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INDUCED AMPLITUDES $Z' \to Z\gamma$ AND $Z' \to ZZ$

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ABSTRACT

The amplitudes $Z' \to Z\gamma$ and $Z' \to ZZ$ in models with an additional $U'(1)$ gauge group beyond the standard $SU(2) \times U(1)$ electroweak theory are induced via fermion loops of ordinary states u, d, e, ν *etc.* and heavy exotic states which will not cancel among themselves though the anomaly condition is satisfied. An E_6 model is used as an example for the illustration. The new heavy neutral gauge boson Z' could decay into $Z\gamma$ and ZZ with branching ratios: $\Gamma(Z' \to Z\gamma)/\Gamma(Z' \to e^+e^-) \sim 10^{-6}$ and $\Gamma(Z' \to ZZ)/\Gamma(Z' \to e^+e^-) \sim 10^{-5}$.

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The presently accepted $SU(2) \times U(1)$ electroweak theory is expected to be embedded in a larger gauge group. The pattern of symmetry breaking may give an extra $U'(1)$ symmetry beyond the standard model at low energy. In such a case, there would exist an extra neutral gauge boson, the Z'. Also, since the ordinary fermions u, d, e, ν, ... couple to Z' with different charges Q' which may not satisfy the anomaly condition[1] among themselves, additional exotic fermions must exist to balance the anomaly. In this letter, we study the induced amplitude $Z' \to Z\gamma$ and $Z' \to ZZ$ through the triangle diagram of fermion loop. If all fermion masses including those of the exotic states were negligible at the energy scale of Z', the amplitude would be tiny because of the anomaly condition. However, the exotic fermions may be much heavier so that both processes could occur. The observations of $Z' \to Z\gamma$ and $Z' \to ZZ$ are interesting in relating the couplings of various fermions.

To be specific, we study the E_6 model inspired from the Superstring Theory.[2] An extra $U'(1)$ exists[3] below the grand unifying scale,

$$E_6 \to SU(3)_{color} \times SU(2)_L \times U(1) \times U'(1) \; .$$

There is a new quark h of charge $-\frac{1}{3}$, a charged heavy lepton E^-, and neutral leptons ν_E, n. These new fermions and the Z' boson may have significant implications for experiments[4] at supercolliders at very high energy and high luminolsity.

In the E_6 model the couplings of $Z$and Z' bosons to the fermions f have the form

$$\mathcal{L} = g_Z(Q_V^Z Z_\mu + \sqrt{x_W}Q'_V Z'_\mu)\bar{f}\gamma^\mu f + g_Z(Q_A^Z Z_\mu + \sqrt{x_W}Q'_A Z'_\mu)\bar{f}\gamma^\mu\gamma_5 f \quad , \qquad (1)$$

where the coefficients Q_i^Z and Q'_i with $i = V$ (vector), A (axial vector) are given in Table I. Here $g_Z = e[x_W(1-x_W)]^{-\frac{1}{2}}$ and $x_W = \sin^2\theta_W$ as usual. The Z,Z' mixing[4] is known to be small and is neglected here.

There is no tree-level nonabelian tri-gauge-boson coupling among $Z'ZZ$ or $Z'Z\gamma$. To the lowest order of g^3, the amplitude is induced from the fermion loop of vertex combinations AVV or AAA types.

Following the treatment by Adler[8] , the amplitude $Z'(\mathbf{e}_{Z'}) \to Z(k_Z, \mathbf{e}_Z) + \gamma(k_\gamma, \mathbf{e}_\gamma)$ has the following structure

$$\begin{aligned}\mathcal{R} = & k_Z \cdot k_\gamma A_3 \epsilon(k_Z, \mathbf{e}_Z, \mathbf{e}_\gamma, \mathbf{e}_{Z'}) - (k_Z \cdot k_\gamma A_3 + k_Z^2 A_4)\epsilon(k_\gamma, \mathbf{e}_Z, \mathbf{e}_\gamma, \mathbf{e}_{Z'}) \\ & + A_3[k_Z \cdot \mathbf{e}_\gamma \epsilon(k_Z, k_\gamma, \mathbf{e}_Z, \mathbf{e}_{Z'}) - k_\gamma \cdot \mathbf{e}_Z \epsilon(k_Z, k_\gamma, \mathbf{e}_\gamma, \mathbf{e}_{Z'})] \quad ,\end{aligned} \tag{2}$$

with $\epsilon(a,b,c,d) \equiv \epsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma$. The transition probability is

$$\sum_{pol.} |\mathcal{R}|^2 = \frac{1}{2} z(1-z)^2(1+z)|A_3 - A_4|^2 M_{Z'}^6 \quad , \tag{3}$$

with $z = M_Z^2/M_{Z'}^2$. The probability vanishes as $z \to 0$ as a manisfestation of Yang's Theorem. A_3 and A_4 contain the dynamical information of the fermion loop of mass m_f

$$A_3 - A_4 \sim \sum_f (Q'_A Q_V^Z Q)_f \;\; \mathcal{I}(z, \frac{m_f^2}{M_{Z'}^2}) \quad , \tag{4}$$

and

$$\begin{aligned}\mathcal{I}(z,\eta) &= \mathcal{I}^{11}(z,\eta) - \mathcal{I}^{01}(z,\eta) + \mathcal{I}^{02}(z,\eta) \quad , \\ \mathcal{I}^{ab}(z,\eta) &= \int_0^1 dx \int_0^{1-x} dy \;\; x^a y^b [zy(1-y) + (1-z)xy - \eta + i0^+]^{-1}.\end{aligned} \tag{5}$$

The above $\mathcal{I}$ functions can be given analytically,

$$\begin{aligned}\mathcal{I}^{01}(z,\eta) &= 2\mathcal{I}^{02}(z,\eta) = (1-z)^{-1}[G(\eta) - G(\eta/z)] \\ \mathcal{I}(z,\eta) &= \frac{1}{1-z}\left(\frac{1}{2} + \frac{\eta}{1-z}[F(\eta) - F(\frac{\eta}{z})] - \frac{1}{2(1-z)}[G(\eta) - G(\frac{\eta}{z})]\right) \quad ,\end{aligned} \tag{6}$$

with

$$G(x) = \int_0^1 ln[1 - x^{-1}y(1-y) - i0^+]dy + 2$$
$$F(x) = \int_0^1 y^{-1} ln[1 - x^{-1}y(1-y) - i0^+]dy \quad . \tag{7}$$

For heavy fermions $x > \frac{1}{4}$,

$$F(x) = -2\arcsin^2\sqrt{\frac{1}{4x}}$$
$$G(x) = 2(4x-1)^{\frac{1}{2}}\arcsin\sqrt{\frac{1}{4x}} \quad ; \tag{8}$$

and when $x < \frac{1}{4}$ for not so massive fermion, the function $\arcsin\sqrt{\frac{1}{4x}}$ in the above expression would be replaced by its complex continuation $i[\text{arcosh}\sqrt{\frac{1}{4x}} - \frac{\pi}{2}i]$, also $(4x-1)^{\frac{1}{2}}$ becomes $-i(1-4x)^{\frac{1}{2}}$. If the fermion mass can be neglected as in the case of the known states u, d, e, *etc.*, then

$$\mathcal{I}(z,0) = \frac{1}{2(1-z)}\left[1 + \frac{ln(z)}{1-z}\right] . \tag{9}$$

Also, we know that heavy fermions decouple: $\mathcal{I}(z,\eta) \to 0$ as $\eta \to \infty$. There is no $Q'_V Q^Z_A Q$ contribution in the E_6 model for $Z' \to Z\gamma$ when m_d and m_e are neglected. So, the branching ratio is simply given by

$$\frac{\Gamma(Z' \to Z\gamma)}{\Gamma(Z' \to e^+e^-)} = \frac{45}{4}(\frac{\alpha}{\pi})^2 \frac{z(1-z^2)(1-z)^2}{x_W(1-x_W)}$$
$$\times \left|(1 - \frac{8}{3}x_W)\mathcal{I}(z,0) + \frac{4}{15}(1 - \frac{8}{3}x_W)[\mathcal{I}(z, \frac{m_t^2}{M_{Z'}^2}) - \mathcal{I}(z,0)]\right. \tag{10}$$
$$\left. + \frac{2}{9}x_W \sum_i \mathcal{I}(z, \frac{m_{h_i}^2}{M_{Z'}^2}) + (-\frac{1}{3} + \frac{2}{3}x_W)\sum_i \mathcal{I}(z, \frac{m_{E_i}^2}{M_{Z'}^2})\right|^2$$

The first term within the absolute signs comes from the known fermions of 3 families including the top quark of negligible mass. The second term gives the top quark mass correction. The last two terms are the contributions from the exotic states h_i and E_i (family index i=1,2,3). Fig. 1 shows the branching ratio for various values of $M_Z/M_{Z'}$ and the top quark mass. Here we use $\alpha \simeq \frac{1}{128}$ and $x_W \simeq 0.23$ at the weak scale. We assume the exotic states are heavy enough that they decouple. The ratio can be as large as $4 \cdot 10^{-6}$ at $M_Z/M_{Z'} \simeq 0.3$ and $m_t \simeq 30\ GeV$.

Now we study the process $Z'(\mathbf{e}_{Z'}) \rightarrow Z(k_1, \mathbf{e}_1)Z(k_2, \mathbf{e}_2)$. Contributions from $Q'_A Q^Z_A Q^Z_A$ terms introduce some complication. The amplitude is given by

$$\begin{aligned} \mathcal{R} = &(k_1 \cdot k_2 A_3 + M_Z^2 A_4 + A_0)\epsilon(k_1 - k_2, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_{Z'}) \\ &+ A_3[k_1 \cdot \mathbf{e}_2 \epsilon(k_1, k_2, \mathbf{e}_1, \mathbf{e}_{Z'}) - k_2 \cdot \mathbf{e}_1 \epsilon(k_1, k_2, \mathbf{e}_2, \mathbf{e}_{Z'})] \quad . \end{aligned} \tag{11}$$

The transition probability is

$$\sum_{pol.} |\mathcal{R}|^2 = z(1-4z)^2 |A_3 - A_4 - A_0/M_Z^2|^2 M_{Z'}^6 \quad . \tag{12}$$

$$A_3 - A_4 \sim \sum_f Q'_{A\ f} (Q_V^{Z\ 2} + Q_A^{Z\ 2})_f \quad J(z, \frac{m_f^2}{M_{Z'}^2}) \quad , \tag{13}$$

$$A_0/M_Z^2 \sim \sum_f (Q'_A Q_A^{Z\ 2})_f \ (m_f^2/M_Z^2) K(z, \frac{m_f^2}{M_{Z'}^2}) \quad , \tag{14}$$

with

$$\begin{aligned} J(z,\eta) &= J^{11}(z,\eta) - J^{01}(z,\eta) + J^{02}(z,\eta) \\ K(z,\eta) &= J^{00}(z,\eta) - 2J^{01}(z,\eta) \\ J^{ab}(z,\eta) &= \int_0^1 dx \int_0^{1-x} dy\ x^a y^b [zx(1-x) + zy(1-y) + xy(1-2z) - \eta + i0^+]^{-1}. \end{aligned} \tag{15}$$

These functions can be simplified as follows

$$\begin{aligned}
J^{00}(z,\eta) &= \frac{1}{1-4z}\left[(\frac{1}{2}-z)H(\frac{z^2}{1-4z},\eta) - H(\frac{z}{1-4z},\eta)\right] \\
J^{01}(z,\eta) &= \frac{1}{1-4z}\left[G(\eta) - G(\frac{\eta}{z}) - zJ^{00}(z,\eta)\right] \\
J(z,\eta) &= \frac{1}{1-4z}\left[\eta J^{00}(z,\eta) - (1-z)J^{01}(z,\eta) + \frac{1}{2}G(\eta) - \frac{1}{2}G(\frac{\eta}{z}) + \frac{1}{2}\right] \ ,
\end{aligned} \tag{16}$$

with

$$H(u,x) = \int_0^1 dy \frac{ln[1 - x^{-1}y(1-y) - i0^+]}{u + y(1-y)} \ , \tag{17}$$

which is related to the Spence function[6] .

We have a compact expression for the branching fraction

$$\begin{aligned}
\frac{\Gamma(Z'\to ZZ)}{\Gamma(Z'\to e^+e^-)} = \frac{45}{4}(\frac{\alpha}{\pi})^2\frac{z(1-4z)^{\frac{5}{2}}}{x_W^2(1-x_W)^2}\Bigg|&(1-2x_W+\frac{8}{3}x_W^2)J(z,0) \\
+\frac{1}{5}(1-\frac{8}{3}x_W+\frac{32}{9}x_W^2)[(J(z,\frac{m_t^2}{M_{Z'}^2}) - J(z,0)] &- \frac{m_t^2}{10M_Z^2}K(z,\frac{m_t^2}{M_{Z'}^2}) \\
-\frac{1}{6}\sum_i J(z,\frac{m_{\nu_{E_i}}}{M_{Z'}}) - \frac{1}{6}(1-2x_W)^2\sum_i J(z,\frac{m_{E_i}}{M_{Z'}}) &- \frac{2}{9}x_W^2\sum_i J(z,\frac{m_{h_i}}{M_{Z'}})\Bigg|^2 \ .
\end{aligned} \tag{18}$$

The numerical result is shown in Fig. 2. The branching ratio can be as large as $4\cdot10^{-5}$ at $M_Z/M_{Z'}\simeq 0.15$ and $m_t \simeq 30GeV$.

In summary, we have studied the the induced amplitudes $Z'\to Z\gamma$ and $Z'\to ZZ$ with a finite branching ratio which, if observable, can relate the couplings of various fermions. The above study is applicable for the general case besides the E_6 example which we use as an illustration.

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TABLE I: Vector and Axial vector couplings of the fermions to the Z and Z' bosons.

Fermion	Q_V^Z	Q_A^Z	Q'_V	Q'_A
u	$\frac{1}{4} - \frac{2}{3}x_W$	$-\frac{1}{4}$	0	$-\frac{1}{3}$
d	$-\frac{1}{4} + \frac{1}{3}x_W$	$+\frac{1}{4}$	$+\frac{1}{4}$	$-\frac{1}{12}$
e	$-\frac{1}{4} + x_W$	$+\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{12}$
h	$\frac{1}{3}x_W$	0	$-\frac{1}{4}$	$+\frac{5}{12}$
E	$-\frac{1}{2} + x_W$	0	$+\frac{1}{4}$	$+\frac{5}{12}$
ν_e	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{3}$
ν_E	$\frac{1}{2}$	0	$+\frac{1}{4}$	$+\frac{5}{12}$

FIGURE CAPTIONS

Fig. 1 Branching ratios $\Gamma(Z' \to Z\gamma)/\Gamma(Z' \to e^+e^-)$ *vs.* $M_Z/M_{Z'}$. The solid, dotted, dashed and dash-dotted curves are for the case of $m_t/M_Z = 0.3,\ 0,\ 0.9$ and 1.5 respectively.

Fig. 2 Branching ratios $\Gamma(Z' \to ZZ)/\Gamma(Z' \to e^+e^-)$ *vs.* $M_Z/M_{Z'}$. The solid, dotted, dashed and dash-dotted curves are for the case of $m_t/M_Z = 0.3,\ 0,\ 0.9$ and 1.5 respectively.

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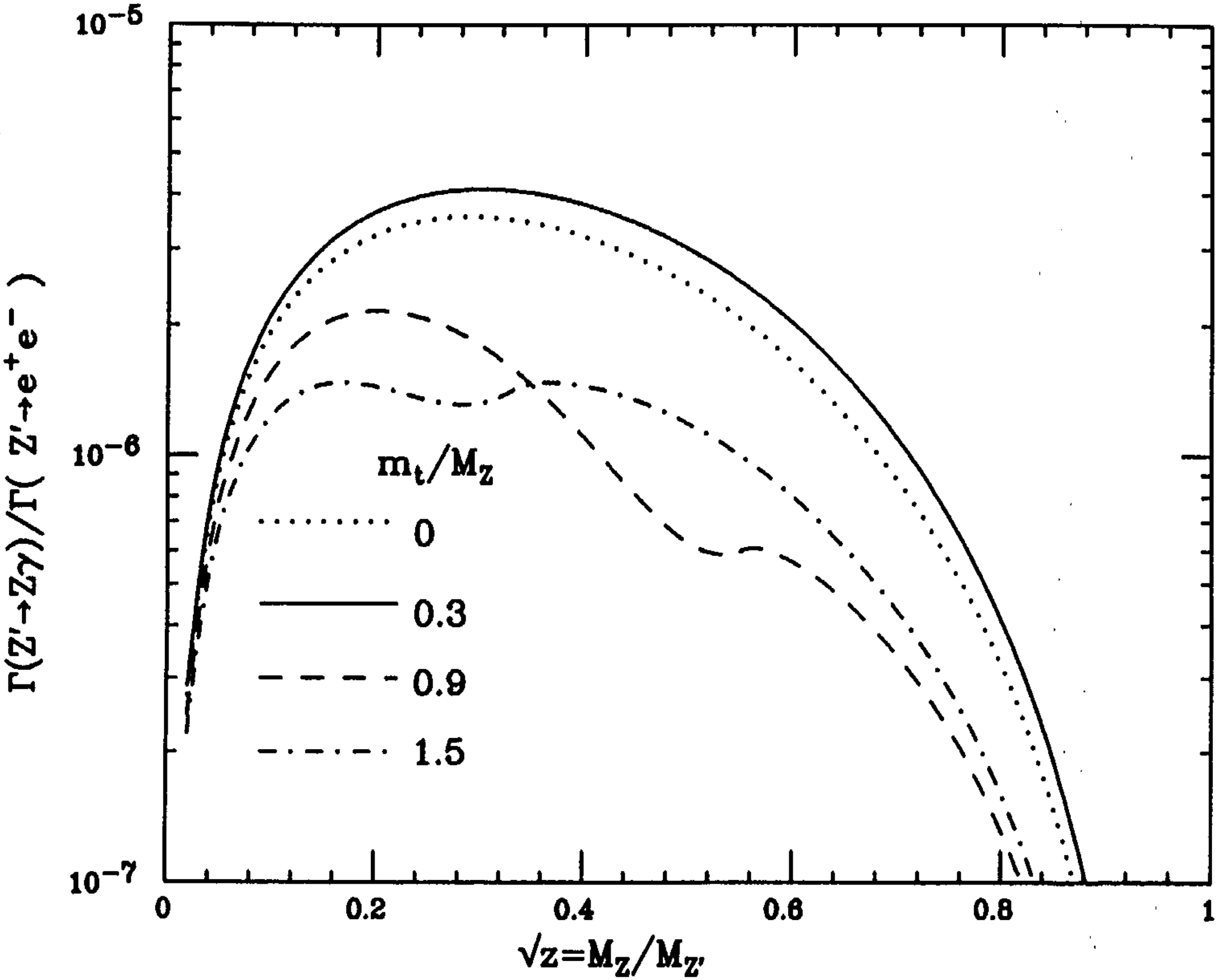
10^{-5}
10^{-6}
10^{-7}
$\Gamma(Z'\to Z\gamma)/\Gamma(Z'\to e^+e^-)$
m_t/M_Z
0
0.3
0.9
1.5
0
0.2
0.4
0.6
0.8
1
$\sqrt{z}=M_Z/M_{Z'}$

Fig. 1

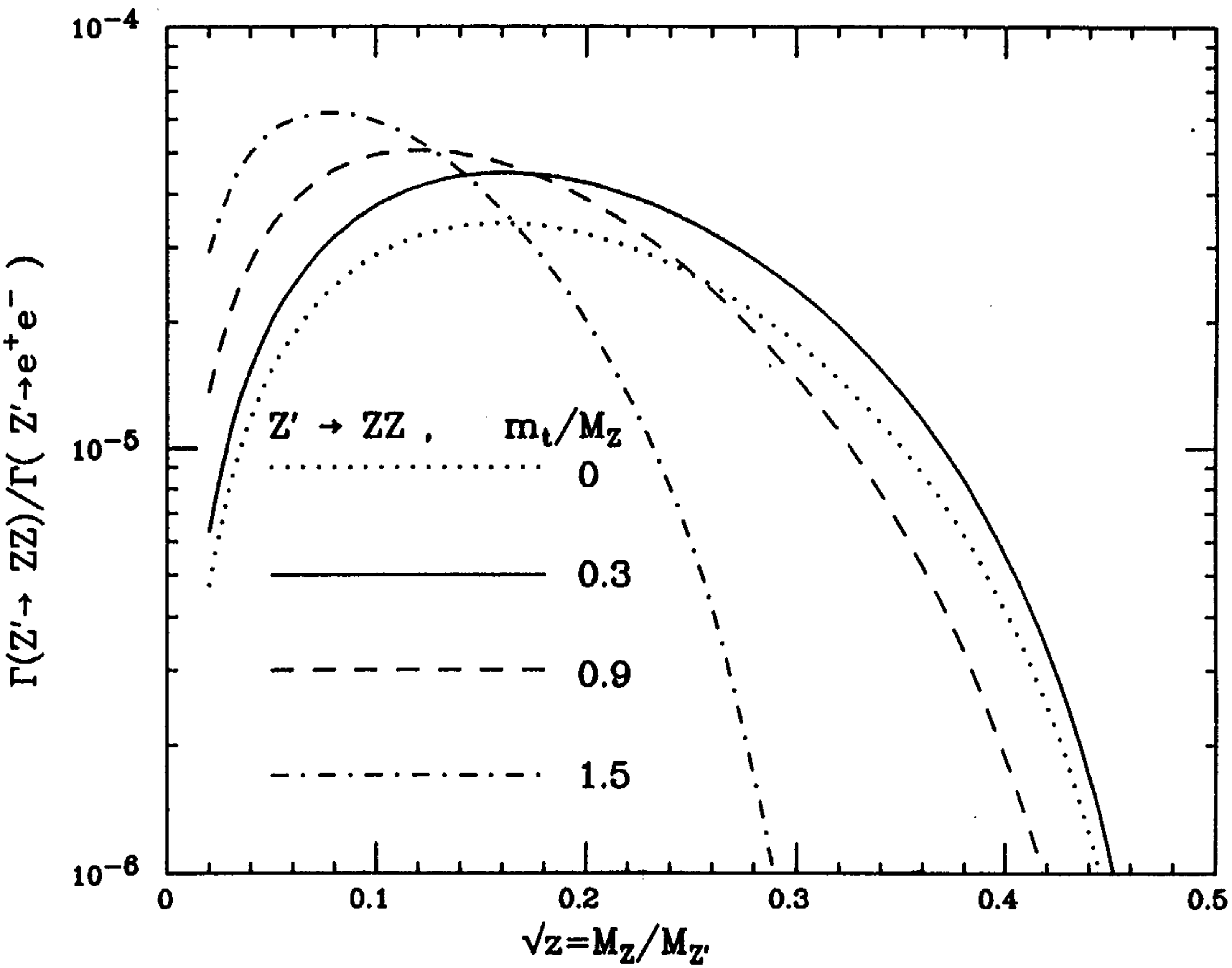

Fig. 2