_C, $a_m < a_2$ and $a_m = a_*$. For λ_D and λ_E, $a_m > a_2$ and $a_m = k/m_A(T_m)$. For λ_D, $m_A(T_m) = m_A(\Lambda/T_m)^p$ while for λ_E, $m_A(T_m) = m_A$ where m_A is the zero-temperature mass of the axion. Again, the dominant contribution to the energy density in gravitationally produced axions comes from the mode for which $a_2 \simeq a_m$. This mode (not shown) lies between modes C and D.

Table I - Expressions for the scale factor (a) and scalar curvature (R) for de Sitter (dS), reheating (RH), radiation-dominated (RD), and matter-dominated (MD) phases in an inflationary Universe model expressed in terms of the conformal time η. In joining one phase to the next one must require that a and its first derivative be continuous. It is not necessary that η be continuous.

Table I				
	dS	RH	RD	MD
a	$-1/H_o\eta$	$A\eta^2$	$B\eta$	$C\eta^2$
a^2R	$12/\eta^2$	$12/\eta^2$	0	$12/\eta^2$

Table II - Expressions for $F = (d\rho_k/\rho_{tot}|_{a=a_m})/(d\rho_k/\rho_{tot}|_{a=a_1})$ with a_2 and $a_m = min(k/m, a_*)$ occurring during the epochs of reheating (RH), radiation domination (RD), or matter domination (MD). For the cases where both a_2 and a_m occur during the same epoch, the first entry is for $a_m > a_2$ and the second entry is for $a_2 > a_m$. For example, for both a_2 and a_m occurring during RD *and* $a_2 > a_m$, $F = (a_*/a_2)^2$. Note that F is maximal for $a_2 = a_m = a_*$.

Table II			
	a_2 in RH	RD	MD
a_m in RH	(a_2/a_m) \ (a_*/a_2)	$(a_*/a_{RH})(a_{RH}/a_2)^2$	$(a_*/a_{RH})(a_{RH}/a_{eq})^2(a_{eq}/a_2)$
RD	(a_2/a_{RH})	1 \ $(a_*/a_2)^2$	$(a_*/a_{eq})^2(a_{eq}/a_2)$
MD	$(a_2/a_{RH})(a_{eq}/a_m)$	(a_{eq}/a_m)	(a_2/a_m) \ (a_*/a_2)

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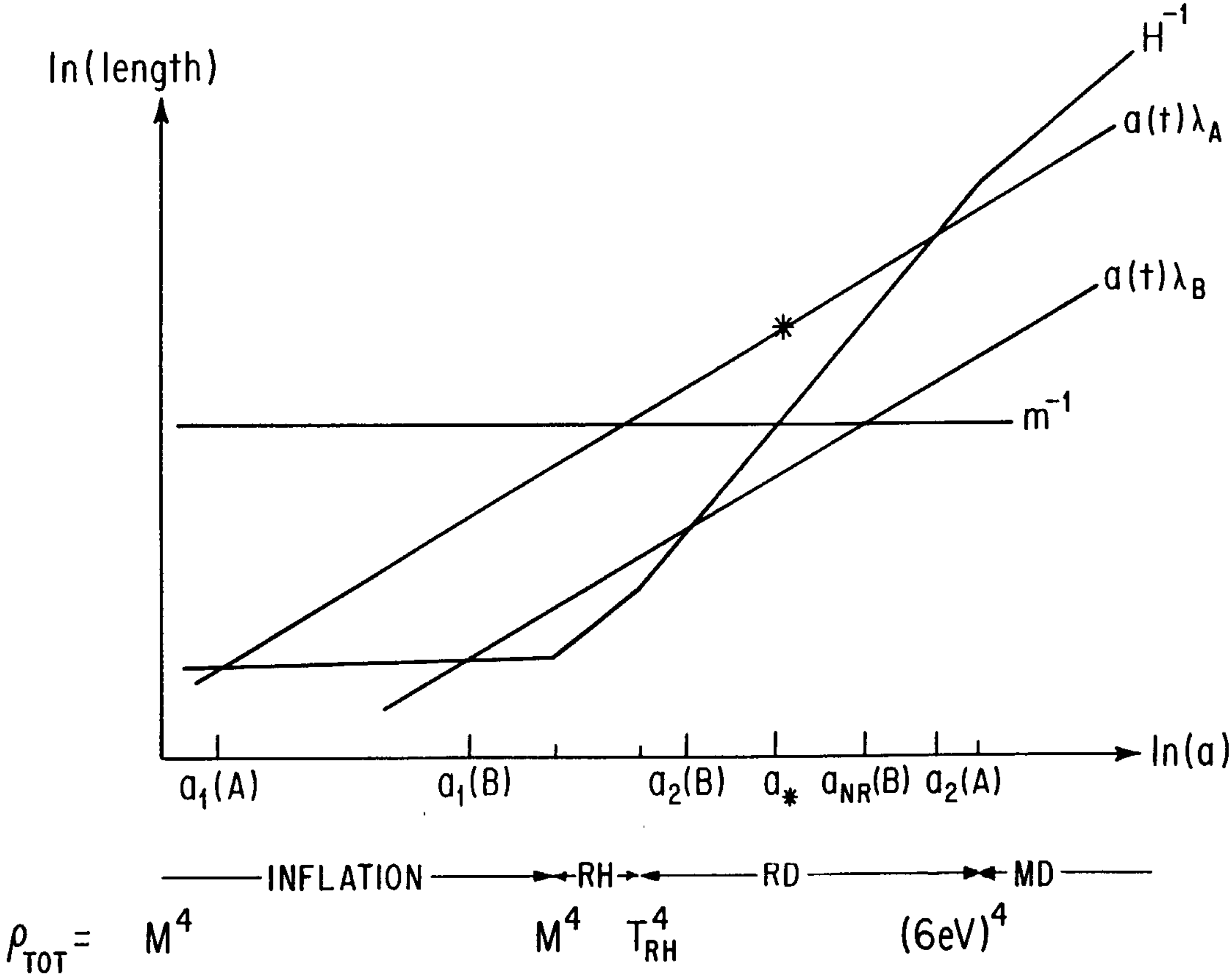
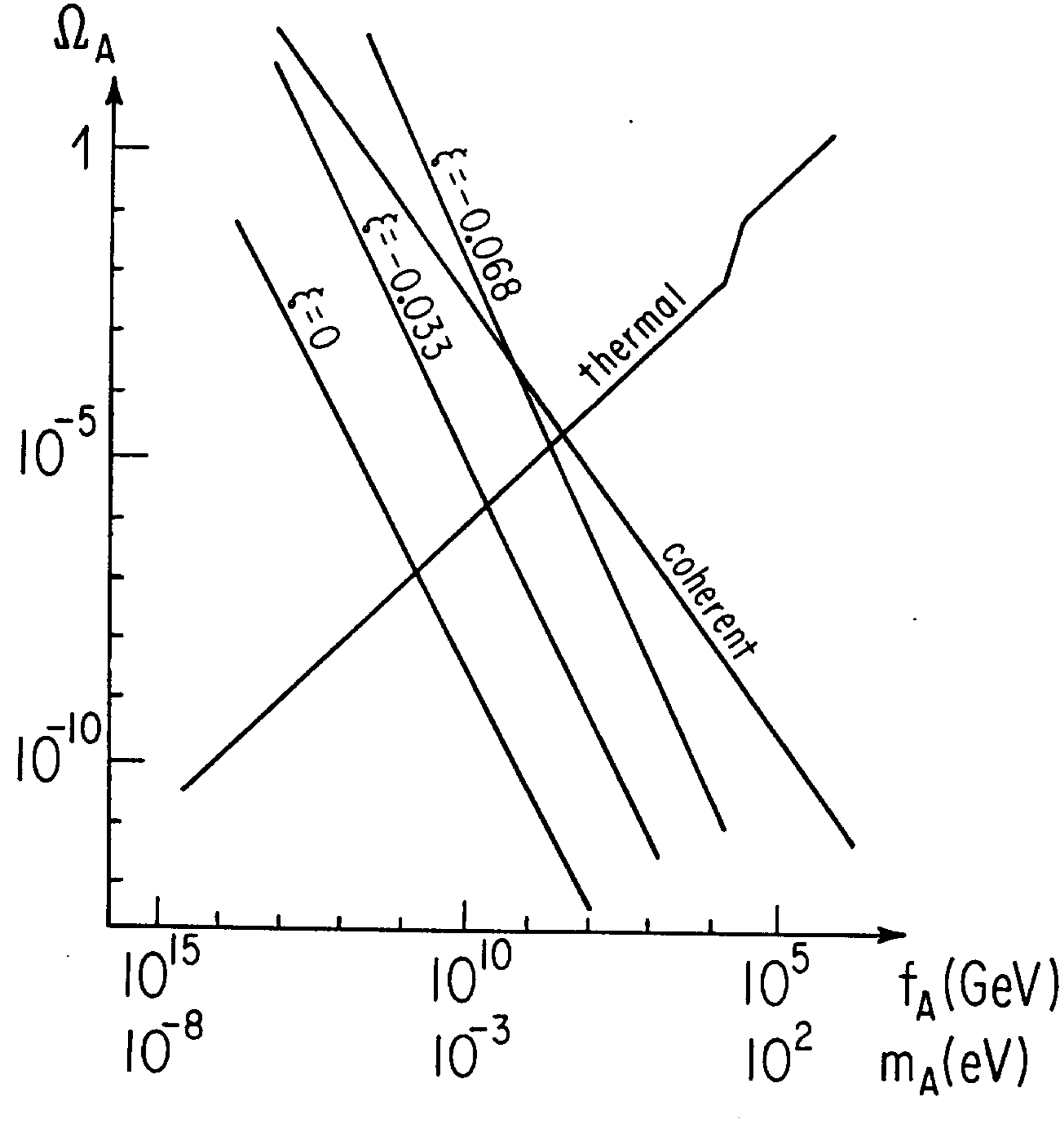


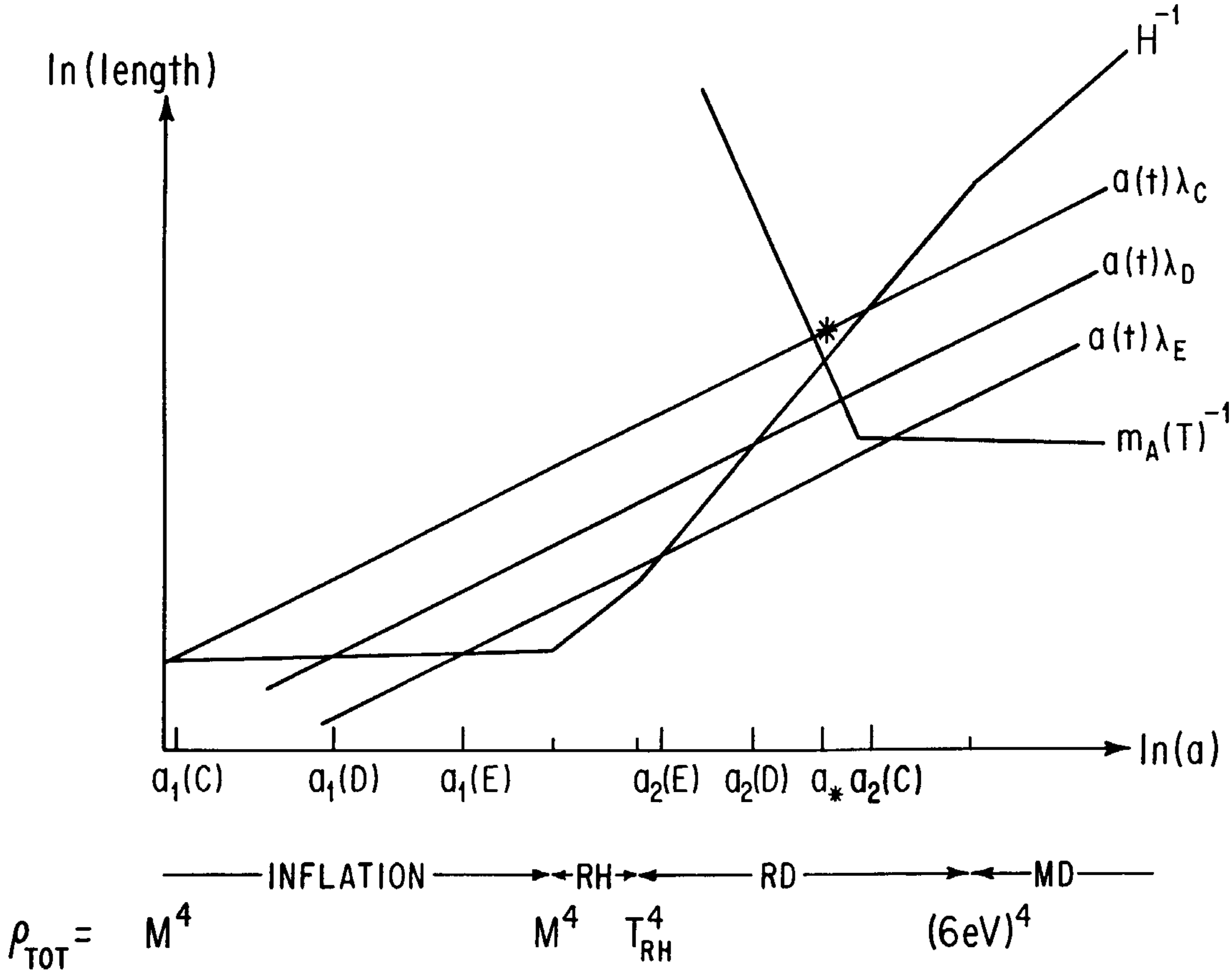
ln(length)
H^{-1}
$a(t)\lambda_A$
$a(t)\lambda_B$
m^{-1}
ln(a)
$a_1(A)$
$a_1(B)$
$a_2(B)$
a_*
$a_{NR}(B)$
$a_2(A)$
INFLATION
RH
RD
MD
$\rho_{TOT} =$
M^4
M^4
T_{RH}^4
$(6eV)^4$

– FIG. 1 –



Ω_A
1
10^{-5}
10^{-10}
$\xi=0$
$\xi=-0.033$
$\xi=-0.068$
thermal
coherent
10^{15}
10^{10}
10^{5}
f_A(GeV)
10^{-8}
10^{-3}
10^{2}
m_A(eV)

- FIG. 2 -


ln(length)
H⁻¹
a(t)λC
a(t)λD
a(t)λE
mA(T)⁻¹
ln(a)
a1(C)
a1(D)
a1(E)
a2(E)
a2(D)
a*
a2(C)
INFLATION
RH
RD
MD
ρTOT =
M4
M4
T4RH
(6eV)4

- FIG. 3 -