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A Chiral Symmetry Order Parameter, The Lattice, and Nucleosynthesis

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Abstract

I discuss an order parameter for the chiral symmetry restoration phase transition which may be useful in computations of big bang nucleosynthesis, a phenomenon which requires finite baryon number density. This parameter is strictly speaking an order parameter in the large N limit, and distinguishes between a parity doubled and a massless fermion realization of chiral symmetry restoration. This order parameter may be evaluated at zero net baryon number density at finite temperature, and is useful so long as the baryon chemical potential, μ is much less than the temperature T.

Recent work on the hadronization phase transition in cosmology has shown that if there is a first order chiral transition, then it may be possible that this transition can affect nucleosynthesis.[1-3] A proper treatment of this problem shows that it may be possible to quantitatively explain the abundances of H^2, He^3, and He^4 for a variety of values of Ω, unlike the case for a conventional computation of element abundances. Here Ω is the fraction of matter compared to the amount needed for closure. These element abundances may therefore be quantitatively explained, and the value of Ω may be chosen to be one without recourse to schemes which involve non-baryonic, weakly interacting dark matter. This new description of nucleosynthesis as yet fails however to explain the observed abundance of Li^7.

The basic physics of this description involves the formation of a mixed phase of chiral symmetric quark-gluon plasma, and hadron matter in a first order chiral symmetry breaking phase transition. As pointed out by Witten[4], if the effective mass of baryons in the quark-gluon plasma is small compared to what it is in the hadron gas, then the baryon number concentrates in the region of quark-gluon plasma. Originally it was thought that these regions of quark-gluon plasma might make strange quark matter nuggets,[4-5] but detailed computations have since shown that such nuggets are likely to diffuse away in the subsequent evolution of the universe[6-7]. Nevertheless, large scale density fluctuations may survive until the time of nucleosynthesis,[1-3] and may affect the computation of element abundances.

The degree of generation of density fluctuations in big bang cosmology depends crucially on the relative abundance of baryon number in the quark-gluon plasma compared to that in the hadron gas at baryon number chemical potential μ small compared to temperature T, $\mu/T \sim 10^{-9}$. If we define the net baryon number density to be ρ_B^{CS} in the chiral symmetric phase and ρ_B^{CB} in the symmetry broken phase, then the quantity of interest is

$$r = \rho_B^{CS}/\rho_B^{CB} \tag{1}$$

Although the numerator and denominator of this expression both depend upon μ, the ratio r is finite in the limit μ approaches zero.

We can understand the physics of the parameter r using the example of an ideal gas of hadrons and of quark-gluon plasma for one flavor of quark. In an ideal quark-gluon plasma, where the fermions have effective masses which are small compared to the temperature

$$\rho_B^{CS} = \frac{4}{\pi^2}\mu T^2 \tag{2}$$

(We should be careful to note that even in the chiral symmetric phase of a quark-gluon plasma at high temperatures, in perturbation theory quarks acquire a mass $m \sim gT$.[8] This can occur and be consistent with the vanishing of $\overline{\Psi}\Psi$ because at finite T the fermion propagator does not have a Lorentz invariant form.) In the hadron gas phase, we have

$$\rho_B^{CB} = \frac{1}{2\pi^{3/2}}(2mT)^{3/2}\frac{\mu}{T}e^{-\beta m} \tag{3}$$

For example, if we take $T \sim 150 Mev$, and $m \sim 1 Gev$, then $r \sim 10^2$

This example shows that r may be a useful parameter for distinguishing between the chiral restored and broken phases. We can see this most simply in the large number of colors, N_c, limit. In this limit, baryons acquire a mass proportional to N_c since baryons contain N_c quarks. In an assymptotically free gas of quarks and gluons, the quarks have small masses. Therefore in the large N_c limit, we might expect that

$$r \sim e^{N_c} \tag{4}$$

This large N_c example shows that the order parameter r is most properly thought of as an order parameter for the confinement-deconfinement phase transition present in large N_c. The difference between this order parameter and the Wilson line or Polyakov loop order parameter is essentially that this order parameter exists for light mass quarks avoiding the artifact of introducing heavy static test quarks. Of course the Wilson or Polyakov loop may be introduced into theories with or without dynamical low mass quarks, but the order parameter r is nice since its physics is that arising from the reaction of light mass quarks. It also has obvious implications for probing the nature of chiral symmetry restoration, and has a simple physical interpretation when chiral symmetry is restored at finite N_c.

The result of the previous equation can be evaded by several obvious mechanisms. The first such mechanism requires a second order transition in the large N_c limit. In this mechanism, the nucleon masses vanish at the second order transition in the hadron gas phase. In this case, $r = 1$, but if we considered the ratio of baryon number density just above the transition temperature to the baryon number density some finite temperature below, then the ratio would go as e^{N_c}. The ratio of these baryon number densities would be very rapidly varying near the transition temperature. The physical application we have in mind for this order parameter exists only for first order phase transitions. In finite N_c QCD for small mass quarks it is believed that this transition is first order,[9], as also seem to be the case in the large N_c limit. We shall therefore not consider this possibility.

Another possibility is that there is a chiral transition at a temperature below the deconfinement transition temperature. In this case, the chiral symmetric phase may be composed of a gas of parity doubled nucleons, each nucleon with N_c quarks in them, and of possibly some massless non-parity doubled nucleons. In this case $r \sim 1$ if there are no zero mass baryons in the chirally symmetric phase, since we expect that the parity doubled nucleon masses are of order N_c, if the mass shift of the nucleons is continuous across the phase transition. If the nucleons in fact suffer a finite mass shift and the masses decrease in the chirally symmetric phase, or if there are massless baryons in the chirally symmetric phase, then $r \sim exp(N_c)$. In large N_c, we however expect that the chiral symmetry and confinement phase transitions should occur at the same temperature.[10]

It is also possible that the deconfinement temperature could occur below the chiral transition temperature, in which case we would expect that there would be a transition between a phase of massless quarks and massive ones. If the chiral transition were first order, we would expect $r \sim 1$ since the order parameter involves only unconfined quarks, and should roughly be given by an expression of order that in Eq. 2. Put another way, in the chiral transition within a deconfined phase, there is no requirement that the quarks be lumped together in baryons and therefore no need for a singular order parameter.

The example of large N_c shows that the parameter r may be useful to disentangle the two possible realizations of chiral symmetry breaking, either massive parity doubled states, or massless fermions. It also appears that the most likely possibility for r to be large across the phase transition is when the confinement-deconfinement phase transtion is at the same temperature as that of the chiral transition, or if the chiral symmetry restoration temperature is below that of deconfinement. In this case if the transition is first order, then $r \sim e^{N_c}$. For the example of the nucleon gas compared to a quark-gluon plasma, this result seems quite quantitatively plausible. At finite N_c, the concept of a confinement-deconfinement phase transition in the presence of fermions becomes a bit cloudy. Here what we presumably mean is that there is a good deal of delocalization or lack of clustering of quarks into localized color singlet states across a first order chiral symmetry restoration phase transition.

The difficulty about quantitatively estimating r is that it seems necessary to compute in a region where perturbation theory has broken down, and naive estimates are obviously invalid. We might try to compute the baryon number on the lattice, but at first sight this seems dificult since there is as yet no fully consistent scheme for including finite baryon number density in lattice Monte-Carlo simulations.

We should notice however that we only need compute the baryon number density in the limit of $\mu << T$. In this limit, we expect analyticity of the baryon number density for small μ, and if so, together with the fact that the baryon number density is an odd function of μ, these conditions require that

$$\rho_B = \mu F(T) \tag{5}$$

where $F(T)$ is some arbitrary function of T. If this is true then

$$\rho_B = \mu(\partial/\partial\mu\ \rho_B)\ |_{\mu=0} \equiv \mu\kappa_B \tag{6}$$

We shall return to the issue of analyticity in the following paragraphs, which is essentially the issue of the finiteness of the above expression.

To obtain an expression for $(\partial/\partial\mu\ \rho_B)\ |_{\mu=0}$, we use that

$$\rho_B = \frac{Tr\ \rho_B\ exp(-\beta H + \beta\mu N)}{Tr\ exp(-\beta H + \beta\mu N)} \tag{7}$$

Therefore, we obtain

$$\kappa_B = \beta \int d^3x\ < \rho_B(x)\rho_B(0) > \tag{8}$$

where the expectation of the baryon number density correlation function is to be evaluated at zero baryon number chemical potential. The baryon charge operators are to be evaluated at equal time. This quantity is a fermion four point function evaluated at finite temperature, and can be computed by standard Monte-Carlo methods. Notice that the value of r is computed as the ratio of κ's.

A representation for κ_B which is useful for studying its analyticity properties is

$$\kappa_B = \beta \; lim_{\vec{k}\to 0} \sum_{k^o} \Pi^{oo}(k^o, \vec{k}) \tag{9}$$

In this expression, $\Pi^{\mu\nu}$ is the baryonic current polarization tensor. Using the transversality properties of $\Pi^{\mu\nu}$, we see that the zero temperature contribution to κ_B vanishes as it must, since at zero temperature

$$\Pi^{\mu\nu}(k) = (k^2 g^{\mu\nu} - k^\mu k^\nu)\Pi(k^2) \tag{10}$$

Taking the time-time components and $\vec{k} \to 0$, we get zero.

At finite temperature, the tensor decomposition of $\Pi\mu\nu$ is more complicated. Nevertheless, perturbative studies have shown that as $\vec{k} \to 0$, $\Pi^{\mu\nu}$ is finite for finite k^o, and as $k^o \to \infty$, $\Pi^{\mu\nu}$ rapidly approaches its value in the vacuum. At zero k^o, we might expect an infrared singularity. For fermion loop insertions nevertheless, there is no such infrared singularity since the fermion propagators always carry at least a time like momentum of order T. In general, we expect that there will be no infrared singularity in the baryon number correlation function integrated over d^3x. At large distances, we expect that the equal time correlation function is damped since there are no long range forces which couple to baryon number. We therefore expect that κ_B is a well defined function of temperature, and our analyticity conditions are satisfied.

Finally, I show how the expression for κ_B in terms of $\Pi^{\mu\nu}$ gives the correct result to one loop in perturbation theory. To verify this to one loop, we write

$$\sum_{k^o} \Pi^{oo}(k^o, \vec{k}) = \sum_{k^o, l^o} \int \frac{d^3 l}{(2\pi)^3} \frac{tr(\gamma^o(m - l\cdot\gamma)\gamma^o(m-(k-l)\cdot\gamma)}{(l^2+m^2)((k-l)^2+m^2)} \tag{11}$$

Explicitly subtrcting the vacuum contribution to Π, and replacing the summation over Matsubarra frequencies by Sommerfield-Watson contour integrals, and taking the limit $\vec{k} \to 0$, shows that after a good deal of algebra

$$lim_{\vec{k}\to 0}\Pi^{oo}(k^0,\vec{k}) = 4\int \frac{d^3l}{(2\pi)^3}\frac{e^{\beta E}}{(e^{\beta E}+1)^2} \tag{12}$$

This is precisely the result one can derive for κ_B by directly writing out the expression for baryon number density and expanding to first order in μ. The expression for κ_B therefore adequately reproduces first order pertubation theory.

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