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A No Hair Theorem for R^2 models

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Abstract

We show that the $R + \epsilon R^2$ theory for homogeneous spacetimes, satisfies all the energy conditions required by the NHT and so it inflates. As the Universe approaches this phase it has already become sufficiently flat and isotropic, leaving the production of density perturbations to the de Sitter phase. When there is an explicit Λ term i.e. $R+\epsilon R^2 - 2\Lambda$ the model naturally leads into double inflation, where the R^2 inflationary phase is followed by the Λ driven phase but this has a non-standard vacuum energy. We investigate the constraints on the initial conditions due to the finite duration of the R^2 inflation and find that all but a small fraction of the available phase space leads to sufficient inflation.

1. Introduction. Inflationary Cosmologies [1] provide, if not the only, certainly the best dynamical explanation as to why the Universe we observe is so homogeneous and isotropic on very large scales. For the same token Inflation is capable of producing the density perturbations required to generate structure in the Universe. Until recently inflation had stood as the only theory capable of producing structure in the universe, today an alternative scenario can be provided by the Cosmic String Theory and it is only fair to say that the winner is yet to be found (see Turner in [1] for a review on Inflation and [2] for one on Cosmic Strings). The area where Inflation stays unchallenged (at least at the classical level) is that of *initial conditions*. It may provide us with a scenario where initial conditions for the Universe are almost unimportant, or their specification does not have to be perfectly accurate.

Among the half dozen or so inflationary models that we have now, those based on higher derivative gravity stand out as they do not require the addition of an arbitrary scalar field introduced solely to generate the inflationary expansion. In the original model proposed by Starobinsky [3] the de Sitter phase was driven by the trace anomaly of the energy momentum tensor, while in the more flexible R^2 model, the so called *improved Starobinsky model* [4,5] based on a lagrangian of the form,

$$\mathcal{L} = -\frac{1}{16\pi G}(R + \epsilon R^2) \tag{1}$$

there is an inflationary phase where the Hubble parameter H decays linearly in time. The study of this model [5] has shown that in order to have sufficient inflation we require ϵ to satisfy $10^{11}G < \epsilon < 10^{15}G$, and $H_i > H_{horizon} \sim 10^{-5} m_{pl}$.

Just like any other inflationary model, the R^2 model has been formulated within the realm of the spatially flat homogeneous and isotropic Robertson-Walker (RW) background. As it stands it can be the subject of the same criticism as the standard RW inflationary model concerning the strong restrictions imposed on the geometry. That is, we could question whether the inflationary features are not particular to, and dependant upon, the high degree of symmetry of the model. In the case of standard General Relativity it has been shown that inflation will always occur for open or flat anisotropic models regardless of the type of inflation. i.e. either old Inflation, a la Guth [1,6], new inflation [1,7] or chaotic inflation [8,9]. There are even formal proves of this statement as in the work of Wald [10]. In the context of inhomogeneous models several exact solutions exist [11] that show inflation will occur under very general conditions and even a formal proof of the so called No Hair Theorem (NHT) has been put forward by Jensen and Stein-Schabes [12]. The theorem states the following:

Any solution of Einstein's equation with a positive cosmological constant that i) accepts a synchronous coordinate system, ii) has a non positive three curvature, iii) has an energy-momentum tensor satisfying the Strong and Dominant Energy Conditions (SEC and DEC respectively), will become asymptotically de Sitter (at least on patch).

If the models considered are homogeneous, then the theorem holds globally. Before we proceed a few comments are in order. The *matter fields* that obey the energy conditions are not the ones producing inflation, this should be obvious since the cosmological constant violates one of these. Any contribution to the stress tensor coming from matter that satisfies the energy conditions would rapidly be redshifted away. The remanent stress tensor, the one that violates the energy conditions, will eventually dominate the dynamics as a slowly varying effective cosmological constant. For closed universes, those with ${}^3R > 0$, the de Sitter phase could be prevented altogether if the universe recollapses very early on (see [13] for an interesting account of closed models). Therefore, condition (ii) excludes the closed 3-geometries (like Bianchi IX).

The purpose of this letter is to show that within the context of the higher derivative theories, and restricting ourselves to open or flat homogeneous spacetimes, an equivalent NHT holds and inflation will almost always take place. Furthermore, we shall show that this happens with or without a cosmological constant Λ. If $\Lambda > 0$ then the model naturally leads into double inflation. This has been suggested by Turner and Silk and Turner *et al* [14] to account for the Large Scale Structure (LSC) of the Universe.

The proof will follow in spirit the one for homogeneous spacetimes [10], and its extension to inhomogeneous cases [12]. Moreover, it will be shown that this problem, which at first sight might appear rather formidable, is actually no more complex than that for Einstein's gravity. The limits imposed due to the finite duration of the R^2 inflationary phase will be examined in Sec. 3 and as a consequence of this we will conclude that within the classical domain almost all models will inflate regardless of the initial conditions. Of course, to give a full answer to the problem of initial conditions we will need to use Quantum Cosmology.

2. The Energy Conditions. Case I: $\Lambda = 0$. We will show that the higher derivative gravity model satisfies all the conditions required for the applicability of the NHT. This can can be done by transforming the Lagrangian (1) into that of Einstein's gravity coupled to a *scalar field* [15]. Then we shall show that the energy momentum tensor derived for this scalar field satisfies iii) and then conclude that these models will inflate.

Let us start by making the following conformal transformation [15],

$$g_{\alpha\beta} \rightarrow \tilde{g}_{\alpha\beta} = (1+2\epsilon R)g_{\alpha\beta} \tag{2}$$

and introduce a new field $\psi = log(1+2\epsilon R)$. Then the action constructed out of (1) and (2) becomes,

$$S = -\frac{\mu^2}{6}\int d^4x\sqrt{-\tilde{g}}\left[(\tilde{R}-2\tilde{\Lambda}) - \frac{3}{2}(\partial_\mu\psi\partial^\mu\psi + \tilde{V}(\psi))\right] \tag{3}$$

where,

$$\tilde{V}(\psi) = 4\tilde{H}_0^2 e^{-\psi}(e^{-\psi}-2) \tag{4}$$

and $\mu^2 \equiv \frac{3}{8\pi G}$, $\tilde{\Lambda} \equiv 3\tilde{H}_0^2 = \frac{1}{8\epsilon}$, and $\tilde{R}$ is the Ricci scalar curvature constructed using the conformally transformed metric $\tilde{g}_{\alpha\beta}$. This we shall call, the conformal picture. If $g_{\alpha\beta}$ satisfies (i) and (ii), so does $\tilde{g}_{\alpha\beta}$. We will restrict our attention to the case $1+2\epsilon R > 0$, (this is necessary for the conformal transformation not to invert the signature of the metric, and for ψ to be well defined). The potential $\tilde{V}(\psi)$ (including the effective cosmological constant $\tilde{\Lambda}$) is shown in Fig.1.

The energy momentum tensor derived from the previous action is

$$\tilde{T}_{\mu\nu} = \frac{\mu^2}{3}\left[\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}\tilde{g}_{\mu\nu}\left(\partial_\sigma\psi\partial^\sigma\psi + \tilde{V}(\psi)\right)\right] \tag{5}$$

As stated before we will concentrate on homogeneous spacetimes where the metric can be written as

$$ds^2 = -dt^2 + h_{ij}(t)w^i w^j \tag{6}$$

where the w^i are the one forms satisfying the group relations $dw^i = C^i_{jk}w^j w^k$ with C^i_{jk} the structure constants of the Bianchi Models [16]. The equations get greatly simplify in these cases since the Ricci scalar R becomes only a function of time.

The DEC states that $T_{\mu\nu}t^\mu t^\nu \geq 0$ and $|T_{\mu\nu}t^\mu| > 0$ for any timelike t^μ ($t^\mu t_\mu < 0$) [17] The first part of the condition becomes

$$\left((t^0)^2 - \frac{1}{2}\right)\dot{\psi}^2 + V(\psi) \geq 0 \tag{7}$$

while the second part gives

$$\left((t^0)^2 - \frac{1}{8}\right)\dot{\psi}^2 + \left((t^0)^2 + \frac{1}{2}\right)\dot{\psi}V(\psi) - V(\psi)^2 \geq 0 \tag{8}$$

The SEC is given by the following inequality $T_{\mu\nu}t^\mu t^\nu \geq -\frac{1}{2}T$ for any timelike vector t^μ. In our case it reduces to

$$\left((t^0)^2 + \frac{1}{4}\right)\dot{\psi}^2 - 2V(\psi) \geq 0 \tag{9}$$

From Fig.1 and the explicit form for the potential it is clear that the SEC is always satisfied, due to the fact that the potential is negative. The DEC will be also satisfied for large values of the curvature, i.e. ψ large. This is a peculiar situation, as in general when one of these is violated, it is the SEC and not the DEC, but since these conditions are independent [17] there there in no conflict. Furthermore, the characteristic time scale on which the field varies in this case is of order $\tilde{\tau} \leq \tilde{\tau}_* \sim \sqrt{\epsilon R}\,\tilde{H}_0 - 1$, which translates into $\tau_* \sim \sqrt{\epsilon}$; since one expects processes at high curvature to vary on Planck time-scales, it is plausible that the DEC is indeed satisfied. As we go towards lower curvature, $|\tilde{V}(\psi)|$ increases and the DEC, as it stands, is in general violated. This is because the ψ field acts as an inflaton. The remedy is simple: we deprive the ψ field of its potential by defining

$$\tilde{\Lambda}(\psi) = \tilde{\Lambda}(1 - e^{-\psi})^2 \tag{10}$$

The DEC is now trivially satisfied but the attractor is now $\tilde{\Lambda}(\psi)$. The shear, spatial curvature and kinetic term all decay as $\exp(-\tilde{N})$, where,

$$\tilde{N} = \int d\tilde{t}\,\sqrt{\frac{\tilde{\Lambda}(\psi)}{3}} \tag{11}$$

It is precisely the $\tilde{\Lambda}(\psi)$ dominated phase that corresponds to the de Sitter-like R^2 inflation with linearly decreasing Hubble parameter. Thus, we can conclude that the R^2 inflation behaves as an attractor for any open or flat homogeneous background.

Case II: $\Lambda > 0$. In this case we assume the existence of a non-zero cosmological constant. This could be thought as coming from some extra scalar field that would produce inflation in the standard scenario. The starting point is the Lagrangian

$$\mathcal{L} = -\frac{1}{16\pi G}(R + \epsilon R^2 - 2\Lambda) \tag{12}$$

The argument is almost identical to the used before. The new potential is now given, in the conformal picture by,

$$\tilde{V}(\psi) = 4\tilde{H}_0^2 e^{-\psi}\left[e^{-\psi}(1 + \frac{\Lambda}{\tilde{\Lambda}}) - 2\right] \tag{13}$$

we could also define a curvature dependent cosmological constant by

$$\tilde{\Lambda}(\psi) = \frac{4}{3}\left[\tilde{\Lambda}\left(1 - e^{-\psi}\right)^2 + \Lambda e^{-2\psi}\right] \tag{14}$$

this is also depicted in fig 1 (again including the effective $\tilde{\Lambda}$ term). The SEC is easily satisfied for all curvatures. However, the DEC obeys a similar equation as in case I and as a result the attractor is driven by $\tilde{\Lambda}(\psi)$ rather than by $\tilde{\Lambda}$. At high curvatures, when $\psi \to \psi_{pl}$, we have $\tilde{\Lambda}(\psi) \approx \tilde{\Lambda}$, and the R^2 term dominates. Thus, if the universe starts to the far right of the global minimum, it will undergo inflation driven by the R^2 term. As the curvature drops, the universe rolls towards the minimum where it may undergo inflation for a second time. Since at the minimum the curvature has the de Sitter value $R = 4\Lambda$, this second phase could be thought of as being driven by the Λ term alone. However, the total vacuum energy at the global minimum is in general larger than Λ, so reflecting the possible strong influence of the R^2 term. One finds, (in physical space-time) that,

$$\Lambda_{min} = \Lambda(1 + \frac{\Lambda}{\tilde{\Lambda}}) \tag{15}$$

Following Barrow and Ottewill [18], we can see that the maximally symmetric space satisfies the equations of motion. Thus, the final state of the $R + \epsilon R^2 - 2\Lambda$ theory, that sits at the global minimum of the potential, is indeed the de Sitter space-time, despite the fact that the vacuum energy is different. The horizon will be present and the generation of perturbations will proceed in the usual way. Of course, we then have to somehow make the global minimum of the potential go to zero. This is no problem if this second cosmological constant comes from some scalar field potential. Thus in the R^2 model with a cosmological constant we have a realization of the so called double inflation whose potential in explaining the large scale structure in the present universe has been pointed out in [14]. Some basic properties of the multiple inflationary phase in the presence of R^2 term have been outlined in the last two references in [4]. Here, we have established that such a phase will be present in any open or flat homogeneous background.

To finalize this section, let us observe that the dynamics of this inflationary phase depends strongly on the ratio $\Lambda/\tilde{\Lambda}$. As ψ approaches its ground state it will oscillate with a characteristic frequency

$$\omega^2 = \frac{\tilde{H}_0{}^2}{1 + \frac{\Lambda}{\tilde{\Lambda}}} \tag{16}$$

If $\Lambda \ll \tilde{\Lambda} \approx (10^{15} GeV)^2$, the R^2 inflation and the Λ driven inflation are well separated. However, if $\Lambda \gg \tilde{\Lambda}$, the minimum would be rather shallow and the oscillations would sweep over a broad range, lasting many Hubble times ($\omega^{-1} \sim \frac{\Lambda}{\tilde{\Lambda}} \tilde{H}_0{}^{-1}$), which would then modify the end of the R^2 phase. This could be important, as during this late stage, the scales of interest are pushed outside of the horizon.

3. Preinflationary phase. In a model with a true cosmological constant the de Sitter phase is infinitely long and any initial anisotropy or spatial curvature will be eventually diluted. In inflationary models however, not only the vacuum energy relaxes to zero after some finite time, but there are values of the curvature below which inflation will not take place. In particular in the R^2 model this limit is given by $R_e \sim 1/\epsilon$, or more precisely, $H_e \sim \frac{1}{6\sqrt{6\epsilon}}$ [5]. On the other hand, the space of initial data can not be arbitrarily large as we can use the classical description only when $R \leq m_{pl}^2$, and $|\frac{\dot{R}}{R}| \leq m_{pl}$. In the conformal picture (for the homogeneous case) it translates into

$$\tilde{\sigma}^2,\ {}^3\tilde{R},\ \tilde{K}^2 \leq \frac{1}{\epsilon}, \tag{17}$$

$$\left(\frac{d\phi}{d\tilde{t}}\right)^2 \leq \frac{m_{pl}^2}{12\pi\epsilon}, \tag{18}$$

where (since we are using a synchronous gauge), $d\tilde{t}^2 = (1 + 2\epsilon R)dt^2$, $\tilde{\sigma}^2$ stands for the shear, $\tilde{K}$ is the extrinsic curvature, and $\phi = \frac{1}{\sqrt{2}}\mu\psi$. The classical evolution that is responsible for the present day isotropy and flatness takes place during the stage $R_e \leq R \leq m_{pl}^2$. We shall say that the universe enters an inflationary phase at $R = R_i$ if,

$$\tilde{\Lambda}(\psi_i) > \tilde{\Lambda}(\psi_e), \tag{19}$$

and

$$\tilde{\sigma}_i^2,\ {}^3\tilde{R}_i,\ 12\pi G\dot{\phi}_i^2 \leq\ 12\pi G\dot{\phi}_{inf}^2, \tag{20}$$

where $|\dot{\phi}_{inf}| = \frac{\mu}{\sqrt{27\epsilon}} exp(-\frac{\sqrt{2}\phi}{\mu})$, is the slide-down velocity during the R^2 inflation.

Thus, the inflationary phase starts when all exponentially decaying quantities are at most of order $\mathcal{O}(1/(\epsilon R)^2)$, and thereby subdominant with respect to the vacuum energy $\tilde{\Lambda}(\psi)$ which contains a (negative) correction to the cosmological constant $\tilde{\Lambda}^{-1} = 8\epsilon$ of order $\mathcal{O}(1/\epsilon R)$. If any of the exponentially decaying quantities fall below $\dot{\phi}_{inf}^2(\phi_i)$ only at some $R_i < R_e$, then there will be no inflationary phase at all. The vacuum energy would level off with the kinetic term and particle production would take place through the oscillations of the spacetime background, which in general may still have some finite anisotropy or spatial curvature. The reheating phase will be then very different from the one that follows the inflationary phase. The Universe may eventually become isotropic (due to particle production), but will not have the cosmological perturbations characteristic to inflation, i.e. an almost flat spectrum. The task of generating the large scale structure would then fall on cosmic strings or some yet unknown mechanism.

If $R_i \sim R_e$, the universe will become flat and isotropic by this time. The reheating phase will be the same as in the R^2 inflation, but again there will be no perturbations present on scales larger than the horizon.

For $R_i \geq R_{horizon} \sim 10^{-9} m_{pl}^2$, the universe would go through an inflationary phase that generates perturbations on scales up to the present horizon size and beyond. If $R_i \geq R_* \sim 10^{-5} m_{pl}^2$, then eternal inflation takes place [19]. For $R_{horizon} \geq R_i \geq R_e$ the perturbation spectrum would have a cut-off at some very large scale given by [5]

$$\lambda_i = \left(\frac{H_{horizon}}{H_i} e^{-18\epsilon H_{horizon}^2} \right) H_0^{-1}, \tag{21}$$

where H_0 is the current Hubble parameter, $R_{horizon} = 12H_{horizon}^2$, and $R_i = 12H_i^2$.

What is apparent from this discussion is that the presence of the (effective) cosmological constant might indeed be responsible for the present isotropy and flatness of the Universe, but not because there is inflation but because of the approach to inflation: the anisotropy and spatial curvature are smoothed out during this *preinflationary phase.* Here by inflation we mean the isotropic de Sitter expansion with $|\dot{H}| \ll H^2$. It is possible to think of inflation in anisotropic models as a phase where all three directions expand exponentially but with different rates and the shear decays exponentially. However, there is no clean, general notion of "inflationary" expansion in the context of an arbitrary homogeneous models, and it becomes especially murky when we come down to calculating the perturbations. On the other hand, the attraction towards the cosmological constant driven phase is a completely general feature and an efficient mechanism of isotropisation. Thus, we will choose here to separate the two evolutionary phases into a preinflationary phase, with its decay of anisotropy and spatial curvature, and the inflationary phase with its calculable spectrum of density perturbations.

We have seen above that the $\tilde{\Lambda}(\psi)$ driven phase is an attractor. To translate this into a statement about the likelihood of inflation we should identify the range of initial data for which $R_i \geq R_{horizon}$, and similarly for $R_{horizon} \geq R_i \geq R_e$.

An analysis of the classical evolution (within the restrictions imposed by the RW background) for various initial conditions have been done in [5]. Here, it will be useful to carry a discussion in the conformal picture as one can readily extend it to arbitrary open or flat homogeneous backgrounds. The technique used is similar to that used in [9,20], for the case of a massive scalar field. Using the fact that the behaviour of the shear and the spatial curvature are bounded [10,12], it is not difficult to extend the analysis to anisotropic models and to consider cases other than $R_i = R_e$.

First we shall examine the case of a RW background. The configuration space is spanned by ϕ and $\dot{\phi}$. In terms of suitable chosen dimensionless variables $x = 1 - e^{-\psi}$ and $y = \dot{\phi}\sqrt{\epsilon}/\mu$, the classically allowed domain is given by,

$$\frac{1}{12}x^2 + y^2 \leq \frac{1}{9} \tag{22}$$

If the kinetic term dominates the energy, then the classical trajectory will be given by

$$y = y_{inf}(x_i)\left(\frac{1-x}{1-x_i}\right)^{\frac{3}{2}} \tag{23}$$

where $y_{inf}(x_i) = \frac{1}{\sqrt{27}}(1 - x_i)$, and we have assumed that the trajectory passes through the point $(x_i, y_{inf}(x_i))$. Here, x_i represents the extreme points on the phase plane where R^2 inflation will still work. This are essentially given by the bounds on ϵ obtained in [5]. We wish to know when do classical trajectories that pass through the extreme points $(x_e, y_{inf}(x_e))$ and $(x_{horizon}, y_{inf}(x_{horizon}))$ hit the *quantum boundary*. Using eqs. (22) and (23) we find that for the non-inflationary trajectories, the boundary is hit at a curvature of order $R_0 \sim R_e$, while for the best possible scenario in which we get at least 70e-folds we get $R_0 \sim 10^2 R_{horizon}$. This is of course nothing more than just an explicit solution for the exponential bound on the decay law. If we add the shear and spatial curvature, the configuration space becomes four-dimensional. The classically allowed domain is cut by $\tilde{K}^2 \leq \frac{1}{2}\epsilon$, via the constraint equation.

From eq. (17) we can see that these two new degrees of freedom obey a similar bound on the exponential law and have essentially the same restrictions on their possible initial values (see also [21]). The classical trajectory can be projected onto the planes (ψ, y), $(\psi, \epsilon_3\tilde{R})$, $(\psi, \epsilon\tilde{\sigma}^2)$. The difference is that due to the constraint equation, the maximal initial values for y should be less than in the RW case.

Since an explicit solution for the classical trajectory is not available one might worry that perhaps the vacuum energy decays too fast and can never dominate the other three contributions, despite their own (exponential) decay. However one finds,

$$\frac{1}{\tilde{\Lambda}(\psi)}\left|\frac{d\tilde{\Lambda}(\psi)}{d\tilde{N}}\right| \approx \left(\sqrt{\frac{8\pi G}{\epsilon}}\frac{|\dot{\phi}|}{4\tilde{\Lambda}}\right)\frac{1}{\sinh^2(\psi/2)} \tag{24}$$

which is less than one for $R > 4R_e$, and as small as 10^{-3} for $R = R_{horizon}$. Thus, again we have almost "vertical" trajectories, where the kinetic term, the shear and spatial curvature relax violently while the vacuum energy undergoes little changes. Thus, we may conclude as before: if $R_0 \geq R_e$, we should have $R_i \geq R_e$, and the universe should become isotropic and flat with or without passing through the exponentially expanding inflationary phase (as defined above); if $R_0 \geq 10^2 R_{horizon} \sim (10^{15}GeV)^2$, we will have $R_i \geq R_{horizon}$ and the universe will undergo inflation of at least ~ 70 e-folds. The approximate relationship is $\epsilon R_0(R_i) \approx (\epsilon R_i)^{\frac{5}{3}}$. In terms of ϕ_0, this leaves more than half the configuration space as favorable to inflation and almost all that leads to the isotropic and flat universe (because an effective cosmological constant is present, even if not dominant). We may conclude that the situation is now reversed: one has to fine tune the initial conditions in the homogeneous but anisotropic universe in order to *avoid* inflation.

4. Conclusions We have analyzed a Lagrangian of the form $\mathcal{L} = R + \epsilon R^2 - 2\Lambda$ for the case of homogeneous but anisotropic metrics, and showed that the model undergoes inflation (except for maybe Bianchi IX). We prove this by using a conformal transformation of the metric that turns the above mentioned Lagrangian into one representing a scalar field minimally coupled to gravity.

When $\Lambda = 0$, the model undergoes R^2 inflation, characterized by the linear decay of the Hubble parameter. In this case we showed that the stress tensor constructed for this theory, when the $\tilde{\Lambda}(\psi)$ term is subtracted satisfies both the SEC end DEC (see also [22]). When $\Lambda > 0$, the model naturally contains two inflationary phases, the final state of which is de Sitter with some unusual vacuum energy. This is important as it puts the R^2 inflation on the same footing as other inflationary models and it shows that the possible explanation of the Large Scale Structure using piecewise scale invariant spectrum [14] can be found in it as well.

We would like to add at this point a technical note about the closed models, in particular Bianchi IX (or its isotropic version, the closed Friedmann model), for which the NHT is in principle [10,12], not applicable. It is clear that if the three-curvature of the spatial sections is not to large then there might be a good chance that the model will inflate. Following Wald [10] we can write this condition as, $R > 2R_{sphere}$ where R_{sphere} stands for the curvature of a three-sphere of equal volume as the Bianchi IX in question. If this condition is satisfied then the closed model will also undergo inflation, otherwise it will recollapse.

Finally, we examined the constraints imposed on the initial conditions due to the finite duration of the R^2 inflationary phase. The results found in eqs.(22-25) are similar to those found in ref. [20] and [23] for an RW cosmology of a massive scalar field: the major portion of the phase space leads to at least 70 e-folds of inflation. One should keep in mind though, that when this statements are maid it is assumed that the actual probability distribution over the set of initial conditions are randomly assigned. The true distribution could be far from uniform. Presumably, the proper way to find these would be to use Quantum Cosmology, but so far little has been done in this respect [see for example Page [24]). Only after that, we can have a full statement about the likelihood of the inflationary phase.

Since the anisotropy and three curvature have already been diluted during the preinflationary phase, the requirement on the duration of the inflationary phases will depends on the choice of the length scale up to which we want to have the scale invariant spectrum of cosmological perturbations.

This results can be extended to inhomogeneous spacetimes when the conditions for the NHT are satisfied locally [12]. Since the typical number of e-folds is much larger than 70, even a very small patch where the conditions can be satisfied, will most presumably be blown to the size of the observable universe and beyond.

After the completion of this work we learn of some independent work by Maeda [25] and by Starobinsky and Schmidt and Schmidt [26] where a similar problem is addressed and similar conclusions are reached.

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Figure Caption

Fig. 1.- The Potential $\tilde{V}(\psi)$ for zero and nonzero cosmological constant is plotted as a function of ψ where $\psi = \ln(1 + 2\epsilon R)$.

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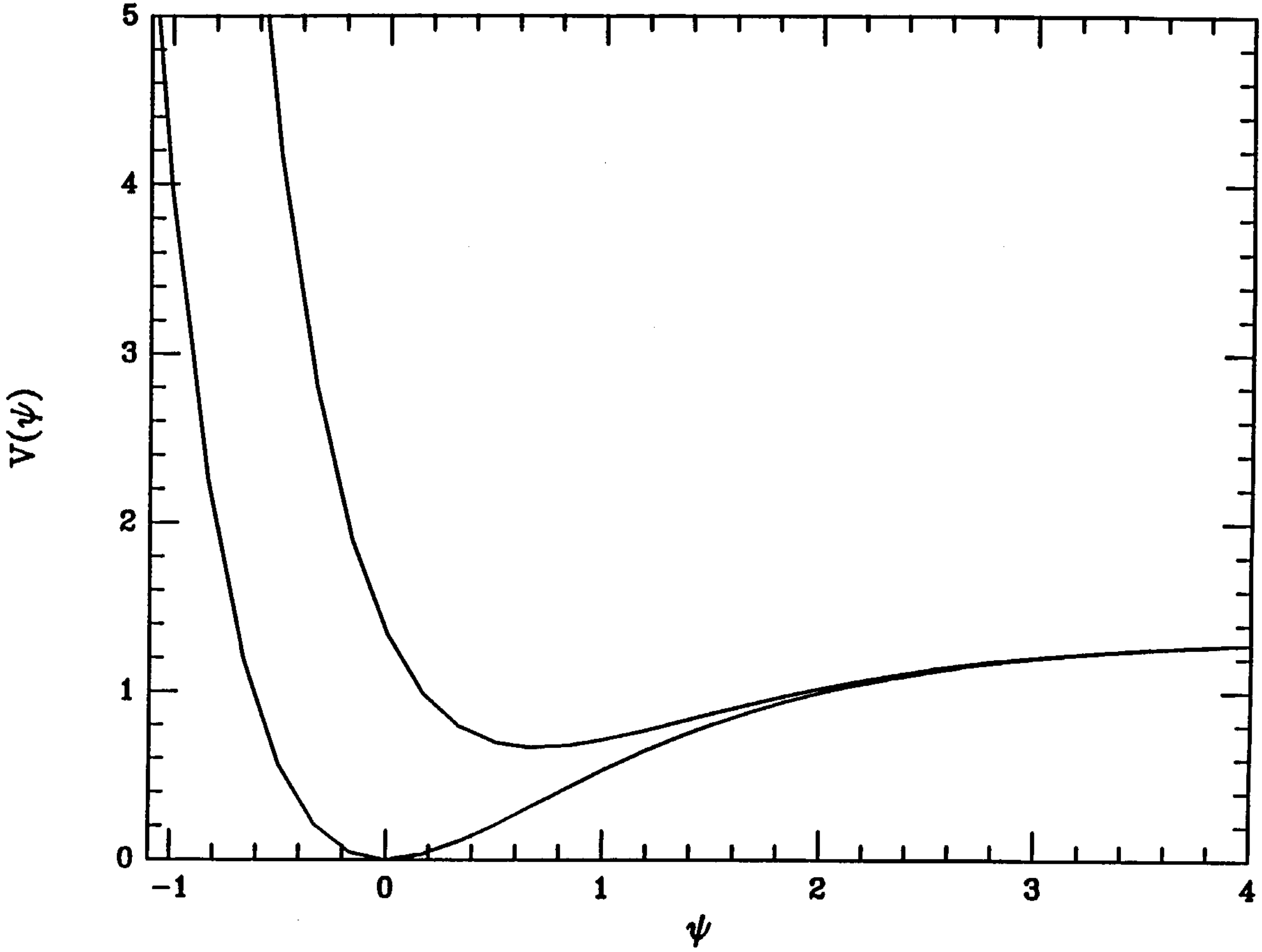

V(ψ)
ψ
5
4
3
2
1
0
−1
0
1
2
3
4