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Non–Perturbative Unification à la Maiani - Parisi - Petronzio at Intermediate Energies

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Abstract

A non–asymptotically free gauge theory with many couplings is shown to exhibit a "trapping" mechanism, in the sense that, as soon as one (or more) of the couplings grows large, the rest of the couplings will follow suit. This mechanism can help achieve a non–perturbative "unification" of the standard model à la Maiani - Parisi - Petronzio at relatively "low" energies (TeV scales). Two scenarios are given, one of which is just the standard $SU(3)_c \times SU(2)_L \times U(1)_y$ and the other one involves, in addition, Technicolor interactions. Predictions for $\sin^2\theta_w(\Lambda_F \simeq 250\text{ GeV})$ and $\alpha_3(\Lambda_F)$ are presented for both scenarios. Both scenarios make use of $SU(2)_L$–singlet heavy fermions which carry charge, color (Model I) and technicolor (Model II). Some experimental implications are also discussed.

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Up to what scale(s) can one trust perturbative calculations in the standard model? Is there a cut–off scale, and if so, can it be much smaller than the Planck mass? What would be its implications? These are the questions we would like to address in this note.

Let us briefly recall the conditions under which a perturbation expansion ceases to be valid. From the renormalization group equation $\frac{d\alpha}{dt} = \beta(\alpha)$, where $t = \ell n \frac{Q^2}{\mu^2}$ and μ is some subtraction point, one obtains $t = \int_{\alpha(\mu)}^{\alpha(Q^2)} \frac{d\alpha}{\beta(\alpha)}$. If the theory is non–asymptotically free, $\beta(\alpha) > 0$ (no fixed points), there is a value of t for which $\alpha \to \infty$, namely $t_c = \int_{\alpha(\mu)}^{\infty} \frac{d\alpha}{\beta(\alpha)} < \infty$. The perturbation expansion breaks down for $t > t_c$. If perturbation theory is to be valid for all energy scales, it follows that $\alpha(\mu) = 0$, in which case $t_c = \infty$. The value $\alpha(\mu) = 0$ is the infrared (IR) stable fixed point. If $\alpha(\mu)$ is small, but not equal to zero, i.e., it is near the IR fixed point, there is a finite physical cutoff $\Lambda(t_c = \ell n \frac{\Lambda^2}{\mu^2})$ beyond which non–perturbative methods will have to be employed. The situation is different for asymptotically free theories where no such cutoff is present because $\alpha(\Lambda) \to 0$ as $\Lambda \to \infty$.

Exploiting the above facts, Maiani, Parisi and Petronzio[1] (MPP) have proposed an interesting scheme in which the standard $SU(3)_c \times SU(2)_L \times U(1)_y$ model becomes non–asymptotically free above $\Lambda_F \simeq 250$ GeV. Since $\alpha_i(\Lambda_F) \neq 0, i = 1,2,3$, the cutoff scale mentioned earlier is finite. The original proposal is to identify that scale with $M_{\mathbf{P}} \simeq 10^{19}$ GeV. In subsequent works by Cabibbo and Farrar[2], and by Grunberg[3], the cutoff is found to be approximately $10^{15} - 10^{17}$ GeV. The couplings at that cutoff are large and of the same order, though not necessarily equal. It was found that the low–energy couplings $\alpha_i(\Lambda_F)$ are insensitive to how large the high–energy couplings are, and depend primarily on the particle content of the theory. Predictions for $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$ were given with the constraint that the number of families (some of them have masses of $0(\Lambda_F)$) is between 8 and 10. No unifying gauge group was necessary in this scheme.

In the MPP scenario[1], a desert is also assumed to exist between 250 GeV and $\sim 10^{15}$ GeV. Although, it is still a matter of speculation, there is a distinct possibility that structures can exist at intermediate energies. For instance, Technicolor interactions[4] may be responsible for a dynamical breakdown of $SU(2)_L \times U(1)_y$ and they would certainly yield new physics in the TeV region. In fact, various works have pointed out the possibility that the standard model with Higgs scalars can only be an effective theory up to $\sim$ 1 TeV.

Since there is a possibility that there is no desert after all, one may ask whether or not the cutoff scale in the gauge sector of the standard model could, in fact, be much lower than the value obtained by Refs. (1,2,3). In fact, could it be in the TeV region? Can one still predict $\sin^2\theta_w(\Lambda_F)$ then? It is claimed in this note that this possibility exists with the help of a phenomenon called the trapping mechanism.

It is shown below that, for a non-asymptotically free gauge theory with many couplings, as soon as one (or more) of the couplings grows large the rest of the couplings will follow suit because of the influence of the first type(s) on the others through the effects of fermion loops. Let us briefly recall what the situation was with just $SU(3)_c \times SU(2)_L \times U(1)_y$. There, it was found [1,2,3] that, as long as one is dealing with standard quark and lepton families, in the process of lowering the non-perturbative unification scale (by increasing the number of families), $\sin^2\theta_w(\Lambda_F)$ increases and $\alpha_3(\Lambda_F)$ decreases. Consistency with experiment requires that $\mathcal{M} \sim 10^{15} - 10^{19}$ GeV with the number of families situated between 8 and 10. It is impossible to make $\mathcal{M} \lesssim 0(1000$ TeV) in the usual MPP scenario.

In the first version of the two scenarios presented here, the MPP picture is modified by adding a large number of colored, $SU(2)_L$-singlet charged fermions of mass of $0(\Lambda_F)$ so that the $SU(3)_c$ coupling constant becomes large at an intermediate scale (is the TeV region). The trapping mechanism will force the $SU(2)_L$ and $U(1)_y$ couplings to grow strong at approximately the same scale. It is then seen that $\sin^2\theta_w(\Lambda_F)$ is rather insensitive to the value of that scale and depends only on the number of these new fermions. $\alpha_3(\Lambda_F)$ is also computed.

The second version deals with a gauge group $G_{TC} \times SU(3)_c \times SU(2)_L \times U(1)_y$ where G_{TC} is the Technicolor group. Here the extra heavy families encountered in the MPP scenario are interpreted as Technifermions. One advantage of this picture is the fact that one can do away with fundamental scalars and avoid the naturalness and triviality problems. At the Technicolor scale $\Lambda_T(\simeq 1$ TeV), a condensate is formed when $\alpha_{TC} = g_{TC}^2/4\pi = \pi/3C_2(R)$, where R is the representation of Technifermions. This is the input value which can be computed. Above Λ_T, G_{TC} is asymptotically free and α_{TC} decreases. We now introduce a set of $SU(2)_L$-singlet, colored, charged Technifermions of mass $M_I > \Lambda_T$. It will be seen that this set is not too large ($\sim$ 10) and also one can easily form a gauge-invariant mass term for these fermions (we will speculate on the possible origin of these masses at the end of this note). For $E > M_I$, both G_{TC} and $SU(3)_c$ couplings begin to grow and become large at a scale $\mathcal{M}$ of 0(200–300 TeVs). They, in turn, trap the $SU(2)_L$ and $U(1)_y$ couplings and the same scenario as the one described above is repeated. Notice that for $E > \Lambda_T, SU(2)_L$ (and, of course, $U(1)_y$) is non-asymptotically free. The value of $\mathcal{M}$ of 0(200–300 TeVs) hints at an extended Technicolor picture[5]. Again, $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$ can be determined. To show the trapping behavior, we shall first discuss an example of two arbitrary gauge groups G_1 and G_2.

Let two gauge groups G_1 and G_2 be vector-like (a prototype of color and technicolor interactions) and chiral (e.g., $SU(2)_L$) respectively. In a more realistic theory, proper care is required so that G_2 is anomaly free. In this note, fermions are chosen to transform as fundamental representations of $G_1 \times G_2$ (a generalization to higher dimensional representations is straightforward). Let us denote by $f_i, i = 1,2$, the fundamental representations and by $d(f_i)$ their respective dimensions. For the purpose of illustration, we take the following set of left and right-handed fermion fields: $\{n(f_1,f_2)_L,\ nd(f_2)(f_1,1)_R, \bar{n}(1,f_2)_L\}$. In addition, we can add another set which can be either $\{n_s(f_1,f_2)_L,\ n_s(f_1,f_2)_R\}$ or $\{\bar{n}_s(f_1,1)_L,\ \bar{n}_s(f_1,1)_R\}$ or both. The reason we want this second set of fermions is the fact that they can have a large explicit mass term of the order of $M_I > \Lambda_T$ without breaking $G_1 \times G_2$ gauge invariance. The need for those fermions will be made clear below.

With $\alpha_i = g_i^2/4\pi, i = 1,2$, the renormalization group equations are now written as

$$\frac{d\alpha_i}{dt} = b_i\alpha_i^2 + \sum_{j=1}^{2} c_{ij}\alpha_i^2\alpha_j + 0(\alpha_i^4), \tag{1}$$

where b_i and c_{ij} are the one– and two–loop coefficients respectively, and $t \equiv \ell n(\mu^2/M^2)$. Explicitly, one has[7]

$$b_1 = \frac{1}{12\pi}\left\{-11C_2(G_1) + 2d(f_2)\,n + \Delta_1\right\}, \tag{2a}$$

$$b_2 = \frac{1}{12\pi}\left\{-11C_2(G_2) + d(f_1)\,n + \bar{n} + \Delta_2\right\}, \tag{2b}$$

$$c_{11} = \frac{1}{48\pi^2}\left\{\left[10C_2(G_1) + 6C_2(f_1)\right]d(f_2)\,n + \Delta_{11} - 34\left[C_2(G_1)\right]^2\right\}, \tag{2c}$$

$$c_{21} = \frac{1}{16\pi^2}\left\{C_2(f_1)\,d(f_1)\,n + \Delta_{21}\right\}, \tag{2d}$$

where

$$\Delta_{1,2} = \begin{cases} 0 \quad \text{for } \Delta_T < E < M_I \quad , \\ \{2d\,(f_2)\,n_s \text{ or } 2\bar{n}_s\}\,, \{d\,(f_1)\,n_s \text{ or zero}\,\} \text{ for } E > M_I, \end{cases} \tag{3a}$$

$$\Delta_{11,21} = \begin{cases} 0 \quad \text{for } \Delta_T < E < M_I \quad , \\ \{[10C_2\,(G_1) + 6C_2\,(f_1)]\,(d\,(f_2)\,n_s \text{ or } \bar{n}_s)\}\,, \\ \{C_2\,(f_1)\,d\,(f_1)\,n_s \text{ or zero}\,\} \text{ for } E > M_I, \end{cases} \tag{3b}$$

and where only coefficients of interest are exhibited. Here $C_2(G_i)$ and $C_2(f_i), i = 1, 2$, are the Casimir invariants of the adjoint and fundamental representations respectively. In what follows, it is assumed that, for the energy range considered, $\alpha_1 \gg \alpha_2$ and, at the two–loop level, one can neglect $c_{12}\alpha_1^2\alpha_2$ and $c_{22}\alpha_2^3$ as compared with $c_{11}\alpha_1^3$ and $c_{21}\alpha_2^2\alpha_1$ respectively. Also, we are neglecting mass threshold effects here.

Depending on the model considered, n and $\bar{n}$ can also be chosen so that $b_1, c_{11} < 0$ and $b_2 > 0$ for $\Delta_T \leq E < M_I$. The assumption $b_2 > 0$ is, in general, not necessary since $c_{21}\alpha_2^2\alpha_1$ dominates over $b_2\alpha_2^2$ for large α_1. For $E > M_I, n_s$ or $\bar{n}_s$ can be chosen so that $b_1, c_{11} > 0$. The coefficient c_{21} is always positive. The picture which emerges is the following one: For $\Delta_T < E < M_I, G_1$ is asymptotically free and α_1 decreases with increasing E, while for $E > M_I, G_1$ becomes non–asymptotically free and α_1 again increases. Also, for $E > \Delta_T, G_2$ is non–asymptotically free.

Let us solve Eq. (1) for the interval M_I to $\mathcal{M}$. For α_1, one has to use the full two–loop equation to obtain

$$\ell n \frac{\mathcal{M}^2}{M_I^2} \simeq \frac{1}{\bar{b}_1\bar{\alpha}_1} + \frac{\bar{c}_{11}}{\bar{b}_1^2}\ell n \left(\frac{\bar{c}_{11}\bar{\alpha}_1}{\bar{b}_1 + \bar{c}_{11}\bar{\alpha}_1}\right), \tag{4}$$

where the limit $\alpha_1^{-1}(\mathcal{M}) \to 0$ has been taken in (4), with $\bar{b}_1$ and $\bar{c}_{11}$ being the one– and two–loop coefficients for $M_I \lesssim E \lesssim \mathcal{M}$. Notice that what we mean by the coupling becoming large at $\mathcal{M}$ is really the assumption that $\alpha_1^{-1}(\mathcal{M}) \ll \alpha_1^{-1}(M_I)$. Here $\bar{\alpha}_1 = \alpha_1(M_I)$. From Eq. (4), one can determine $\mathcal{M}$ for a given M_I and $\bar{\alpha}_1$. Eq. (4) will later be used in the computation of $\alpha_3(\Lambda_F)$. Examples given below show that typically $\mathcal{M}/M_I \overset{<}{\simeq} 0(10)$. As for α_2, the one–loop value is given by $\bar{\alpha}_2(\mathcal{M}) = \alpha_2(M_I)/(1 - \alpha_2(M_I)b_2\ell n\frac{\mathcal{M}^2}{M_I^2})$. It will also be seen below that, because $\mathcal{M}/M_I \overset{<}{\simeq} 0(10)$, the quantity $\alpha_2(M_I)b_2\ell n\frac{\mathcal{M}^2}{M_I^2}$ is much less than one and the one–loop value $\bar{\alpha}_2(\mathcal{M}) \approx \alpha_2(M_I)$ is small because $\alpha_2(M_I)$ is assumed to be small. As it turns out, at the two–loop level, the situation changes drastically because the effects of G_1 on G_2 will start to show up there.

Let us define $x \equiv \ell n \frac{\mathcal{M}^2}{Q^2}$. Up to two loops, the solution to Eq. (1) for α_1 is given by

$$\alpha_1(x) = \frac{1}{b_1 x} - \frac{c_{11}}{b_1^3}\left(\frac{\ell n\ x}{x^2}\right) + 0\left(\frac{\ell n^2 \frac{1}{2} x}{8x^3}\right), \tag{5}$$

valid for $\alpha_1(x) < 1$ and $\alpha_1(0) \gg 1 (Q^2 = \mathcal{M}^2)$. Here $b_1, c_{11} > 0$ for $E > M_I$. Using Eq. (5), the R.G. equation for α_2 now reads

$$\frac{d\alpha_2}{dx} = -\left(b_2 + \frac{c_{21}}{b_1 x} - \frac{c_{21}c_{11}}{b_1^3}\left(\frac{\ell n x}{x^2}\right)\right)\alpha_2^2 + \cdots, \tag{6}$$

Integrating Eq. (6) from M_I to some scale $\mathcal{M}'$, we obtain

$$\alpha_2^{-1}(M_I) = \alpha_2^{-1}(\mathcal{M}') + b_2 \ell n \frac{\mathcal{M}'^2}{M_I^2} + f(\mathcal{M}, \mathcal{M}'), \tag{7}$$

$$f(\mathcal{M}, \mathcal{M}') = \frac{c_{21}}{b_1}\ell n\left[\ell n\left(\frac{\mathcal{M}^2}{M_I^2}\right)/\ell n\left(\frac{\mathcal{M}^2}{\mathcal{M}'^2}\right)\right]$$
$$+\frac{c_{21}c_{11}}{b_1^3}\left[-\left(\ell n \ell n\left(\frac{\mathcal{M}^2}{\mathcal{M}'^2}\right)+1\right)/\ell n\left(\frac{\mathcal{M}^2}{\mathcal{M}'^2}\right) + \left(\ell n \ell n\left(\frac{\mathcal{M}^2}{M_I^2}\right)+1\right)/\ell n\left(\frac{\mathcal{M}^2}{M_I^2}\right)\right]. \tag{8}$$

From Eq. (7), one obtains

$$\alpha_2(\mathcal{M}') = \alpha_2(M_I)/\left[1 - \alpha_2(M_I)\left(b_2 \ell n \frac{\mathcal{M}'^2}{M_I^2} + f(\mathcal{M}, \mathcal{M}')\right)\right]. \tag{9}$$

By examining more closely Eq. (9), one discovers that, as $\mathcal{M}'$ approaches $\mathcal{M}$, the function $f(\mathcal{M}, \mathcal{M}')$ becomes large and $\alpha_2(M_I) f(\mathcal{M}, \mathcal{M}' \to \mathcal{M}) \to 1$ which, in turn, implies that $\alpha_2(\mathcal{M}' \to \mathcal{M}) \simeq 0(1)$ or larger (strong couplings). Notice that $f(\mathcal{M}, \mathcal{M}')$ is only logarithmically dependent on $\mathcal{M}'$ and hence a precise value is not needed as long as it is close to $\mathcal{M}$.

The above discussion illustrates a general phenomenon involving a theory with many couplings, at least one of which grows strong at some scale $\mathcal{M}$. Intuitively speaking, the fact that one of the couplings becomes strong indeed indicates that there is an enhancement of the fermionic contributions to the β–functions of the other couplings. It is similar to (but not the same as) adding more fermions to the one–loop coefficients b_i. This is what we call the "trapping" mechanism.

The above analysis is carried out up to the two–loop level and one may wonder about the importance of three loops and higher ones. First, it is well–known that the first two terms in the β–function are <u>renormalization–scheme independent</u> and, therefore, have physical significance. Beyond two loops, we have to rely on a particular renormalization scheme. For instance, using the $\overline{MS}$ scheme, the authors of Ref. (8) have calculated $\beta(\alpha)$ up to three loops for an arbitrary Yang–Mills theory. Take the example of QCD with n_f flavors. Ref. (8) gives $\beta_{QCD}(\alpha_c) = \frac{\alpha_c^2}{4\pi}(-11 + \frac{2}{3}n_f) + \frac{\alpha_c^3}{16\pi^2}(-102 + \frac{38}{3}n_f) + \frac{\alpha_c^4}{64\pi^3}(-\frac{2857}{2} + \frac{5033}{18}n_f - \frac{325}{54}n_f^2)$. For $n_f = 18$ so that $b = \frac{1}{4\pi} > 0$, it is seen that $\beta_{QCD} > 0$. In fact, the coefficient of the third term is positive as long as $n_f \leq 40$. This simple example shows that even within a particular renormalization scheme such as the $\overline{MS}$ scheme, it is possible to make $\beta > 0$, up to three loops, in order for the enhancement effect discussed earlier to be additive. We conjecture that the induced strong effect found at two loops hold to all orders with perhaps an additional constraint on n_s. A more detailed discussion of this point for the problem at hand will be presented in a separate paper. Let us give a few concrete examples.

From here on, we will assume that at $\mathcal{M}' \simeq 0(\mathcal{M})$, all couplings are of the <u>same</u> order and that $\alpha_i^{-1}(\mathcal{M}') \ll \alpha_i^{-1}(\Lambda_F)$ for $i = 1,2,3$. It is equivalent to the assumption that $\alpha_i(\mathcal{M}')$ are large enough so that the low energy couplings are relatively insensitive to $\alpha_i(\mathcal{M}')$ and one would get predictions rather then bounds on $\alpha_i(\Lambda_f)$.

The trapping mechanism is illustrated by two models. Model I is described by $SU(3)_c \times SU(2)_L \times U(1)_y$. There are n families of conventional fermions $\{(3,2,\frac{1}{6})_L, (3,1,\frac{2}{3})_R, (3,1,-\frac{1}{3})_R, (1,2,-\frac{1}{2}), (1,1,-1)_R\}$, n_s fermions of the type $\{(3,1,y_s)_L, (3,1,y_s)_R\}$. In addition, we can have an arbitrary number of Higgs doublets. In the final results given below, we shall explain the origin of this choice. The fermions $\psi^s_{L,R} = (3,1,y_s)_{L,R}$ as well as parts of the conventional families will be given a mass of $0(\Lambda_F)$. This procedure is analogous to that employed by Refs. (1,2,3). We only explore the mass range of $0(\Lambda_F)$ because it is interesting phenomenologically.

Model II is described by $SU(4)_{T.C.} \times SU(3)_c \times SU(2)_L \times U(1)_y$. For simplicity, we choose to have three families of conventional technicolor–singlet fermions (a generalization to a larger number is straightforward). There are <u>no</u> Higgs fields here. We have <u>one</u> family of Technifermions

$\{(4,3,2,\frac{1}{6})_L,(4,3,1,\frac{2}{3})_R,(4,3,1,-\frac{1}{3})_R,(4,1,2,-\frac{1}{2})_L,(4,1,1,(-1,0))_R\}$. In analogy with Model I, we now include N_s fermions of the type $\{(4,3,1,y_s)_L,(4,3,1,y_s)_R\}$. These latter fermions will be chosen to have a gauge–invariant mass term of the form $M_I\bar{\psi}_L^{TS}\psi_R^{TS}+h.c.$ This is the mass scale M_I mentioned above.

It turns out that, for both models $\psi^{S,TS}$ will have to be $SU(2)_L$ singlets otherwise $\sin^2\theta_w$ becomes too large.

Let us first concentrate on Model II because the physics which comes out is quite interesting. To determine $\alpha_3(\Lambda_F)$, the full too–loop equation is used for the energy range $\Lambda_{TC}\lesssim E\lesssim \mathcal{M}$. For $\Lambda_F\lesssim E\lesssim\Lambda_{TC}$, the one–loop result $\alpha_3^{-1}(\Lambda_F)=\alpha_3^{-1}(\Lambda_{TC})-\frac{21}{6\pi}\ell n\left(\frac{\Lambda_{TC}}{\Lambda_F}\right)$ is used since the two–loop term $c_3\alpha_3^3$ is small compared with the one–loop term in that region. The following coefficients are needed: For $\Lambda_{TC}\lesssim E\lesssim M_I$, one has $b_{TC}=\frac{28}{12\pi},b_3=-\frac{5}{12\pi},c_{TC}=-\frac{134}{48\pi^2},c_3=\frac{226}{48\pi^2}$ whereas for $M_I\lesssim E\lesssim\mathcal{M}$, one has $\bar{b}_{TC}=(-28+6N_s)/12\pi,\bar{b}_3=(-5+8N_s)/12\pi,\bar{c}_{TC}=(615N_s-536)/192\pi^2,\bar{c}_3=(152N_s+226)/48\pi^2$. In addition to Eq. (4), the following one is used

$$\ell n\frac{M_I^2}{\Lambda_{TC}^2}=\frac{1}{b}\left(\alpha_0^{-1}-\bar{\alpha}^{-1}\right)+\frac{c}{b^2}\ell n\left[(b\alpha_0+c\bar{\alpha}\alpha_0)/(b\bar{\alpha}+c\bar{\alpha}\alpha_0)\right], \qquad (.10)$$

where c and b are the coefficients for $\Lambda_{TC}\lesssim E\lesssim M_I$ and $\alpha_0=\alpha_i(\Lambda_{TC}),i=TC,3$. The results are listed in Table 1 for $N_s=8$ and 10. As we shall see below, N_s is constrained to be $8\le N_s\le 10$ in order for $\sin^2\theta_w(\Lambda_F)$ to come out right. From Table 1, one notices that $\mathcal{M}/M_I$ is at most of 0(20). Typically, $\mathcal{M}/M_I\lesssim 0(10)$ and $\mathcal{M}$ ~100 TeV - 500 TeV while $M_I\sim$ 20 TeV - 40 TeV. For that mass range $0.09\lesssim\alpha_3(\Lambda_F)\lesssim 0.12$. In obtaining these results, the following inputs are used: $\Lambda_F\simeq 250$ GeV, $\Lambda_{TC}\simeq 1\,TeV$, $\alpha_{TC}(\Lambda_{TC})=\pi/3C_2(f_{TC})=8\pi/45\simeq 0.56$.

To compute $\sin^2\theta_w$, we need to know the coefficients of the following β–functions: $\beta_{TC}=b_{TC}\alpha_{TC}^2c_{TC}+\alpha_{TC}^3+c_{T3}\alpha_{TC}^2\alpha_3+\cdots,\beta_3=b_3\alpha_3^2+c_3\alpha_3^2+\bar{c}_{3T}\alpha_3^2\alpha_{TC}+\cdots,\beta_2=(b_2+c_{23}\alpha_3+c_{2T}\alpha_{TC})\alpha_2^2+\cdots,\beta_1=(b_1+c_{13}\alpha_3+c_{1T}\alpha_{TC})\alpha_1^2+\cdots$. The following relevant coefficients are listed: $c_{T3}=(2+N_s)/2\pi^2,\bar{c}_{3T}=\{45(2+N_s)\}/48\pi^2,c_{23}=7/4\pi^2,c_{2T}=30/16\pi^2,c_{13}=\{154(1+\frac{144}{77}y_s^2N_s)\}/72\pi^2,c_{1T}=\{50(1+\frac{9}{5}y_s^2N_s)\}/16\pi^2$. For $\alpha_3\simeq\alpha_{TC}$ and with the $N_s\sim 8-10$, one can ignore c_{T3} and $\bar{c}_{3T}$ as compared with c_{TC} and c_3. Furthermore, since the function $f(\mathcal{M},\mathcal{M}')$ defined by Eq. (8) is of the form $f_i(\mathcal{M},\mathcal{M}')=-\sum_{j=3,Tc}\frac{c_{ij}}{b_j}\ell n\ell n\frac{\mathcal{M}^2}{\mathcal{M}'^2}-\sum_{j=3,Tc}\left(\frac{c_{ij}c_j}{b_j^3}\right)\left(\ell n\ell n\frac{\mathcal{M}^2}{\mathcal{M}'^2}/\ell n\frac{\mathcal{M}^2}{\mathcal{M}'^2}\right)+g_i(\mathcal{M},M_I)$, with $i=1,2$, it is seen that, as $\mathcal{M}'\to\mathcal{M}$, the second term dominates.

We obtain ($i = 1,2$)

$$\lim_{\mathcal{M}' \to \mathcal{M}} f_i(\mathcal{M}, \mathcal{M}') \simeq \sum_{j=3,Tc} \left(\frac{c_{ij} c_j}{\bar{b}_j^3}\right) |K|, \tag{11a}$$

$$|K| = |\ell n \ell n \frac{\mathcal{M}^2}{\mathcal{M}'^2} / \ell n \frac{\mathcal{M}^2}{\mathcal{M}'^2}|. \tag{11b}$$

Neglecting $\alpha_{1,2}^{-1}(\mathcal{M}')$, one has $\alpha_{1,2}^{-1}(\Lambda_F) \simeq \bar{b}_{1,2} \ell n \frac{\mathcal{M}'^2}{\Lambda_T^2} + b_{1,2} \ell n \frac{\Lambda_T^2}{\Lambda_F^2} + f_{1,2}(\mathcal{M}, \mathcal{M}')$ where, in the limit $\mathcal{M}' \to \mathcal{M}$, the first two terms are negligible compared with $f_{1,2}(\mathcal{M}, \mathcal{M}')$. A word of caution is needed here. Since these heavy fermions also have $U(1)_y$ quantum numbers, one may wonder whether or not $\alpha_1(\mathcal{M})$ can already be large at one loop. The one-loop result gives $\alpha_1(\mathcal{M}) = \alpha_1(M_I)/[1 - \alpha_1(M_I)(\frac{140}{3} + 48 y_s^2 N_s) \ell n \left(\frac{\mathcal{M}}{M_I}\right)/6\pi]$. and for $|y_s| = \frac{1}{3}$ (smallest unit of charge known), $N_s = 10, \alpha_1(M_I) \sim 1/100$, it turns out that $\alpha_1^{one-loop}(\mathcal{M}) \simeq 1.2 \alpha_1(M_I)$. So even for α_1, one can safely ignore the first two terms in $\alpha_1^{-1}(\Lambda_F)$. The final result is, for $i = 1,2$,

$$\alpha_i^{-1}(\Lambda_F) \simeq \left(\sum_{j=3,Tc} \frac{c_{ij} c_j}{\bar{b}_j^3}\right) |K| \tag{.12}$$

We obtain

$$\begin{aligned} \tan^2 \theta_w(\Lambda_F) &= \frac{\alpha_1(\Lambda_F)}{\alpha_2(\Lambda_F)}, \\ &= \left(\frac{0.818}{1 + (144 y_s^2 N_s/77)}\right) \left[(1 + 1.46 R_1)/(1 + 1.07 R_1 R_2)\right], \end{aligned} \tag{.13}$$

with $R_1 = 4\left[\frac{-28+6N_s}{-5+8N_S}\right]^3 \left[\frac{152N_s+226}{615N_s-536}\right], R_2 = \frac{1+(9y_s^2 N_s/5)}{1+(144 y_s^2 N_s/77)}$. One also has

$$|K| = \alpha_{e.m.}^{-1}(\Lambda_F) / \sum_{i=1,2} \sum_{j=3,Tc} \left(c_{ij} c_j / \bar{b}_j^3\right). \tag{.14}$$

Using $\alpha_{e.m.}(\Lambda_F) \simeq 1/128$, one can determine the magnitude of $|K|$.

From Eq. (14), one notices two things. First, in the limit $y_s = 0$, one has $\tan^2 \theta_w(\Lambda_F) = 0.823$ corresponding to $\sin^2 \theta_w(\Lambda_F) \simeq 0.45$, for $N_s = 10$. Also, $\sin^2 \theta_w(\Lambda_F) \to 0.39$ for $N_s \to \infty$. This is clearly an unacceptable value and that is the reason why we need those heavy fermions to be charged. Secondly, if we take $|y_s| = \frac{1}{3}$ which is the smallest unit of charge, we have the following predictions which are independent of $\mathcal{M}'$

$$\sin^2\theta_w(\Lambda_F) \simeq 0.248,\ 0.232,\ 0.218\ ,0.205\ ,0.154 \text{ for } N_s = 7,8,9,10,11, \qquad (.15)$$

which implies that $8 \leq N_s \leq 10$. Also, from Eq. (16), if we take $N_s = 10$, we obtain $|K| \simeq 37.4$ which implies, for all practical purposes, $\mathcal{M}' \simeq \mathcal{M}$, because a precise value is not needed because $\sin^2\theta_w$ is independent of $|K|$. Since the $SU(2)_L$–singlet fermions have a mass which is not coming from the electroweak breaking, they decouple for $E < M_I$ and will not affect the phenomenological ρ–parameter although they have $U(1)_y$ quantum numbers.

Model I is particularly simple and also interesting. One has, for $E \gtrsim \Lambda_F, b_3 = (-33_4 n + 2n_s)/12\pi, c_3 = (76n + 38n_s - 306)/48\pi^2, c_{23} = n/4\pi^2, c_{13} = (22n + 792y_s^2 n_s/11)/72\pi^2$. Here, $\tan^2\theta_w(\Lambda_F) = c_{23}/c_{13}$, i.e.,

$$\tan^2\theta_w(\Lambda_F) = \frac{9n}{11n + 36y_s^2 n_s} \qquad (.16)$$

The two–loop equation is again used to determine $\alpha_3(\Lambda_F)$. The results are summarized in Table 2. The assumption $|y_s| = \frac{1}{3}$ is made here. From Eq. (18), one notices the following amusing coincidence, namely for $n = n_s$, one has $\sin^2\theta_w = \frac{3}{8}$, <u>independently</u> of n or n_s. However, this value of $\sin^2\theta_w$ is evaluated at $\Lambda_F = 250$ GeV and should not be confused with the same GUT value which is valid at $\Lambda_G \simeq 10^{15}$ GeV. Another special value is obtained when $n_s = 5n$ giving $\sin^2\theta_w(\Lambda_F) = 9/40 = 0.225$. In fact, if n_s is a multiple of n, i.e., $n_s = \lambda n$ then $4 \leq \lambda \leq 6$ giving $0.25 \geq \sin^2\theta_w(\Lambda_F) \geq 0.205$.

As for $\alpha_3(\Lambda_F)$, it turns out that the more one increases n and n_s, the more $\mathcal{M}$ decreases. For instance, if $n = 11$ and $n_s = 50(\sin^2\theta_w \simeq 0.236)$, then one obtains $\alpha_3(\Lambda_F) \simeq 0.086$ for $\mathcal{M} \simeq 1$ TeV (and decreases for $\mathcal{M} > 1$ TeV). This value (1 TeV) is dangerously close to Λ_F. For this reason, plausible bounds on n and n_s are $3 \leq n \leq 11$ and $15 \leq n_s \leq 50$. The lower bound on n_s comes from $\sin^2\theta_w$. Table 2 deals with three particular cases, all obeying the relationship $n_s = 5n(\sin^2\theta_w = 0.225)$, for $n = 3,7,8$. The case $n = 3$ (standard families) and $n_s = 15$ is particularly interesting since the only extra heavy fermions are the $SU(2)_L$–singlet charged, colored fermions of mass $\sim 0(\Lambda_F)$. These fermions should give rise to a spectacular increase in the R–ratio (in the event of a construction of an e^+e^- machine with C.M. energy > 500 GeV). In fact, $R = \sum_i Q_i^2 = \frac{5}{3}n + \frac{1}{3}n_s = 10$

for $n = 3, n_s = 15$. Note that the case $n = 3$ and $n_s = 15$ is also most attractive because the ρ-parameter is negligibly affected by the $SU(2)_L$-singlet fermions even if they have mass of $0(\Lambda_F)$. The induced shift in the Z boson mass[8] is roughly proportional to $\frac{\alpha}{4\pi}(\frac{\sin\theta_w}{\cos\theta_w})^2 \times y_s^2 n_s \ell n \frac{\Lambda_F^2}{M_Z^2} \simeq 0(10^{-4})$ for $|y_s| = \frac{1}{3}$ and $n_s = 15$. For $n = 7$ or 8, it is assumed that the heavy standard families have mass of $0(\Lambda_F)$ and, in order for the ρ-parameter not to be much affected, the new doublets will have to be almost degenerate, a rather unusual situation.

The "trapping" mechanism pointed out in this note is a way to lower the domain of validity of perturbation theory in the standard model beyond which new physics will certainly enter. The point here is the possibility that it happens, not near the Planck scale, but at intermediate energies. By "new", we mean new interactions and not just new particles. The prediction for $\sin^2\theta_w(\Lambda_F)$ is preserved and is not unique to specific grand unified theories.

The last point we would like to point out is the possibility of identifying the non-perturbative "unification" scale $\mathcal{M}$ in Model II with the extended Technicolor scale. One might then have a tumbling scenario in which, perhaps some gauge group would tumble down to $G_{TC} \times SU(3)_c \times SU(2)_L \times U(1)_y$. It is interesting to see if such scenario can occur. It could perhaps explain the origin of the heavy fermion ψ_s and their masses.

The ideas of unification with Technicolor and the $SU(2)_L$ singlet fermions were developed in collaboration with Georges Grunberg. A joint paper dealing with a non-perturbative unification with Technicolor along the line of Refs. [1,2,3] without the trapping mechanism is in preparation.

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Table 1

M_I (TeV)	$\mathcal{M}$ (TeV)	$\alpha_3(\Lambda_F)$
15	140	0.115
	72	0.129
20	241	0.105
	114	0.118
30	517	0.095
	218	0.105
40	879	0.087
	343	0.1

Table 2

$\mathcal{M}$ (TeV)	$\alpha_3(\Lambda_F)$	$n = n_s/5$
1	0.126	7
	0.111	8
1.5	0.103	7
	0.09	8
2.0	0.092	7
	0.08	8
300	0.126	3
1000	0.115	3
4000	0.103	3
10000	0.096	3

Table Captions

Table 1: Predictions for Model II. The upper (lower) value in each now corresponds to $N_s = 8(10)$ for $\sin^2\theta_w(\Lambda_F) = 0.232$ and 0.205 respectively.

Table 2: Predictions for Model I corresponding to the case $n_s = 5n$ with $\sin^2\theta_w(\Lambda_F) = 0.225$.

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