Response Maps of the CDF Central Electromagnetic Calorimeter with Electrons

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ABSTRACT

We have measured response maps of the CDF central electromagnetic calorimeter with a 50 GeV electron beam. We present the results of these measurements in terms of the similarity and uniformity module-to-module and tower-to-tower. We derive the uniformity correction functions applicable to all 48 calorimeter modules, insuring uniformity at the 1% level.
1. INTRODUCTION

The calibration of the CDF central calorimeter has been performed in the NW beam at Fermilab. Studies of the linearity and resolution in energy, electron/pion separation, response maps and so on were involved in the calorimeter calibration.[1] For the absolute energy calibration, all 48 wedge modules (plus 2 spares) of central electromagnetic (EM) calorimeter were exposed to 50 GeV electrons at the center of each tower, coordinated with Cs-137 radioactive source runs.[2] 5 wedge modules were used to investigate the precise relative response maps over the entire region of 10 towers. Each tower was actually scanned by 50 GeV electrons over the tower face with about 4 cm spanned in both of the horizontal and vertical directions, while the beam size was about 2.5 cm in diameter. The response at each point was measured with a statistical error of less than 1.0 %. By obtaining the response maps from the 5 sampling wedge modules we have attempted to deduce response and correction functions applicable to all 48 modules, insuring uniformity at the 1 % level, as required by the CDF physics goals. The present report is concerned only with the study of the response maps. It is to be noted that an independent mapping study was also made with cosmic ray muons for almost all wedge modules[3], which provides a useful complementary correspondence.

2. RELATIVE RESPONSE MAPS

We study the relative response maps with respect to the response at the tower center of each tower for which the absolute energy calibration was established.
2.1 Coordinate System for Response Maps

The device which determines the position of electron hit in the central EM calorimeter is a strip-wire proportional chamber[4] embedded between the 8th and 9th lead-scintillator layers. The strip-wire coordinate system defines the electromagnetic shower center in the lateral shower profile. This chamber system determines the particle hit position on the chamber plane to an accuracy of 2 mm. However, the chamber system does not cover the whole area of wedge module. The effective region which the strip chamber covers in a module is $|x| \leq 22.5$ cm (in $\phi$) and $6.0 \text{cm} \leq z \leq 239.4\text{cm}$ (in $\theta$), whereas the whole range of the active area extends to $|x| \leq 23.1$ cm and $4.2$ cm $\leq z \leq 246.0$ cm on the strip chamber plane. Thus there remains the region outside the strip chamber coverage, i.e. the regions near $\phi$-cracks in $x$ and $\theta$-boundaries in $z$ at 90° and 45°. (The definition of this coordinate system will be given later.)

In the beam test, the position of beam particles was also measured by the beam chamber system. The experimental configuration is shown in Fig.1. The beam chamber coordinate is defined with the beam axis and beam hit position via encoder readout of the rotation angle of the turn table on which the wedge module was mounted. The coordinate thus covers the regions around $\phi$-cracks and $\theta$-boundaries. We therefore use both coordinates as a hybrid system.

a) Hybrid Coordinate System

The hybrid coordinate system is a combined system of the strip-wire coordinates for $|x| \leq 20$ cm and $10 \text{cm} \leq z \leq 235$ cm and the beam chamber coordinates for the outer area. The beam chamber coordinates for the outer area were calibrated with the strip-wire coordinates for the central area.
The axes x, y, and z are defined as shown in Fig. 2(a). The precision in determining the coordinates was about 4 mm.

b) Reference Points

The reference point is a normalization point to be used for making response maps. The reference point is defined to be the physical tower center at the depth of the strip chamber for each tower, except for tower 9. The shower response in tower 9 is different from that in other towers due to a non-negligible longitudinal shower leakage. We define the reference point for tower 9 to be the point of minimum longitudinal shower leakage, z=222 cm. Fig.2(b) shows the reference points on the strip chamber plane.

c) Tower Coordinate

The hybrid coordinate (x,z) is convenient in describing the response map in a global area of the wedge module. Since we observed that the response map has specific characteristics tower-to-tower, we also define the local tower coordinate (x,z') whose origin is the reference point for each tower. The tower coordinate system is shown in Fig.2(c).

2.2 Response

The light response from an electron hit in a tower was measured from the outputs of two phototubes. The actual response at a tower was obtained by adding the outputs from both sides of neighboring towers, taking into account the lateral shower leakage. A typical raw response map for a tower is shown in Fig.3, which appears as a saddle with a saddle point at the tower center. That is, significant variations of the light response exist as a function of (x,z). The response increases approaching
the point near the tower edge in \(x\) by 5-10 \% and decreases approaching the tower boundaries in \(z\) by 2-8 \%. The characteristics are similar module-to-module and tower-to-tower except for towers 0 and 9. The sources of non-uniformity can be classified into two kinds. One is the shower leakage in the process of energy deposition and the other is the variation of light collection efficiency in the process of light transmission through the scintillators and wave length shifters.

The non-uniformity around \(\phi\)-cracks and \(\theta\)-boundaries is mainly due to lateral shower leakage; while that at the 45° \(\theta\)-boundary is due to additional longitudinal shower leakage.

Several factors causing the variations of the light collection can be categorized: (1) the light attenuation in the scintillator, (2) the light reflection at the edge of scintillators, and/or backing aluminum plates used for the compensation of non-uniform light transformation in the wave length shifters, and (3) the light leakage at tower and module boundaries.

2.3 Parametrization of Response Maps

2.3.1 Single Tube Response

For convenience, we call a phototube mounted on a light guide viewing the positive side in \(x\) as the left tube, and the right tube viewing the negative side, as shown in Fig.2(a).

Typical light responses in \(x\) of the left and right tubes at \(z'=0\), normalized by the response at the reference point, are shown in Fig.4(a). Dashed curves in the figure are best fits with a single component attenuation with a parameter \(\rho_1\) or \(\rho_2\). Those response curves apparently
do not fit well to straight lines, indicating that a single linear parameter is not sufficient for parametrizing the response map precisely. Two major reasons for the deviation of the response from the straight line can be considered: (1) wave length dependence of the light attenuation in the scintillator as a function of the distance from a hit position, (2) light reflection at both edges of scintillators contacting with the wave length shifters.

2.3.2 Ratio of Left to Right Tube Responses

The ratios of the left to right tube responses as a function of x are plotted in Fig.4(b). In this case, the data fit well to a straight line, which can be expressed as,

$$\ln(L(x,z')/R(x,z')) = \ln(A) + 2x/\lambda(z'),$$  \hspace{1cm} (1)

where L and R refer to the light responses of left and right tubes. A is a left-right equalization factor at x=0, which is within 1 % of unity. $\lambda(z')$ corresponds to the attenuation length at z'. Thus the above linear relation can be used to determine the single particle hit position in x, for example. The relation is also useful for monitoring the gain variations of phototubes in a tower with on-line monitoring during the physics run.

2.3.3 Sum of Left and Right Responses

The sum of left and right tube responses normalized at the reference point, which is used for the actual energy determination, is shown in Fig.4(c). The response curve is well fit to an expression,

$$S(x,z') = S(0,z') \cosh(x/\omega(z')),$$

where $\omega(z')$ is a z' dependent parameter which characterizes the x dependence of the response. The function $S(0,z')$ represents a z'
dependence of the response at $x=0$ (tower center in $\phi$). The dashed curve in Fig. 4(c) is given by Eq. (2) with $\omega(0) = 44.2$ cm at $z'=0$.

### 2.3.4 Parameters $\lambda$ and $\omega$ at $z'=0$

Parameters $\lambda(0)$ and $\omega(0)$ introduced in Eqs. (1) and (2) respectively were obtained by fitting only to the data within $|x| \leq 19$ cm and $|z'| \leq 2$ cm to avoid an effect of lateral shower leakages. Figures 5(a) and (b) show the distributions of $\lambda(0)$'s and $\omega(0)$'s for all towers of 5 modules. The values of $\lambda_\varepsilon(0)$ and $\omega_\varepsilon(0)$ for electrons are summarized in Table 1; the values for the same parameters from the cosmic ray muon test [3], $\lambda_\mu(0)$ and $\omega_\mu(0)$ are also presented in this table for comparison.

The average of $\lambda_\varepsilon(0)$ and $\omega_\varepsilon(0)$ for all towers over 5 modules is $88.5 \pm 3.7$ cm and $44.3 \pm 3.7$ cm respectively. The tower-to-tower deviations of $\lambda_\varepsilon(0)$ and $\omega_\varepsilon(0)$ for 5 modules are 2.3 % and 4.2 %, while the module-to-module deviations are 3.8 % and 10.4 % respectively. This indicates that the deviations module-to-module for both $\lambda_\varepsilon(0)$ and $\omega_\varepsilon(0)$ are about twice as large as those tower-to-tower.

The correlation between $\lambda_\varepsilon(0)$ and $\omega_\varepsilon(0)$ is plotted in Fig. 5(c); no significant correlation is seen. While, the data from the cosmic ray test [3] with 46 modules indicate that there is a fairly significant correlation.

### 2.3.5 Lateral Shower Leakages

It is known that the lateral distribution of electromagnetic showers can be expressed approximately as an exponential function with a lateral attenuation length of the order of a few times the Moliere unit (1.63 cm
for lead). In other words, the lateral extension of electromagnetic showers is so small that the measurement of energy deposition with full confinement can be made by detecting the shower at only one tower. It is obviously not true when an electron hit is close to the tower boundary and/or module edge where lateral shower leakage takes place.

There are two different types of lateral shower leakage in the present calorimeter configuration. One is shower leakage between neighboring towers in a module in the z-direction, i.e., at tower ϑ-boundaries. The other is shower leakage with adjacent modules sharing response in the x-direction, i.e., at φ cracks.

a) Shower leakage at ϑ-boundaries

There is no neighboring tower on one side of tower 0 (90° side) and tower 9 (45° side). The other towers neighbor each other with a gap of 0.6 cm between each wave shifter.

The response due to the lateral shower leakage into a non-active area, such as the gap between tower boundaries, can be expressed in the form,

\[ F(z') = \alpha (1 + \beta e^{-\gamma z'})^{-1}, \]  

if the lateral shower distribution is approximately represented by a single exponential function.

In the present study, as stated previously, the light response at a tower is measured by summing the outputs of neighboring towers on both sides. The parametrization of the response at boundaries is thus made for the output already summed, except for towers 0 and 9 where adjacent tower did not exist.
The above treatment is practical for the case of a single particle hit near the $\theta$-boundaries. However, when electron and photon clusters occur near $\theta$-boundaries it is necessary and possible to decompose the overlapping response by solving linear equations with the response function and the leakage function in the form of Eq.(3) with different parameters, knowing each shower's hit position by means of the strip chamber, for example. This task is to be done in the program of the electron clustering algorithm.

b) Shower leakage at $\phi$-cracks

The geometrical configuration of the gap between adjacent modules is shown in Fig. 6. The parametrization of the response at $\phi$-cracks can, in principle, be made in a similar way as with the $\theta$-boundaries. In this case, the gap between adjacent modules consists of steel skins, wave shifters and so on, and the size is about 2.24 cm. Since the size is larger, more drastic change in the response map is observed at $\phi$-cracks. The present mapping study is made with every single wedge module. Thus the mapping and its parametrization are made without measuring the output from the adjacent module, contrary to the case of $\theta$-boundaries. We have, however, studied the response at $\phi$-cracks using a particular set of two wedge modules.[5]
be compatible with the data of this two-wedge study.

Figure 7 shows the response of one module as a function of hit position, viewed by the two phototubes in that module’s tower, as the beam is moved out of this wedge and into an adjacent wedge. As is seen in Fig. 7, the response decreases rapidly at $|x| > 22$ cm and reaches a level less than 10%. Then there is a slight enhancement at the position of the steel skin ($|x| = 24.2$ cm) with depletions at the positions of the wavelength shifters ($|x| = 23.5$ cm and 24.9 cm). In the region $|x| > 25.3$ cm (i.e., active region in the next module), the response remains as a tail. The tail is the fractional shower leakage observed from the next module, which must correspond to the mirror image of the shower leakage in the module under observation.

The response curve in the module under observation can be fitted to the form in Eq. (3), replacing $z'$ and $x$. The response function near $\phi$-cracks can be obtained from the response in a single module, with a boundary condition given from the two-wedge study. The present parametrization of the response at $\phi$-cracks is made in this manner.

In order to reproduce the entire curve in Fig. 7, in particular the tail in the next module, an additional function with the form of Eq. (3) with different values for the $\beta$ and $\gamma$ parameters is needed. (See a note in Table 10.)
3.) This complication might be related to a simplified assumption that the lateral shower distribution is expressed as a single component function. The modified two component function is of practical use only when the separation of sharing response at $\phi$-cracks is required from the electron clustering algorithm.

In this study the response in the region of $\phi$-cracks (2.24 cm wide) is not dealt with because a crack chamber (3.5 cm wide) consisting of a uranium preradiator and wire chambers [5] was installed after the beam test used in this study.

2.3.6 Z-Dependence of Response Map at x=0 ($S(0,z')$)

The function $S(0,z')$ in Eq.(2) represents the $z'$ dependence of the response at $x=0$. The response in $z'$ shows a slightly complicated structure as is seen in Fig.8. There is a decrease in response at tower boundaries by 2-8% compared to that at the reference point (Fig.8(a)). Also, as seen in Figs.8(b) and (c), the response on the both 90° and 45° sides rapidly decreases approaching the edges.

Also seen in Fig.8(a) is a non-zero slope in $z'$. The response seems to be decreasing with increasing $z'$. This effect might be due to an asymmetric tower geometry and/or incomplete uniformity correction due to the reflective backing used to compensate the non-uniformity of the
wave length shifter.

The function $S(0,z')$ representing the $z'$ dependence of the response maps can be expressed with two independent terms, one relating to the shower leakage as in Eq.(3), and the other relating to the slope in $z'$,

$$S(0,z') = a(1 + \beta e^{\gamma|z'|})^{-1}(1 + \delta z' + \varepsilon z'^2), \quad (4)$$

where $\alpha, \beta, \gamma, \delta$ and $\varepsilon$ are parameters.

The best fitting curves are shown in Figs. 8 (a), (b) and (c). It is noted here that the form of Eq.(4) is capable of expressing the $z'$ dependence of the responses at $x=0$ for not only towers 1-8 but also the special towers 0 and 9. Also, the above feature of $z$-dependence is present not only at $x=0$, but at any $x$. Thus, the $z$-dependence will be recast in a more general form.

3 AVERAGE MAP, RESPONSE FUNCTION AND UNIFORMITY CORRECTIONS

3.1 Average Map

In the present analysis a wedge module is divided into 50x250 cells of an area of 1cm x 1cm. For each cell the mean response and its standard deviation are calculated for each wedge module. Then the average and its deviation for each cell over 5 modules are obtained by taking the weighted mean as,

$$<S_i> = \sum_m (S_{mi}/\sigma_{mi}^2)/\sum_m (1/\sigma_{mi}^2), \quad (5)$$

where $i$ and $m$ refer to the cell number and module number respectively. The map of $<S_i>$'s is hereafter called the average map. The deviation of $S_{mi}$ from the average response $<S_i>$ for each cell is calculated as
\[ \varepsilon_{mi} = \frac{(S_{mi} - \langle S_i \rangle)}{\langle S_i \rangle}. \quad (6) \]

In order to examine the module-to-module deviation in detail, some different geometrical regions were assigned as categories by grouping cells as shown in Fig. 9.

For each category, we examine the distribution of the deviation \( \varepsilon_{mi} \)'s.

In each case, the distribution was fit to a Gaussian function to derive the standard deviation. The width of the distribution is a convolution of both the module-to-module deviation and the statistical error. In order to obtain the intrinsic module-to-module deviation we subtract the contribution of statistical error as follows. The average variance corresponding to the statistical error is given by

\[ \sigma_{\text{stat}}^2 = \frac{N}{\Sigma(1/\sigma_i^2)}, \quad (7) \]

where \( N \) is the number of cells grouped into a category and \( \sigma_i \) is the statistical error in measuring \( S_i \). The net module-to-module deviation is obtained from the observed deviation as

\[ \sigma_{\text{mmd}}^2 = \sigma_{\text{obs}}^2 - \sigma_{\text{stat}}^2 = \sigma_{\text{obs}}^2 - \frac{N}{\Sigma(1/\sigma_i^2)}. \quad (8) \]

These results are tabulated as "similarity" in Table 2 for each category.

As is shown in the table, the overall deviation for the entire region is
3. 2 Response Function and Uniformity Corrections

Parametrization of the response functions has been previously discussed in part as in Eqs (2), (3) and (4). The overall response function for the average map can be expressed in a more general form as,

\[ S(x,z') = F_1 F_2(z') F_3(z') F_4(x,z') F_5(x,z'), \]  \hspace{1cm} (9)

where

\[ F_1 = p_1 \]
\[ F_2 = (1 + p_2 e p_3 |z'|)^{-1} \]
\[ F_3 = (1 + p_4 z' + p_5 z'^2) \]
\[ F_4 = \cosh(x/\omega(z')) (1 + p_8 z' + p_9 z'^2) \] \hspace{1cm} with $\omega(0) = p_6 \cdot p_7$ for $\omega(z')$
\[ F_5 = (1 + p_{10} e |x| (p_{11} + p_{12} |z'|))^{-1}, \]

with 12 parameters for each tower of towers 0 to 9. (For tower 0, $p_3 |z'|$ should be changed to $p_3 z'$.)

The parameter $p_1$ is an effective normalization factor and the value is close to 1.00. $F_2$ fits the effect of leakage on the response at $\theta$-tower boundaries. $F_3$ represents the non-uniformity in $z'$ due to asymmetric
tower geometry in $z'$ and non-uniform response in the wave length shifter. (In this term, the parameter $p_5$ is used for only tower 0 and tower 9 (i.e., $p_5=0$ for others)). $F_4$ expresses an $x$ dependence of the response and the parameters $p_8$ and $p_9$ are used to fit the data in the region within $|x| \leq 20$cm. $F_5$ is for the lateral shower leakage at $\phi$-cracks.

The parameters $p_6$ and $p_7$ are determined in two different ways:

1) the average of $\omega_\epsilon(0)$ from 5 modules, 

$$p_6 = \langle \omega_\epsilon(0) \rangle_5, \text{ and } p_7 = 1.0,$$

2) $\lambda_\mu$ from cosmic ray muon data,

$$p_6 = \lambda_\mu(0), \text{ and } p_7 = \langle \omega_\epsilon(0)/\lambda_\mu(0) \rangle_5.$$

Details of case 2) will be discussed in Section 4.6. (The parameters $p_6$ and $p_7$ are fit to the data within $|x| \leq 20$cm to avoid shower leakage into the $\phi$-cracks.)

The parameters in Eq.(9) are tabulated in Table 3. Typical $x$ dependence of the response function is shown in Figs.10 (a), (b) and (c) together with data. Also shown in Figs. 11 (a), (b) and (c) are typical $z'$ dependence of the resultant response function and maps at several different $x$. Figure 12 shows a typical three dimensional presentation of the response function $S(x,z')$. Also shown in Fig. 13 is the corrected $z$-response from the raw data at $x=0$ cm, for example, in which the average rms deviation in reproducibility is 0.77%.

The uniformity correction function is an inverse of the response
function. In practice, the response function $S(x,z')$ and related parameters are implemented using a database, and a correction factor relative to the absolute energy calibration constant obtained at tower center is applied in the data reduction procedure.

4. RESULTS AND DISCUSSION

In this section, we summarize the results of mapping and evaluate the uniformity corrections.

4.1 Non-Uniformity

We have observed non-uniformity in the raw data of response maps.

The non-uniformity of the response is defined as the fractional difference of the response in each cell from the overall average. Figure 14 shows the distribution of deviations in non-uniformity for all cells. The main peak is mainly consisting of the contributions from the central area. On the other hand, the higher side of the distribution is from the outer area and the lower side is from the region around $\theta$-tower boundaries and $\phi$-cracks. The non-uniformity in the overall area is 3.9%. The non-uniformity at $\theta$-boundaries and $\phi$-cracks increases typically to 5% and 7% respectively. The non-uniformity of tower 9 is about twice as large as that of other towers. Details of non-uniformity for each tower and category are shown in Table 2.

4.2 Similarity

We have observed similar characteristics in the raw response maps module-to-module and tower-to-tower.

The similarity for each cell is defined as the percentage deviation of
the response in each cell with respect to the average obtained from the 5 modules.

The distribution of overall deviations in similarity (for category I as defined in Fig. 9) is shown in Fig. 15. The r.m.s. deviation of the distribution is 0.8%. The similarity varies from 0.7 to 0.9% depending on the tower number. (See Table 2.) At the tower boundaries the similarity increases to 1.0%. The similarity at $\phi$ cracks ($|x|\geq20$cm) is larger, being 1.8%.

Study of the similarity with a large number of sample modules was made with cosmic ray muons.[3] 41 modules were tested with a cell size of 4 cm x 4 cm. The similarity over the entire module is 0.95%±0.47%, which is consistent with electron data.

The small rms deviation in the similarity for all modules insures that a single response correction function can be used for all the 48 modules.

4.3 Local Reproducibility

The deviation of responses from the local response function $S(x,z')$ for each module is denoted the local reproducibility. Table 2 represents the local reproducibility for 5 modules for each tower and various categories described in Section 3.1.

The local reproducibility is 0.8-1.0% for towers 0 to 5, and 1.1-1.4% for towers 6 to 9. The reproducibility in the central region (i.e., categories G and H in Fig. 9) is less than 0.7%, and that at tower $\Theta$-boundaries (category C) and $\phi$ cracks (category D) is 1.4% and 2.2%, respectively. The trend relates to the fact that the similarity at $\Theta$-boundaries and $\phi$-cracks is worse than that in the central region, and in the fitting the procedure starts from the central region (categories G and
H) and propagates toward the tower edges.

The local response function $S(x,z')$ is able to fit to the data, on the average, within 1.0% over the entire region. A histogram with solid line in Fig.16 is the distribution of deviations in local reproducibility over the entire region.

### 4.4 Global Reproducibility

The percentage deviation of responses from the average global response function over 5 modules $\langle S(x,z') \rangle_5$, is denoted the global reproducibility. The global reproducibility for the average response maps is 1.1% over the central region of $|x| \leq 20$ cm and $|z| \leq 234$ cm (84% of the entire region), and 1.5% over the entire region. The reproducibility for each tower and various categories is summarized in Table 2. The overall reproducibility ranges from 1.2% to 1.4% for towers 1-5 and 7, and 1.5-2.0% for other towers. A histogram with dashed line in Fig.16 is the distribution of deviations in global reproducibility over the entire region.

It should be noted that the global reproducibility with respect to the simple response function of the form $p_1 \cosh(x/p_6)$ over the entire region is 2.4%. The distribution of the deviations of reproducibility in this case is shown in Fig. 17.

### 4.5 Errors Associated with Data Reduction

The number of events accumulated in each cell varied from several to 100. The precision of the magnitude of light response at each cell is estimated to be 0.2 to 2.0%, taking into account the energy resolution of 2.0% at 50 GeV. For mapping studies, a cut was applied to the cells with
statistical errors larger than 1 %.

The systematic error in making response maps was estimated from the comparison of the results of two different electron calibration runs made for 21 modules. The comparison indicates that the systematic error in making response maps is 0.36±0.13%.

We estimate the causes of systematic errors. The time variation of temperature during the measurement would affect to the gain variation of phototubes by 0.14%. The instability of high voltages might result in 0.12% error for the phototube gains. As for the momentum tagging system, the uncertainty of magnetic fields was 0.2% and the uncertainty in determining the particle hit position was 0.1%. We may neglect the errors associated with electronics system (PM AVC and ADC etc.) The overall systematic error is estimated to be about 0.3%, which is consistent with that obtained from the comparison of two different electron calibration runs above mentioned.

Thus we added an empirical systematic error of 0.36% to the statistical error in the response for each cell before reproducing the response maps.

4.6 Applicability of Correction Function

The remaining question is the applicability of the correction function with parameters obtained from 5 sampling modules to the other 45 modules.

The most sensitive parameter in the response function is $\omega$. Figures 18 (a), (b), (c) and (d) show the sensitivity of the reproducibility against the correction factor which is defined as a factor relative to the representative value of $\omega$. It is seen in the figure that the range of change in reproducibility by 0.5 % corresponds to the shift of 10 % of the correction factor. That is, in order to guarantee the reproducibility to be
within 0.5 % from the nominal value, for example, the value of \( \omega \) must lie within 10 % of the representative value obtained from the 5 sampling modules.

Fortunately we have measurements of \( \lambda \) and \( \omega \) for 46 modules with cosmic ray muons.[3] The cosmic ray data indicate that there is a significant correlation between \( \lambda \) and \( \omega \), and the measurement of \( \lambda \) is more reliable. Actually, there exist two groups in the correlation plot, which might be related to different batch numbers during production of the scintillator boards. In the distribution of \( \lambda \)'s, which is not a normal distribution, 80 towers out of 460 towers have greater than 10 % deviation. When the 1 % level of uniformity is required, the values of \( \omega \) for those 80 towers must be modified individually from the representative value which nominally gives 1.5 % global reproducibility. This modification can be made by multiplying a factor \( \lambda_\mu / <\lambda_\mu>_3 \) to the representative value for each individual tower.

Nevertheless, it is estimated that even the use of the representative value of \( \omega \), together with other parameters, is capable of reproducing and correcting the response, on the average, to 1.3 % for the main central region ( \( |x| \leq 20 \text{ cm}, \ z \leq 234 \text{ cm} \) ) and 1.7 % for the entire region.

5. SUMMARY

We summarize the results of the present study of response maps as follows.

1) With the use of 5 sampling wedge modules, we have measured the precise response maps with electrons and described the
results in terms of the non-uniformity and similarity module-to-module and tower-to-tower. We observe in the raw data that the overall non-uniformity is 3.9% and the similarity is 0.8%.

2) The response function is derived using 12 parameters for each tower, i.e., 120 parameters for all towers of all the 48 wedge modules. The overall reproducibility of the response function is estimated to be 1.7% for the entire region of all towers and 1.3% for the main central region ($|x| \leq 20$ cm and $z \leq 234$ cm; 84% of the entire region) for all 48 modules (and 2 spares). Better reproducibility can be obtained when the individual parameters for a particular set of towers are applied.

3) The response correction function, which is an inverse of the response function, is implemented from a data base and is used to scale the energy at any point with respect to the absolute calibration constant at the reference points.

4) The procedure for unscrambling electron and photon clusters for a particular case, i.e. for overlapping showers at the boundaries between modules and between towers was briefly discussed.

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1. Parameters $\lambda(0)$ and $\omega(0)$ obtained from 50 GeV electrons and cosmic ray muons. The average values of $\lambda_e$, $\omega_e$, $\lambda_\mu$ and $\omega_\mu$ are 88.5±3.7 cm, 44.3±3.7 cm, 98.1±8.0 cm and 54.0±2.7 cm, respectively.

2. Non-uniformity, similarity, local reproducibility and global reproducibility for each tower and category described in the text.

3. Parameters in the response function for towers 0 to 9.

FIGURE CAPTIONS

1. Schematics of the NW beam line and experimental configuration.

2. (a) Schematics of the wedge module of the CDF central calorimeter and the coordinate system used for response mapping.  
(b) Reference points used for response mapping.  
(c) Tower coordinate system used for response mapping.


4. (a) Single tube response from the left and right tubes at $z' = 0$ (in logarithmic scale). The dashed curves are $\exp(x/\rho_1)$ and $\exp(x/\rho_2)$.
(b) Plot of ratios of the left and right tube outputs as a function of $x$.
(c) Plot of sums of the left and right tube outputs as a function of $x$.

Dashed, dot-dashed and dotted curves represent $\cosh(x/\omega)$ with $\omega = 44.2$ cm (best for $\omega$), 82.6 cm (best for $\rho$) and 86.6 cm (best for $\lambda$) respectively.

5. (a) Distribution of parameter $\omega_e(0)$'s from 50 towers examined.
(b) Distribution of parameter $\lambda_e(0)$'s from 50 towers examined.
(c) Correlation plot of $\omega_e(0)$ and $\lambda_e(0)$. 

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6. Schematics of $\phi$-cracks region.

7. Typical response map at $\phi$-cracks.

8. Typical $z$-dependence of the response at $x=0$; (a) tower 2, (b) tower 0 and (c) tower 9. Data from the cosmic ray test are also plotted for comparison.

9. Classification of geometrical category for testing similarity, uniformity and reproducibility.

10. Typical $x$-dependence of the response at $z'=0$; (a) tower 2, (b) tower 0 and (c) tower 9. Data from the cosmic ray test are also plotted for comparison.

11. Response function and maps as a function of $Z'$ at (a) $x=0$ cm, (b) $x=10$ cm and (c) $x=15$ cm.

12. Typical three dimensional presentation of the response function.

13. Corrected $z$-response from the raw data at $x=0$ cm. The average r.m.s. deviation in this case is 0.77%.

14. Distribution of the deviations in non-uniformity.

15. Distribution of the deviations in similarity.

16. Distributions of the deviations in local reproducibility (solid line) and global reproducibility (dashed line).

17. Distribution of the deviations in reproducibility with respect to a simple response function $p_1 \cosh(x/p_6)$.

18. Sensitivity of the reproducibility as a function of correction factor for the parameter $\omega$. 