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Computing finite temperature loops with ease

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Abstract

An efficient way of calculating perturbatively at non-zero temperature is to start with a diagram in momentum space, and then Fourier transform each propagator in a loop with respect to the (imaginary) time. Discontinuities are read off from the energy denominators of this non-covariant approach.

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The properties of field theory at a temperature T are of interest in a variety of problems [1- 5]. The standard procedure is to work in Euclidean space-time, with an imaginary time $1/T \geq \tau \geq 0$. As τ is of finite extent, the energies are discrete when $T \neq 0$, so evaluating any loop diagram requires the computation of infinite sums. While these sums can be done by contour integration, it is a tiresome process.

In this work I show that there is a simple way of performing the energy sums at non-zero temperature. With this method, any diagram is directly seperated into a piece equal to that at zero temperature, plus contributions at $T \neq 0$. It is apparent from this seperation that all terms at non-zero temperature are ultraviolet finite. The process is also an expedient means of extracting the discontinuities of diagrams for complex values of the external energy.

The method is not novel. It was introduced, some time ago, by Balian and De Dominicis, Baym and Sessler, and Dzyaloshinski [6]. Later, Cornwall and Norton, Norton [7], and Weldon [8] used it. In the first half of the paper, I emphasize its convienience in practical calculations. In the latter half, I use it to give a general discussion of the analytic structure of amplitudes at non-zero temperature.

The virtues of this technique are common to the real-time formalism of finite temperature field theory [5,9]. One significant difference is that the real-time formalism requires extra degrees of freedom. For example, a single scalar field becomes a two-by-two matrix, with the other components representing unphysical, "ghost" fields. This confusing multiplicity of fields does not arise in the present approach.

The essential trick is to use propagators that are in a momentum representation in space, but in a coordinate representation in time; thus I begin by reviewing known features of these propagators [1,2]. For a spinless boson, consider the "mixed" propagator,

$$\Delta(\tau, \mathbf{p}) = \frac{1}{\beta} \sum_{j=-\infty}^{+\infty} e^{-ip_0\tau} \frac{1}{p^2 + m^2} , \tag{1}$$

where $\beta = 1/T$, $p^2 = p_0^2 + \mathbf{p}^2$; $\mathbf{p}$ is the spatial momentum, and $p_0 = 2\pi j T$ the energy. From its definition, $\Delta(\tau, \mathbf{p})$ satisfies

$$\Delta(\tau - \beta, \mathbf{p}) = +\Delta(\tau, \mathbf{p}) \ , \ \Delta(-\tau, \mathbf{p}) = +\Delta(\tau, \mathbf{p}) \ . \tag{2}$$

To solve for $\Delta(\tau, \mathbf{p})$, instead of doing the sum over j it is preferable to Fourier transform eq. (1) to

$$\frac{1}{p^2+m^2} = \int_0^\beta d\tau\, e^{ip_0\tau}\, \Delta(\tau,\mathbf{p})\,. \tag{3}$$

Over the interval $\beta \geq \tau \geq -\beta$, the solution to eqs. (2) and (3) is

$$\Delta(\tau,\mathbf{p}) = \frac{1}{2E_{\mathbf{p}}} \sum_{s=\pm} f_s(\mathbf{p})\, exp\left(-sE_{\mathbf{p}}|\tau|\right)\ , \tag{4}$$

where $E_{\mathbf{p}} = +\sqrt{\mathbf{p}^2+m^2}$, and

$$f_+(\mathbf{p}) = 1 + n(\mathbf{p})\ ,\ f_-(\mathbf{p}) = n(\mathbf{p})\ ,\ n(\mathbf{p}) = \frac{1}{exp(\beta E_{\mathbf{p}}) - 1}\,. \tag{5}$$

$n(\mathbf{p})$ is the Bose-Einstein distribution function. A helpful identity is

$$e^{-\beta E_{\mathbf{p}}} = \frac{f_-(\mathbf{p})}{f_+(\mathbf{p})}\,. \tag{6}$$

The dependence of the $f(\mathbf{p})$'s, $n(\mathbf{p})$, and $E_{\mathbf{p}}$ on the mass m is implicit, and should be clear from the context. These functions, and $\Delta(\tau,\mathbf{p})$, depend on the spatial momentum $\mathbf{p}$ solely through its magnitude, $\mathbf{p}^2$.

In the following only the propagator forward in time, $\beta \geq \tau \geq 0$, is needed. The distribution function $n(\mathbf{p})$ is Boltzman as $T \to 0$, $n(\mathbf{p}) \approx exp(-E_{\mathbf{p}}/T)$. At zero temperature, $\Delta(\tau,\mathbf{p}) = exp(-E_{\mathbf{p}}\tau)/(2E_{\mathbf{p}})$, and only states with positive energy propagate forward in time. When $T \neq 0$, due to stimulated emission a particle with positive energy has a probability $\sim 1 + n(\mathbf{p})$. At non-zero temperature it is also possible to absorb a particle with energy $E_{\mathbf{p}}$ from the thermal bath. In the propagator this is a state with negative energy, $\sim exp(+E_{\mathbf{p}}\tau)$, whose residue $\sim n(\mathbf{p})$ is the probability for absorption.

In Feynman gauge, the propagators for a gauge field and the Fadeev-Popov ghosts are those of eq. (4), with $m = 0$. In a general covariant gauge, the gauge field propagator can be computed by using a Stuckelberg mass,

$$\Delta^{\mu\nu}(p_0,\mathbf{p}) = \frac{\delta^{\mu\nu}}{p^2} + (\xi - 1)\frac{p^\mu p^\nu}{(p^2)^2}$$

$$= \frac{\delta^{\mu\nu}}{p^2} + p^\mu p^\nu\, lim_{\,m\to 0} \left(\frac{1}{p^2+m^2} - \frac{1}{p^2+\xi m^2}\right)\ , \tag{7}$$

but when $\xi \neq 1$ the detailed form of $\Delta^{\mu\nu}(\tau, \mathbf{p})$ is not very illuminating. In Coulomb-type gauges, the transverse part of $\Delta^{ij}(\tau, \mathbf{p})$ is as in eq. (4); the other components of $\Delta^{\mu\nu}(\tau, \mathbf{p})$, and the ghost propagator, are delta-functions in time, or derivatives thereof.

For fermions, I define a propagator $\tilde{\Delta}(\tau, \mathbf{p})$ as in eq. (1), except that the energy is an odd multiple of πT, $p_0 = (2j+1)\pi T$. It satisfies

$$\tilde{\Delta}(\tau - \beta, \mathbf{p}) = -\tilde{\Delta}(\tau, \mathbf{p}) \; , \; \tilde{\Delta}(-\tau, \mathbf{p}) = +\tilde{\Delta}(\tau, \mathbf{p}) \; . \tag{8}$$

The solution to eq. (3), with $\tilde{\Delta}(\tau, \mathbf{p})$ replacing $\Delta(\tau, \mathbf{p})$, and eq. (8) is

$$\tilde{\Delta}(\tau, \mathbf{p}) = \frac{1}{2E_{\mathbf{p}}} \sum_{s=\pm} \tilde{f}_s(\mathbf{p}) \; exp\left(-sE_{\mathbf{p}}|\tau|\right) \; , \tag{9}$$

with

$$\tilde{f}_+(\mathbf{p}) = 1 - \tilde{n}(\mathbf{p}) \; , \; \tilde{f}_-(\mathbf{p}) = -\,\tilde{n}(\mathbf{p}) \; , \; \tilde{n}(\mathbf{p}) = \frac{1}{exp(\beta E_{\mathbf{p}}) + 1} \; , \tag{10}$$

which obeys

$$e^{-\beta E_{\mathbf{p}}} = -\,\frac{\tilde{f}_-(\mathbf{p})}{\tilde{f}_+(\mathbf{p})} \; . \tag{11}$$

$\tilde{n}(\mathbf{p})$ is the Fermi-Dirac distribution function, with the minus signs in the $\tilde{f}$'s reflecting the Pauli exclusion principle.

The fermion propagator $\Delta_f(\tau, \mathbf{p})$,

$$\frac{1}{-i \not{p} + m} = \int_0^\beta d\tau \; e^{ip_0\tau} \Delta_f(\tau, \mathbf{p}) \; , \tag{12}$$

is constructed from $\tilde{\Delta}(\tau, \mathbf{p})$:

$$\Delta_f(\tau, \mathbf{p}) = \left(-\frac{\partial}{\partial \tau}\gamma_0 + i\mathbf{p} \cdot \boldsymbol{\gamma} + m \right) \tilde{\Delta}(\tau, \mathbf{p}) \; ; \tag{13}$$

$\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$. As required, $\Delta_f(\tau, \mathbf{p})$ is anti-periodic in τ, $\Delta_f(\tau - \beta, \mathbf{p}) = -\Delta_f(\tau, \mathbf{p})$. From the definition of Δ_f, however, it cannot transform with a definite sign under time reflection symmetry, $\tau \rightarrow -\tau$. Like $\tilde{\Delta}$, eq. (8), the part of $\Delta_f \sim i\mathbf{p} \cdot \boldsymbol{\gamma} + m$ is even under this transformation; the rest of $\tilde{\Delta}$, $\sim \partial/\partial\tau \; \gamma_0$, is odd.

At large momentum, both distribution functions assume a Boltzman form, $n(\mathbf{p}) \approx \tilde{n}(\mathbf{p}) \approx exp(-|\mathbf{p}|/T)$. Thus Δ and $\tilde{\Delta}$ are approximately the same at large $\mathbf{p}$, equal to their values at zero temperature: $\Delta(\tau, \mathbf{p}) \approx \tilde{\Delta}(\tau, \mathbf{p}) \approx exp(-E_{\mathbf{p}}\tau)/(2E_{\mathbf{p}})$, up to exponentially small corrections. This is why a non-zero temperature never introduces any counterterms beyond those needed at zero temperature.

The boson and fermion propagators behave very differently in the infrared limit. Let $m \ll T$, and consider small momenta, $E_{\mathbf{p}} \ll T$; then the τ dependence of the propagators is negligible, $\sim E_{\mathbf{p}}\tau \leq E_{\mathbf{p}}/T \ll 1$. For small momenta the Bose-Einstein distribution functions are large, $n(\mathbf{p}) \approx T/E_{\mathbf{p}} + \ldots$, and dominate the propagator, $\Delta(\tau, \mathbf{p}) \approx n(\mathbf{p})/E_{\mathbf{p}} \approx T/E_{\mathbf{p}}^2$. This result for $\Delta(\tau, \mathbf{p})$ is evident from eq. (1), as in the infrared limit the greatest term has zero energy, $p_0 \sim j = 0$ in the sum.

The Fermi-Dirac distribution function is well behaved about zero momentum, $\tilde{n}(\mathbf{p}) \approx 1/2 - E_{\mathbf{p}}/(4T) + \ldots$, but there is a cancellation in the propagator: $\tilde{\Delta}(\tau, \mathbf{p}) \approx (1 - 2\tilde{n}(\mathbf{p}))/(2E_{\mathbf{p}}) \approx 1/(4T) + \ldots$. This occurs because $\tilde{\Delta}(\tau, \mathbf{p})$ is obtained from eq. (1) by summing over energies p_0 that are odd multiples of πT. As p_0 cannot vanish, about zero momentum $\tilde{\Delta}$ is better behaved than Δ by two powers of $E_{\mathbf{p}}$.

Given the mixed propagators, the method of computation is straightforward: starting with an arbitrary loop diagram in momentum space, eqs. (3) and (12) are used to Fourier transform each propagator in a loop with respect to time. After that, it is easy to do the energy sums, and then the integrals over time.

I demonstrate the technique by computing the self-energy of a massless fermion, to one-loop order in a non-abelian gauge theory. In Feynman gauge,

$$\Sigma_f(p_0, \mathbf{p}) = 2g^2 C_f \; \frac{1}{\beta} \sum_{j=-\infty}^{+\infty} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \quad \frac{i}{\not{k}} \frac{1}{(p-k)^2} \; ; \tag{14}$$

the non-abelian coupling $= g$, and $C_f = (N^2 - 1)/(2N)$ for a fermion in the fundamental representation of $SU(N)$. As a fermion self-energy, the energies p_0 and k_0 are odd multiples of πT, $k_0 = (2j+1)\pi T$. Introducing times τ_1 and τ_2 for the fermion and gauge field propagators, the sum over j in eq. (14) just gives a delta-function in time, setting $\tau_1 = \tau_2 \equiv \tau$:

$$\Sigma_f(p_0, \mathbf{p}) = 2g^2 C_f \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int_0^\beta d\tau \; e^{ip_0\tau} \; \Delta(\tau, \mathbf{p} - \mathbf{k}) \left(-\frac{\partial}{\partial\tau}\gamma_0 + i\mathbf{k} \cdot \gamma \right) \tilde{\Delta}(\tau, \mathbf{k}) \; . \tag{15}$$

The integral over τ is elementary, merely a series of integrals over exponentials.

Using $exp(ip_0\beta) = -1$, eqs. (6) and (11),

$$\Sigma_f(p_0, \mathbf{p}) = 2g^2 C_f \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2E_1 2E_2} A_{12} , \tag{16a}$$

where $E_1 = \sqrt{\mathbf{k}^2}$, $\tilde{n}_1 = \tilde{n}(\mathbf{k})$, $E_2 = \sqrt{(\mathbf{p}-\mathbf{k})^2}$, and $n_2 = n(\mathbf{p}-\mathbf{k})$, with

$$A_{12} = \sum_{s_1=\pm} \sum_{s_2=\pm} \left(\tilde{f}_{s_1}(\mathbf{k}_1) f_{s_2}(\mathbf{k}_2) - \tilde{f}_{-s_1}(\mathbf{k}_1) f_{-s_2}(\mathbf{k}_2) \right) \frac{-i \not{k}_{s_1}}{ip_0 - s_1 E_1 - s_2 E_2} , \tag{16b}$$

$\not{k}_{s_1} = -s_1 E_1 \gamma_0 + i\mathbf{k}_1 \cdot \gamma$. Explicitly,

$$A_{12} = (1 - \tilde{n}_1 + n_2) \left(\frac{-i \not{k}_+}{ip_0 - E_1 - E_2} + \frac{i \not{k}_-}{ip_0 + E_1 + E_2} \right)$$

$$+ (\tilde{n}_1 + n_2) \left(\frac{-i \not{k}_+}{ip_0 - E_1 + E_2} + \frac{i \not{k}_-}{ip_0 + E_1 - E_2} \right) . \tag{16c}$$

For arbitrary p_0 and $\mathbf{p}$ the result for Σ_f is involved, so consider the limit of $\mathbf{p} = 0$. In this limit, the the last two terms in eq. (16c) contribute to a term $\sim 1/p_0$:

$$\Sigma_f(p_0, 0) \approx i \frac{\gamma_0}{p_0} m_f^2 + \dots , \quad m_f^2 = C_f \frac{g^2 T^2}{8} . \tag{17}$$

To one-loop order, at $\mathbf{p} = 0$ the renormalized propagator is:

$$-i \not{p} - \Sigma_f \approx -i \frac{\gamma_0}{p_0} \left(p_0^2 + m_f^2 \right) . \tag{18}$$

The self-energy $\Sigma_f(p_0, \mathbf{p})$ is defined only on a discrete set of points, for $p_0 = (2j+1)\pi T$. This is trivially extended to the complex p_0 plane, by letting p_0 assume arbitrary, complex values. This continuation is not unique: given any function $h(x)$ such that $h(1) = 1$, then $h(exp(ip_0/T + i\pi))\ \Sigma_f(p_0, \mathbf{p})$ agrees with the trivial continuation on the points $p_0 = (2j+1)\pi T$. If $h(x) \neq 1$, though, it alters the analytic structure of the self-energy, either by introducing new poles (or zeroes), or by changing the behavior at infinity. Thus the trivial continuation is correct [10].

After analytic continuation, eq. (18) shows that while the bare propagator has a massless pole, at non-zero temperature the pole is off the light cone, for $p_0^2 = -m_f^2$ when $\mathbf{p} = 0$. As this mass is small on the scale set by T, $m_f \approx gT$, the approximation of retaining only the term $\sim 1/p_0$, which lead to eq. (17), is justified.

This temperature dependent fermion "mass" was noted in ref. (3) (footnote (13), up to a typographical error in m_f^2) and by Weldon (11). The appearance of this mass is special to non-zero temperatures: although the pole in the renormalized propagator is off the light cone, the renormalized propagator still anti-commutes with γ_5, and so remains chirally symmetric.

In general, $\Sigma_f(p_0, \mathbf{p})$ is gauge variant. For a physical field, however, the position of a pole in the propagator is a measurable and so a gauge invariant quantity. Thus m_f^2, computed above in Feynman gauge, has the same value in any gauge [11].

Having worked through this example, it is obvious generalizing to any loop diagram. Assume that a diagram has I loop integrations and L propagators. From the diagram in momentum space, use eqs. (3) and (12) to write each propagator in time, with L time variables, $\tau_1, \tau_2 \ldots \tau_L$. Doing the I sums generates a series of delta-functions in time, leaving integrals over $\tau_1, \tau_2 \ldots \tau_{L-I}$. These integrals are all over exponentials, and produce $L - I$ energy denominators, as well as the proper statistical factors of the f's and $\tilde{f}$'s, through the identities of eqs. (6) and (11).

This method is a type of non-covariant perturbation theory. When the temperature vanishes, it is awkward to treat a relativistic system non-covariantly, but at non-zero temperature, it is natural to work in the rest frame of a thermal bath.

The part of each diagram at zero temperature is given by the terms where every statistical factor is f_+ or $\tilde{f}_+$, with $f_+ \approx \tilde{f}_+ \approx 1$. From the definition of the f's, if one subtracts from a diagram its value at zero temperature, each term is accompanied by at least one distribution function, $n(\mathbf{p})$ or $\tilde{n}(\mathbf{p})$, as in eq. (16c). Since these distribution functions are Boltzman at high momentum, $\sim exp(-|\mathbf{p}|/T)$, once any diagram at $T \neq 0$ is subtracted at $T = 0$, the remainder is always ultraviolet finite. Due to the exponential suppression of the Boltzman distribution, this applies to non-renormalizable as well as renormalizable theories (assuming that counterterms eliminate all zero temperature divergences in sub-diagrams to $I - 1$ order).

The trick of transforming propagators to τ space is useful only for loop diagrams: for tree diagrams nothing is gained by Fourier transformation. Also, experience shows that if in a loop diagram there are factors of p_0^2, it is best reducing them as

$$\frac{p_0^2}{p^2} = 1 - \frac{\mathbf{p}^2}{p^2} \, .$$

Otherwise, p_0^2 becomes $-\partial^2/\partial\tau^2$; this is fine as it is, but if two time derivatives are integrated by parts, terms from the boundaries, $\tau = \beta$ and $\tau = 0$, enter. These boundary terms can be neglected in integrating only a single derivative by parts, $p_0 \to i\,\partial/\partial\tau$. The difference occurs because time derivatives of a propagator do not have the same periodicity properties as the propagators themselves. In the bosonic case, $\partial\Delta(\tau,\mathbf{p})/\partial\tau = -1/2$ at $\tau = 0$, but $= +1/2$ at $\tau = \beta$.

The discontinuities of diagrams for complex p_0 are determined by the energy denominators that appear in this non-covariant scheme [2,6–8]. These imaginary parts are related to damping times, and measure generally how rapidly a system near equilibrium approaches it [2,12]. With my conventions,

$$p_0 = -i\omega + \epsilon \; , \tag{19}$$

$\epsilon \to 0^+$. In the complex p_0^2 plane, any cut is along the negative real axis. I assume $\omega \geq 0$, so the cut is approached from below. Discontinuities are then given by using

$$2\; Im \, \frac{-1}{ip_0 - E} = 2\pi \; \delta(\omega - E) \; . \tag{20}$$

Consider a general two-point function for an operator Θ,

$$\Sigma_\Theta(p_0,\mathbf{p}) = \langle 0|\Theta\Theta|0\rangle \; ,$$

cut through an intermediate state that contains L particles, the first L_f of which are fermions. This intermediate state has $L-1$ loop integrals, so I choose the momenta $k_1 \ldots k_{L-1}$ to be the loop momenta, with $k_L = p - k_1 - \ldots - k_{L-1}$. Introducing times $\tau_1 \ldots \tau_L$ for each of the L propagators, the sums over $k_1^0 \ldots k_{L-1}^0$ produces $L-1$ delta-functions in time, which set $\tau_1 = \tau_2 = \ldots = \tau_L \equiv \tau$. The remaining τ integral produces one energy denominator, whose discontinuity is, from eq. (20):

$$2\; Im\; \Sigma_\Theta(-i\omega,\mathbf{p}) = \sum_{cuts} \prod_{i=1}^{L-1} \left(\int \frac{d^3\mathbf{k}_i}{(2\pi)^3}\,\frac{1}{2E_i} \right) \frac{1}{2E_L} \prod_{j=1}^{L} \left(\sum_{s_j=\pm} \right) 2\pi\delta(\omega - s_1E_1 \ldots - s_LE_L)$$

$$\vartheta\; \left(\tilde{f}_{s_1}(\mathbf{k}_1) \ldots \tilde{f}_{s_{L_f}}(\mathbf{k}_{L_f}) f_{s_{L_f+1}}(\mathbf{k}_{L_f+1}) \ldots f_{s_L}(\mathbf{k}_L) - \tilde{f}_{-s_1}(\mathbf{k}_1) \ldots f_{-s_L}(\mathbf{k}_L) \right) \; ; \tag{21}$$

$E_j = \sqrt{\mathbf{k}_j^2 + m_j^2}$, *etc.*. ϑ depends upon the external momentum $(-i\omega,\mathbf{p})$ and the

$L-1$ momenta $(-is_iE_i, \mathbf{k}_i)$. If no fermions are cut, ϑ is a product of form factors,

$$\vartheta = \langle 0|\Theta|L\rangle\langle L|\Theta|0\rangle \; . \tag{22}$$

If fermion lines are cut, the residue for each fermion propagator,

$$-s_jE_j\gamma_0 + i\mathbf{k}_j\cdot\gamma + m_j \; ,$$

in included in ϑ. Eq. (21) is similar to the non-relativistic case, as in eq. (4-23) of Kadanoff and Baym [2]; it is also a small extension of Weldon's eq. (A17) [8].

The physical interpretation of eq. (21) is directly analogous to that at zero temperature. As for the real part of a diagram, the result at $T=0$ is obtained by keeping only terms with f_+ and $\tilde{f}_+$, setting $f_+ \approx \tilde{f}_+ \approx 1$. Coleman and Norton [13] showed that at $T=0$, all discontinuities can be viewed as physical processes, in which intermediate particles are on mass shell, propagating forward in time with positive energy. At finite temperature, the only change is that particles can be either emitted into the thermal bath, with positive energy, or absorbed from it, with negative energy. The sum over the s_j's generates all possible processes of emission and absorption, weighted by the appropriate probabilities for a thermal distribution (the f's and $\tilde{f}$'s); see, *e.g.*, the examples of refs. (8) and (9).

In the real-time formalism, besides discontinuities from physical particles, it is also necessary to include contributions from the finite temperature "ghosts" [9]. Kobes and Semenoff [9] showed that the contribution of the "ghosts" cancel when all external legs are physical particles, as they must to agree with eq. (21). The present approach, which starts directly in imaginary time, has no use for these peculiar, finite temperature "ghosts".

As at zero temperature, on very general grounds Σ_Θ satisfies certain properties. If only physical particles contribute to the discontinuity, causality implies that the imaginary part is positive semi-definite,

$$Im\;\Sigma_\Theta(-i\omega, \mathbf{p}) \geq 0 \; . \tag{23}$$

Secondly, assume that Θ has an even number of fermion fields, so Σ_Θ is a bosonic type of self-energy, with ϑ unchanged when all intermediate energies change sign, $s_j \to -s_j$. For negative ω, from eq. (19) the cut along the negative p_0^2 axis is approached from above. Then while the real part of Σ_Θ is even under $\omega \to -\omega$, the imaginary part is odd:

$$Im\,\Sigma_\Theta(i\omega,\mathbf{p}) = -Im\,\Sigma_\Theta(-i\omega,\mathbf{p})\ . \tag{24}$$

A dispersion relation can be written for Σ_Θ:

$$Re\left(\Sigma_\Theta(-i\omega,\mathbf{p}) - \Sigma_\Theta^{T=0}(-i\omega,\mathbf{p})\right) =$$

$$\frac{2}{\pi}\,P\int_0^\infty \frac{Im\left(\Sigma_\Theta(-i\omega',\mathbf{p}) - \Sigma_\Theta^{T=0}(-i\omega',\mathbf{p})\right)}{\omega'-\omega}\,d\omega' \;+\; \Sigma_\Theta^{tadpole}\ , \tag{25}$$

where eq. (24) has been used to write the integral only over positive ω'; P is the principal value prescription. The value of Σ_Θ at $T=0$ is subtracted from both the real and imaginary parts of Σ_Θ, ensuring that the integral over ω' is convergent. Note that it is necessary to allow for the possibility of tadpole contributions, $\Sigma_\Theta^{tadpole}$: these have no dispersive part, but do contribute to the real part of the amplitude, as some constant times powers of T.

It is interesting to ask if the dispersion relation, and positivity of the discontinuity, can be used to determine the sign of the real part of an amplitude. Consider a scalar field ϕ: if Σ_ϕ is its self energy, the renormalized inverse propagator is $\sim p^2+m^2-\Sigma_\phi$. For a thermal bath to screen, the self-energy at zero momentum, subtracted at $T=0$, must be negative:

$$\Sigma_\phi(0,0) - \Sigma_\phi^{T=0}(0,0) \equiv -m_s^2\ , \tag{26}$$

with $m_s^2 \sim +T^2$ a positive mass squared induced by interactions. This seems unlikely from eq. (25): by eq. (23), the total imaginary part is positive, so naively one expects the same for the real part, with m_s^2 negative.

Let ϕ interact with a massless fermion ψ through a Yukawa interaction,

$$L_{int} = \tilde{\kappa}\,\overline{\psi}\psi\,\phi\ . \tag{27}$$

When $\mathbf{p}=0$,

$$Im\,\Sigma_\phi(-i\omega,0) = +\,\tilde{\kappa}^2\,\frac{\omega^2}{8\pi}\left(1-2\tilde{n}(\omega/2)\right)\ . \tag{28}$$

The total discontinuity, $\sim (1-2\tilde{n})$, is positive at all temperatures, and so satisfies eq. (23). What enters into eq. (25), though, is not the total discontinuity, but only the difference between that at $T \neq 0$ and $T = 0$; this is negative, $\sim -2\tilde{n}$. Thus by eq. (25) $\Sigma_\phi(0,0) - \Sigma_\phi^{T=0}(0,0)$ is negative, with m_s^2 positive:

$$m_s^2 = +\frac{\tilde{\kappa}^2 T^2}{6} , \tag{29}$$

from either eqs. (25) and (28), or direct calculation (there is no tadpole).

If scalars contribute to the discontinuity, instead of $\sim 1-2\tilde{n}$, eq. (28), the total imaginary part is $\sim 1+2n$. Then the difference that enters into the right hand side of eq. (25) is positive, $\sim +2n$, contributing to m_s^2 with a negative sign. For any physical model of interacting scalars, however, invariably tadpoles enter, and ensure that at $p_0 = \mathbf{p} = 0$, the right hand side of eq. (25) is negative, with m_s^2 positive.

For instance, in a scalar theory with interaction $\sim \lambda\phi^4$, the self-energy has no imaginary part to one-loop order, with the real part of Σ_ϕ due entirely to the tadpole, $m_s^2 \sim +\lambda T^2$. A non-zero discontinuity first enters at two-loop order, $\sim \lambda^2$: this contributes to m_s^2 with a negative sign, but is always smaller than positive contributions to m_s^2 from the tadpoles. In short, a general proof that temperature screens — *i.e.*, that quantities like m_s^2 are positive — must follow from more than just dispersion relations.

For eq. (23) to hold, only physical states must contribute to the discontinuity. In a gauge theory, this is true for gauge invariant operators, such as

$$\Theta = tr\left(F_{\alpha\beta}F_{\delta\gamma}\right) \ , \ tr\left(\overline{\psi}\psi\right) \ , \tag{30}$$

etc.. For such Θ, the contribution of ghosts and longitudinal degrees of freedom cancel in the discontinuity, by the same arguments as at zero temperature [14].

All gauge invariant operators are composite, so their renormalization requires special treatment. Single insertions of Θ are standard, and are renormalized multiplicatively. For multiple insertions, though, additive renormalization of the source coupled to Θ is usually needed, even in free field theory [15]. At non-zero temperature this can be ignored by subtracting the diagram at zero temperature.

If Θ is a gauge variant operator, the discontinuities of unphysical states need not cancel, so $Im\,\Sigma_\Theta$ can be negative. Consider an $SU(N)$ gauge theory with N_f flavors of massless fermions in the fundamental representation. To one-loop order in Feynman gauge, the self-energy of the gluon is

$$Im\,\Pi^{ij}(-i\omega,0) = +\frac{g^2}{48\pi}\delta^{ij}\omega^2\left(+2N_f\left(1-2\tilde{n}(\omega/2)\right) - 5N\left(1+2n(\omega/2)\right)\right) \; ; \qquad (31)$$

the other components of $\Pi^{\mu\nu}$ vanish at $\mathbf{p}=0$ [3]. In an abelian theory the photon self-energy is gauge invariant. Thus the total contribution of fermions, which to this order is the same as in the abelian theory, is positive, $\sim +N_f(1-2\tilde{n})$. In a non-abelian theory, though, the contribution of the gluons is negative at any temperature, $\sim -N(1+2n)$. This is of no concern, as for arbitrary p_0 and $\mathbf{p}$ the gluon self-energy is gauge variant.

This point has been neglected. Several authors have computed the two-point function of the color electric field [16,17]. In an abelian theory this is a gauge-invariant and so a physical quantity; thus like the term $\sim N_f$ in eq. (31), the imaginary part always has the "right" sign. In an non-abelian theory, though, correlations of the electric field, or $F_{\mu\nu}$, are not gauge invariant. Thus whether the imaginary part of $\langle E^i E^j\rangle$ has one sign in Coulomb or time-like axial gauges [16], or the opposite sign in covariant gauges [17], merely demonstrates that it is gauge variant.

In a non-abelian gauge theory, the only gauge invariant information contained in the propagators themselves are the positions of singularities, and their behavior about them: *e.g.*, the positions of poles and their residues. As seen for the fermion propagator, eqs. (17) and (18), the mass shell is simplest in the static limit: $\mathbf{p}=0$, with p_0^2 small and negative by analytic continuation. For the gluon self-energy,

$$\Pi^{ij}(p_0\approx 0,0) = \delta^{ij}\, m_{gl}^2 \;\;,\;\; m_{gl}^2 = +\left(N+\frac{N_f}{2}\right)\frac{g^2T^2}{9}\; ; \qquad (32)$$

the other components of $\Pi^{\mu\nu}$ vanish at $\mathbf{p}=0$. To one-loop order, the transverse part of the renormalized gluon propagator is

$$\Delta^{ij}(p_0,\mathbf{p}) \approx \left(\delta^{ij} - \frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\right)\frac{1}{p_0^2+m_{gl}^2}\;, \qquad (33)$$

for $T^2 \gg |p_0^2| \gg \mathbf{p}^2$. m_{gl}^2 is the "plasmon" mass (squared) of the gluon, and is one-third the square of the electric screening mass [3]. The gluon propagator is written in Coulomb gauge, where it is evident that the only propagating mode is transverse. This mode is massive, with the position of its pole, at $p_0^2 = -m_{gl}^2$ for $\mathbf{p} = 0$, gauge invariant. This is unlike unphysical degrees of freedom, such as longitudinal modes and Fadeev-Popov ghosts. In covariant gauges, these unphysical modes have massless poles which contribute to gauge variant discontinuities [16,17]; the unphysical modes do not contribute in Coulomb or axial gauges [16].

To summarize: for physical particles, even if they start off massless, a mass is generated by interactions at non-zero temperature [3]. For scalars, fermions, and transverse gauge fields, this is seen from eqs. (29), (19), and (33), respectively. The sole exception is for Goldstone modes [18,19].

At temperatures high enough to restore any broken symmetry, the discontinuities of physical operators are saturated entirely by massive states.[f1] To one-loop order, the structure of an arbitrary discontinuity is typified by scalar fields with a tri-linear interaction,

$$L_{int} = \kappa\, \phi\, \phi_1\, \phi_2\ . \tag{34}$$

From eq. (21), the process $\phi \to \phi_1\, \phi_2 \to \phi$ contributes to the ϕ self-energy as [8, 9]:

$$2\, Im\, \Sigma_\phi(-i\omega, \mathbf{p}) = \kappa^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \frac{2\pi}{2E_1\, 2E_2} \{(1 + n_1 + n_2)\, (\delta(\omega - E_1 - E_2) - \delta(\omega + E_1 + E_2))$$

$$-(n_1 - n_2)\, (\delta(\omega - E_1 + E_2) - \delta(\omega + E_1 - E_2))\}\ ; \tag{35}$$

where $E_1 = \sqrt{\mathbf{k}^2 + m_1^2}$, $n_1 = n(\mathbf{k})$, $E_2 = \sqrt{(\mathbf{p} - \mathbf{k})^2 + m_2^2}$, and $n_2 = n(\mathbf{p} - \mathbf{k})$. I assume that the masses of ϕ_2 and ϕ_1 satisfy $m_2 \geq m_1 \geq 0$.

The first two delta-functions in eq. (35) represent the only discontinuities possible at zero temperature. For positive ω, this is $\omega = E_1 + E_2$.

The other two delta-functions represents channels that are only possible at non-zero temperature, when it is possible to absorb particles from the thermal bath. Depending upon the values of $\mathbf{p}$ and $\mathbf{k}$, for $\omega > 0$ this is possible for either $\omega = E_2 - E_1$ or $\omega = E_1 - E_2$. For example, $\omega = E_2 - E_1$ represents an intermediate state where ϕ scatters off of a ϕ_1 field in the distribution to produce ϕ_2, and then back again.

I start with zero momentum, $\mathbf{p} = 0$. Then $Im\, \Sigma_\phi(-i\omega, 0) \neq 0$ over two regions: $\omega = E_1 + E_2$ is possible if $\omega > m_1 + m_2$, while $\omega = E_2 - E_1$ is allowed for $m_2 - m_1 >$

$\omega > 0$. For instance, as $\omega \to 0$ it is only possible to satisfy $\omega = E_2 - E_1$ if both ϕ_1 and ϕ_2 have large momentum, $|\mathbf{k}| \sim 1/\omega$:

$$Im\,\Sigma_\phi(-i\omega \approx 0, 0) \approx + \frac{\kappa^2}{4\pi} \; \frac{m_2^2 - m_1^2}{\omega T} \; exp\left(- \frac{(m_2^2 - m_1^2)}{2\omega T} \right) . \tag{36}$$

This vanishes exponentially as $\omega \to 0$, since at high momentum the virtual ϕ_1 and ϕ_2 lie on the tails of a Boltzman distribution. As $\omega \to (m_2 - m_1)^-$, $Im\,\Sigma_\phi(-i\omega, 0) \approx (m_2 - m_1 - \omega)^{1/2}$.

Eq. (36) shows that discontinuities at non-zero temperature differ markedly from those at zero temperature. At zero temperature, the propagation of heavy particles is only damped by their decay into lighter ones. At non-zero temperature, it is also possible for light particles to be damped by their coupling to heavier ones, as the light states scatter off of heavy particles in the thermal distribution. Thus eq. (36) is non-zero (if small) even if the masses m_2 and m_1 are much greater than m: all that is required is that $m_2 > m_1$.

I refer to discontinuities that are kinematically forbidden at zero temperature, but which open up for any non-zero temperature, as examples of "Landau damping" [20]. Of course Landau studied a different problem — he considered classical propagation in a plasma, whereas the above is quantum-mechanical — but the principle is the same. By its nature, any discontinuity due to Landau damping is Boltzman as $T \to 0$, $\sim exp(-M/T)$, with M some mass scale typical of the problem. At non-zero temperature, M diverges if the external energy (or momenta) are small, and again the discontinuity is exponentially small: $M \sim 1/\omega$ in eq. (36), which mimics Landau's result, eq. (17) of ref. (5), as well as eq. (A.8′) of ref. (18), *etc.*.

At non-zero momentum, the cut due to $\omega = E_1 + E_2$ contributes if

$$\omega > \omega_+ = \sqrt{\mathbf{p}^2 + (m_1 + m_2)^2} \, , \tag{37}$$

as is familiar from zero temperature.

To understand the region of Landau damping, note that at large $\mathbf{k}$, $\omega = E_2 - E_1 \approx -|\mathbf{p}| cos(\theta)$. Thus the region below the light cone, $|\mathbf{p}| > \omega > 0$, is always Landau damped. If $m_2 > m_1$, a region above the light cone is also damped:

$$\omega_- > \omega > 0 \;\; , \;\; \omega_- = \sqrt{\mathbf{p}^2 + (m_2 - m_1)^2} \, . \tag{38}$$

To determine ω_-, note that in a frame in which all particles are at rest, clearly $\omega_- = m_2 - m_1$. Transformation to a different frame imparts a momentum $\mathbf{p}$ to ϕ, but by Lorentz covariance the value of $p^2 = -\omega_-^2 + \mathbf{p}^2 = -(m_2 - m_1)^2$ is preserved.

Altogether, to one-loop order $Im\,\Sigma_\phi(-i\omega, \mathbf{p}) \neq 0$ for $\omega > \omega_+$ and $\omega_- > \omega > 0$. This discontinuity vanishes as the boundaries of each region are approached. For example, when $\omega \to 0$ $Im\,\Sigma_\phi(-i\omega, \mathbf{p}) \approx exp(-(\mathbf{p}^2 + m_2^2 - m_1^2)/(2\omega T))$, similar to eq. (36).

As seen for ω_-, the boundaries ω_+ and ω_- can be written in a relativistically invariant form, as lines of constant $p^2 = -\omega^2 + \mathbf{p}^2$. At non-zero temperature, however, the entire region over which Landau damping occurs is *not* relativistically invariant. This is simply because for any $\mathbf{p}$, Landau damping occurs all of the way down to $\omega = 0$, which is not a line of constant p^2. This lack of covariance is unremarkable at non-zero temperature, since a thermal bath selects a preferred rest frame.

This analytic structure for $Im\,\Sigma_\phi(-i\omega, \mathbf{p})$ differs from ref. (8): there the complex p^2 plane was used, so ω_+ and ω_- agree, but not that damping occurs down to $\omega = 0$. For $m_1 = m_2 = 0$, the region of quantum Landau damping at one-loop order, eq. (38), coincides with that classically damped in a background field, as determined by Heinz and Siemens [21].

The regions in which damping occurs depend on the values of the masses m_1 and m_2. If one of the virtual particles is a Goldstone boson, $m_1 = 0$, then $Im\,\Sigma_\phi(-i\omega, \mathbf{p}) \neq 0$ over the entire $\omega - |\mathbf{p}|$ plane [18]. When all masses are non-zero, though, to one-loop order there is always an undamped region. If ϕ has a mass m such that $m_2 + m_1 \geq m \geq m_2 - m_1$, it is not damped on mass shell: the ϕ mass shell is $\omega_m = \sqrt{\mathbf{p}^2 + m^2}$, and for m in this range $\omega_+ > \omega_m > \omega_-$ over all $\mathbf{p}$.

In particular, assume that there is only one type of particle, with a tri-linear interaction as in eq. (34); $\phi = \phi_1 = \phi_2$, and $m = m_1 = m_2$. In this instance, only the region below the light cone is Landau damped, with $\omega_- = |\mathbf{p}|$ from eq. (38); from eq. (37), $\omega_+ = \sqrt{\mathbf{p}^2 + 4m^2}$, so $\omega_+ > \omega_m > \omega_-$ for all $\mathbf{p}$. Thus to one-loop order, a massive particle does not damp itself on mass shell.

The existence of an undamped region, $\omega_+ > \omega > \omega_-$, is special to one-loop order, and is removed at two-loop order. To see this, consider the contribution to $Im\,\Sigma_\phi$ at two-loop order from $\phi \to 3\,\phi \to \phi$. If all intermediate states have positive energy, there is a standard cut which starts at $p^2 = -9\,m^2$. At non-zero temperature, however, from eq. (21) Landau damping can occur by absorption of one ϕ, with the other two ϕ's having positive energy:

$$\omega = E_{\mathbf{p}-\mathbf{k}_1-\mathbf{k}_2} + E_{\mathbf{k}_2} - E_{\mathbf{k}_1} \,. \tag{39}$$

There are five other similar channels; all states have mass m. It is possible to show that any region undamped at one-loop order will be damped by processes such as this. To wit: if $m_1 = m_2 = m$, at one-loop order $Im\,\Sigma_\phi(-i\omega, 0) = 0$ until $\omega > 2m$. I concentrate especially on small $\omega \approx 0^+$, $\mathbf{p} = 0$. Let $\mathbf{k}_1$ and $\mathbf{k}_2$ be large, so by eq. (39) $\omega \approx |\mathbf{k}_1 + \mathbf{k}_2| + |\mathbf{k}_2| - |\mathbf{k}_1|$, and consider $|\mathbf{k}_1| > |\mathbf{k}_2|$, with $\mathbf{k}_2$ approximately anti-parallel to $\mathbf{k}_1$. This constitutes a region of non-zero measure in the space of $\mathbf{k}_1$ and $\mathbf{k}_2$, so integrating over it gives $Im\,\Sigma_\phi(-i\omega, 0) \neq 0$ for arbitrarily small values of ω. Other values of ω and $\mathbf{p}$ are reached by integration over different regions of $\mathbf{k}_1$ and $\mathbf{k}_2$ space.

Thus Landau damping implies that at two-loop order, $Im\,\Sigma_\phi(-i\omega, \mathbf{p})$ is non-zero over the *entire* plane of ω and $|\mathbf{p}|$. In the half three space of complex p_0^2 and $\mathbf{p}^2 > 0$, for any $\mathbf{p}^2$ the cut along negative p_0^2 starts at the origin, $p_0^2 = 0$. Consequently, at non-zero temperature the mass shell for any physical particle is a singularity, off the physical sheet, in the midst of a cut.[f2] As commonly stated, there are no stable asymptotic states at $T \neq 0$: what is not often stated is that it is usually necessary to go to two-loop order to see this.

I use these arguments to estimate qualitatively the lifetimes of the quarks and gluons in the high-temperature phase of QCD [16,17,21]. I start with a non-abelian $SU(N)$ gauge theory without quarks, so the physical excitations are the massive, transverse components of the gluon. To one-loop order, the imaginary part of the gluon tensor is given in eq. (29). At small $\omega \approx m_{gl} \approx g\,T$, the Bose-Einstein distribution functions dominate, as $n(\omega/2) \approx T/\omega \approx 1/g$. Suppressing the vector indices,

$$Im\,\Pi \approx g^2\,\omega^2\,n(\omega/2) \approx g\,\omega^2 \,, \tag{40}$$

For $\omega = m_{gl}$, it appears that $Im\,\Pi/Re\,\Pi \approx g$, an estimate first given by Kajantie and Kapusta [16].

This is an overestimate. It follows by including one-loop effects self-consistently on the external legs, so that at $\mathbf{p} = 0$ the pole of the propagator is at $p_0^2 = -m_{gl}^2$ instead of at $p_0^2 = 0$. Yet if renormalized propagators are used on the external legs, they must also be used on the internal legs which are cut. If this is done, by the arguments above the (massive, transverse) gluon does not damp itself, on mass shell, to one-loop order.

The gluon decays on mass shell at two-loop order, through channels as in eq. (39). This can be estimated from eq. (21), assuming that all intermediate momenta are on the order of $\sim m_{gl}$. From the form of eq. (21), if L lines are cut in an intermediate state, at most $L-1$ distribution functions can appear in the imaginary part. Cutting through three gluon lines at two-loop order,

$$Im\,\Pi \approx +g^4\, m_{gl}^2\, n^2(m_{gl}) \approx +g^2\, m_{gl}^2 \,, \tag{41}$$

so that

$$\left.\frac{Im\,\Pi}{Re\,\Pi}\right|_{mass\ shell} \approx +g^2 \,. \tag{42}$$

This estimate applies exclusively to the transverse components of the gluon, at a point on mass shell, such as $p_0^2 = -m_{gl}^2$: only then do the discontinuities of ghosts and longitudinal modes cancel, with the imaginary part of positive sign, eq. (23). I also assume that infrared divergences cancel when all intermediate states are summed over. The cancellation of infrared divergences at non-zero temperature occurs in all known examples [22], but general proofs, as have been developed at zero temperature, are lacking.

While eq. (41) is nominally $\sim g^4$, the Bose-Einstein distribution functions $n(gT) \sim 1/g$ turn the final result into $\sim g^2$. Nevertheless, since eq. (41) requires integration over three-body phase space, the numerical coefficient in eq. (42) is probably small.

Including quarks, I restrict myself to the physically interesting case of $N = 3$ and $N_f = 2$ or 3. For either value of N_f, from eqs. (17) and (32) it can be seen that the masses m_f and m_{gl} satisfy $m_f < m_{gl} < 2\,m_f$. For quark and gluon masses in this range, by the previous example neither the quark or the gluon can decay on mass shell to one-loop order. At two-loop order, the largest contribution to the gluon decay is if it proceeds entirely through gluons, with the lifetime identical to eq. (42). For the quark self-energy, cutting through a quark plus two gluons gives

$$Im\,\Sigma_f \approx +g^4\, m_f\, n^2(m_{gl}) \approx +g^2\, m_f\;, \tag{43}$$

so that from eq. (17),

$$\left.\frac{Im\,\Sigma_f}{Re\,\Sigma_f}\right|_{mass\ shell} \approx +g^2\;. \tag{44}$$

At infinite temperature QCD is an ideal gas of massless quarks and gluons. The coupling g^2 turns on at finite T: to one-loop order, the quarks and gluons develop poles away from the light cone, with m_f and $m_{gl} \sim gT$. These massive states decay at two-loop order, with lifetimes $\sim 1/(g^2 m) \sim 1/(g^3 T)$. Thus at high temperatures, the plasma of quarks and gluons is very nearly ideal, with the physical excitations having small masses and large lifetimes. Numerical experiments [23] will determine whether this picture persists all of the way down to the deconfining transition.

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Footnotes

f1. Unless the system is supersymmetric at zero temperature: then, as shown by Aoyama and Boyanovsky [19], there is a Goldstone fermion at any temperature.

f2. This includes Goldstone modes, whether bosonic or fermionic. On the light cone, although the real part of the self-energy for a Goldstone mode vanishes, at any $\mathbf{p} \neq 0$ its imaginary part is non-zero, even at one-loop order: see, *e.g.*, the appendix of Itoyama and Mueller [18].

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