Fermi National Accelerator Laboratory

FERMILAB-Pub-87/126-A
July 1987

# GALAXY AND STRUCTURE FORMATION WITH HOT DARK MATTER AND COSMIC STRINGS

R Brandenberger[1), N Kaiser[2), D Schramm[3) and N Turok[4)

1) DAMTP, University of Cambridge, Cambridge CB3 9EW. UK.

2) Institute of Astronomy, University of Cambridge, Cambridge CB3 OHA. UK.

3) University of Chicago, Chicago, IL 60637 U.S.A., and NASA/Fermilab Theoretical Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510, U.S.A.

4) Blackett Laboratory, Imperial College, London, SW7 2BZ. UK.

## Abstract

Galaxy and structure formation in a neutrino dominated universe with cosmic strings are investigated. Unlike in the usual adiabatic scenario strings survive neutrino free streaming to seed galaxies and clusters. The effective maximum Jeans mass is about $1.5 \times 10^{14} h_{50}^{-4} M_{\odot}$, much lower than in the adiabatic scenario. Hence cluster formation is only marginally different than in the cold dark matter (CDM) and strings model. $G\mu$ is slightly larger. Galaxy masses are lower than with strings and CDM. The mass spectrum of galaxies is flatter than with CDM, and the density profile about an individual loop is less steep, in better agreement with observations.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

## Introduction

The standard nucleosynthesis scenario[1] constrains the energy density in baryons to be $\Omega_B \leq 0.15$, but theoretical prejudice insists that the total energy density is $\Omega = 1$. The remainder of the density of the universe must then be non-baryonic. It is called hot if the dark particles have relativistic peculiar velocities at the time $t_{eq}$ of equal matter and radiation and cold if they do not. Massive neutrinos are the best motivated candidate of either kind, and a (hot) tau neutrino with mass $m_\nu$ = 30eV would be consistent with all existing constraints.

Models with linear adiabatic density perturbations and hot dark matter (HDM) are, however, hard to reconcile with observations. In these models perturbations on mass scales smaller than $10^{15} M_\odot$ are wiped out by the free streaming of neutrinos just before $t_{eq}$ [2,3]. Hence galaxies can only form by fragmentation of larger objects. In order to explain observations of quasars at redshifts of about 4, large scale nonlinear structures had to form early, requiring a large amplitude of the primordial perturbations.

With cosmic strings as the source of density perturbations the situation is quite different. The strings survive neutrino free streaming and can seed small scale structure, albeit less efficiently than with cold dark matter.

We emphasize that the essentials of the scenario are the same as for strings and CDM. Loops with the mean separation of galaxies are to be identified with galaxies and similarly for clusters[4,5]. Thus, the scale-free correlation function predicted with strings on large scales[6] is unaffected. What is different is the efficiency of accretion. Small loops accrete less and the mass spectrum of objects $n(M)$ is less steep. The density profile about an individual loop is less steep than with CDM.

Galaxy cores would be primarily baryonic, neutrinos being prevented from clustering on small scales by phase space constraints[7]. Galaxies will have formed recently and thus there will be significant evolution at low redshifts unlike in the CDM scenario.

## Neutrino Accretion

Neutrino accretion may be understood heuristically as follows. The accretion timescale is $t_H$, the Hubble time during which the neutrinos move a distance $\lambda_\nu = v_\nu t_H$, where $v_\nu$ is their velocity. On scales below $\lambda_\nu$ perturbations are washed out but on larger scales they grow in the usual way. Neutrinos decouple while they are relativistic, well before $t_{eq}$, and go nonrelativistic soon before $t_{eq}$ after which their velocity decays as $a^{-1}(t)$ ($a(t)$ is the scale factor). The comoving scale $\lambda_\nu a^{-1}$ (the Jeans length) thus increases as $a(t)$ in the radiation dominated era while the neutrinos are relativistic, is approximately constant (reaching a maximum) as they go nonrelativistic, and then decreases as $a^{-1/2}$ in the matter-dominated era. At $t_{eq}$ the r.m.s. neutrino velocity is $v_\nu \sim 0.17$ and $t_H \sim 50h_{50}^{-2}$Mpc where $h_{50}$ is the Hubble parameter in units of $50\text{kms}^{-1}\text{Mpc}^{-1}$. Therefore, at $t_{eq}$ $\lambda_\nu(t_{eq}) = \lambda_{\nu,eq} \sim 8h_{50}^{-2}$Mpc. In the string model, perturbations on all comoving scales larger than $\lambda_{\nu,eq}$ start growing at $t_{eq}$ whereas scales $\lambda < \lambda_{\nu,eq}$ must wait until $\lambda_{\nu,eq}\left(\frac{a}{a_{eq}}\right)^{-1/2} \sim \lambda$ before growth starts.

Now we proceed to a more precise treatment. Since the neutrinos interact very weakly their phase space density $f(\underline{r},\underline{p})$ is conserved[3]. Liouville's equation in physical coordinates and in the Newtonian approximation reads

$$\frac{\partial f}{\partial t} + \dot{\underline{r}}.\underline{\nabla} f + \dot{\underline{p}}.\nabla_{\underline{p}} f = \frac{\partial f}{\partial t} + \frac{\underline{p}}{m}.\underline{\nabla} f - m\nabla\phi.\nabla_{\underline{p}} f = 0 \qquad (1)$$

where $\phi$ is the Newtonian potential. The neutrino density is

$\rho(\underline{r}) = (2\pi)^{-3}\int d^3p f(\underline{r},\underline{p}) m_\nu$. In an expanding universe. transforming to comoving coordinates $\underline{x} = \underline{r}/a$ and $\underline{q} = a\underline{p} - m\dot{a}\underline{x}$ one finds the unperturbed solution $f_0(\underline{x},\underline{q}) = 2\left[e^{q/T_\nu a} + 1\right]^{-1}$ for neutrinos and antineutrinos - they decouple with a relativistic distribution and this is preserved thereafter.

Our strategy is as follows[8]. We linearize (1) and Fourier transform in $\underline{x}$ ($\underline{k}$ shall denote the conjugate momenta). Each Fourier mode will evolve independently and has a source term given by the Fourier transform of the source density $\rho_s$. For simplicity we approximate a loop of mass $M_l$ by a point mass. Then the Fourier transform is $\delta\tilde{\rho}(\underline{k}) = M_l$. (1) can easily be converted into an integral equation. After evolving the modes we Fourier transform back to find the density profile of neutrinos accreted around the point mass.

Choosing $a(t_{eq}) = 1$ then in terms of a new time variable $z = \frac{1}{2}\left[\sqrt{a+1} - 1\right]$ the resulting integral equation for $\delta_\nu(k) = \delta\rho_\nu(k)/\rho_\nu$ is[9]

$$\delta_\nu(k,z) = 6\int_{z_0}^{z} dz'\left[\delta_\nu(k,z') + \frac{M_l}{\rho_{\nu,eq}}\right]\frac{F(z,z')}{\left[1+\alpha^2F^2(z,z')\right]^2} \qquad (2)$$

with

$$F(z,z') = \ln\left[1 + \frac{1}{z'}\right] - \ln\left[1 + \frac{1}{z}\right] \qquad (3)$$

Here $\alpha = kv_0\tau_*$ with $v_0 = \frac{T_{\nu,eq}}{m_\nu} \sim 0.05$ being a measure of the neutrino velocity at $t_{eq}$ and $\tau_* = \left[8\pi G\rho_{\nu,eq}/3\right]^{-1/2} = 2^{1/2}t_{H,eq}$. $\rho_{\nu,eq}$ is the energy density in neutrinos at $t_{eq}$.

in comoving units $v_0\tau_* = 3.5h_{50}^{-2}$Mpc. Modes with $k \gg \left[v_0\tau_*\right]^{-1}$ are suppressed. (2) is valid right through the radiation-matter transition. $z_0$ is the time when accretion begins. For $k = 0$ (2) yields the usual equation for the growth of perturbations in CDM[10].

We have solved (2) numerically for $\delta_\nu(k,z)$. For large enough $z$ each $k$ mode grows as $z^2 \sim a(t)$. Thus $C(k) = \left[\rho_{\nu,eq}/M_1\right]\delta_\nu(k,z)z^{-2}$ tends to a constant. In Figure 1 we plot $C(k)$ versus $\alpha(k)$ for two different values of $z_0$. An analytical fit which is good for $0 \leq \alpha \leq 100$ is $C(k) = A/(B+\alpha^2(k))$. For $z_0 = 0.01$ (i.e. initial scale factor $a_0 = 0.04$) the constants A and B are $A = 1.1$ and $B = 1.1/6$. For $z_0 < 0.01$ there is no significant change. For $z_0 = 0.1$ (i.e. $a_0 = 0.44$) some growth is lost and $A = 1.1$ with $B = 0.4$.

The mass profile of the accreted neutrinos can be calculated analytically. For $a \gg 1$ the mass inside a comoving radius $x$ is

$$\delta M(<x) = \int_0^x d^3x\,\delta\rho_\nu(x)$$

$$= M_1\,\frac{a}{4}\int_0^x d^3\underline{x}(2\pi)^{-3}\int d^3\underline{k}\,e^{i\underline{k}\cdot\underline{x}}C(k) \tag{4}$$

$$= \alpha M_1 a\left[1 - \left[1 + \frac{x}{L}\right]e^{-x/L}\right]$$

where $\alpha = \frac{A}{4B} = \frac{3}{2}$ for $z_0 \ll 1$ and $\alpha = 0.7$ for $z_0 = 0.1$, and $L = v_0\tau_*/\sqrt{B} \sim 8.4h_{50}^{-2}$Mpc for $z_0 \ll 1$ and $L \sim 5.6h_{50}^{-2}$Mpc for $z_0 = 0.1$. The second term in (4) is the growth suppression due to neutrino free streaming. For $x \gg L$ there is very little suppression, but for $x \ll L$ $\delta M_{<x} \sim x^2$, quite different from CDM. Our answer for L agrees well with the naive estimate and gives the effective maximal Jeans mass $M_J$ (mass inside a ball of radius L) quoted in the abstract.

Baryon Accretion and Loop Decay

Accretion of baryons begins on all scales after baryons decouple from radiation at a redshift of $z_{rec} \sim 1.5.10^3$. This makes little difference on scales $\lambda > L$ since neutrinos are already clustering and the baryons will just track them. However, on small scales neutrino perturbations have not started growing and baryonic clustering is important. The equation governing the fractional density enhancement in baryons $\delta = \delta\rho_B/\rho_B$ in the matter dominated era is[10)]

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2\Omega_B}{3t^2}\delta = 4\pi G\delta\rho_s \qquad (5)$$

where $\delta\rho_s$ is the source perturbation. For a point mass $\delta\rho_s = M_l(t)a^{-3}(t)\delta^3(\underline{x})$, and taking into account the decay via gravitational radiation $M_l(t) \sim M_l(1-\frac{t}{t_d})$ where $M_l$ is the initial mass and $t_d$ is the decay time. (5) can be solved to give (for $\Omega_B$=1/8) the accreted baryonic mass $\delta M_B(a)$

$$\delta M_B(a) = M_l\left[\left[\frac{3}{4} + \frac{3}{20}\left(\frac{a_{rec}}{a_d}\right)^{3/2}\right]\left(\frac{a}{a_{rec}}\right)^{1/4} + \left[\frac{1}{4} - \frac{1}{12}\left(\frac{a_{rec}}{a_d}\right)^{3/2}\right]\left(\frac{a_{rec}}{a}\right)^{3/4} - 1 - \frac{1}{15}\left(\frac{a}{a_d}\right)^{3/2}\right] \quad \text{if } a < a_d$$

$$= M_l\left[\left[\frac{3}{4} + \frac{3}{20}\left(\frac{a_{rec}}{a_d}\right)^{3/2} - \frac{9}{10}\left(\frac{a_{rec}}{a_d}\right)^{1/4}\right]\left(\frac{a}{a_{rec}}\right)^{1/4} + \left[\frac{1}{4} - \frac{1}{12}\left(\frac{a_{rec}}{a_d}\right)^{3/2} - \frac{1}{6}\left(\frac{a_d}{a_{rec}}\right)^{3/4}\right]\left(\frac{a_{rec}}{a}\right)^{3/2}\right] \quad \text{if } a > a_d \qquad (6)$$

happened when $a(t) \sim a_J(x) = \left[\frac{L}{x}\right]^2$.

Loop decay also affects neutrino growth. We have integrated (2) numerically in the matter dominated era with the source mass varying as above. The results are shown in Figure 2 where the ratio $f_\nu$ of the growth factors with and without loop decay is plotted as a function of $z_d/z_J = \left[a_d/a_J\right]^{1/2}$.

Now we calculate the density profile taking loop decay and baryon accretion into account. We can write in a phenomenological manner

$$\delta M(<x) = M_{seed}(a_J(x))\left[\frac{a}{a_J(x)}\right] \tag{7}$$

The seed mass at $a_J(x)$ is the sum of the neutrino mass at $a_J(x)$ in the absence of baryons - this equals $f_\nu M_1$ - and the mass in baryons at $a_J(x)$ - we denote this by $f_B M_1$, where $f_B$ is given by (6). Hence from (4) and (7)

$$\frac{\delta M(<x)}{M} \sim \frac{3}{4}\frac{M_1 a}{M_J}\frac{L}{x}\left[f_B + f_\nu\right](x) \tag{8}$$

## Consequences

Now we turn to the consequences of the above calculations. We normalize $G\mu$ ($\mu$ is the mass per unit length in string, G is Newton's constant) by demanding that loops with a mean separation of Abell clusters have accreted an Abell cluster mass around them. Since $\delta M/M \sim 130$ inside an Abell radius[11] the comoving scale corresponding to the Abell radius $3h_{50}^{-1}$Mpc is $(130)^{1/3}3h_{50}^{-1}$Mpc $\sim 15h_{50}^{-1}$Mpc, much larger than the maximal effective Jeans length L. Thus, only a very small growth factor is lost compared to the cosmic string scenario with CDM.

However galaxies are much smaller than with CDM. For galaxy loops we find $a_d/a_{rec} \sim 10$ and hence $f_B + f_\nu \sim 0.75$ on scales $x \sim 1$Mpc corresponding to masses of $\sim 3.10^{11} M_O$. $f_B + f_\nu$ is very weakly dependent on x, although $f_B \gg f_\nu$ at small x and vice versa at large x.

The galaxy loop mass is given in terms of the cluster loop mass $M_{cloop}$, fixed by the mass $M_c$ of a cluster :

$$M_{cloop} = \frac{\xi}{5}\left(\frac{\bar{\rho}}{\rho_b}\right)^{1/3} \frac{M_c}{1+z_{eq}} . \tag{9}$$

Here $\left(\frac{\bar{\rho}}{\rho_b}\right) \sim 130\left(\frac{\sigma_c}{700}\right)^2$ is the overdensity in a cluster today and $\sigma_c$ is the one dimensional velocity dispersion in $\mathrm{kms}^{-1}$. The factor 5 comes from matching the growth through the matter-radiation transition to that of a spherical collapse model[12], and $\bar{\rho}/\rho_b$ enters because from this one can tell the redshift at turnaround[5]. $\xi$ is a factor representing the loss in growth due to a loop being formed near $z_{eq}$. If a loop is formed exactly at $z_{eq}$ then $\xi \sim 4$. The mass of a galaxy loop is $M_{gloop} = M_{cloop}(d_g/d_c)^2$ where $d_g(d_c)$ is the mean separation of galaxies (clusters).

In the spherical collapse model a shell which collapses reaches its greatest density when $\delta\rho/\rho$ calculated in linear theory reaches 1.58. We shall use this to define, through (8), the total nonlinear mass accreted by a loop. Using $d_g/d_c = 1/11$ and $f_B + f_\nu = 0.75$ we find

$$M_{galaxy} \sim 5.10^{10} M_O\, h_{50}^5 \left(\frac{\sigma_c}{700}\right)^8 \xi_4^3 \left(\frac{11 d_g}{d_c}\right)^6 \tag{10}$$

where $\xi_4$ is $\xi$ in units of 4. This corresponds to a rotation velocity for the shell just collapsed of

$$v_{rot} = \sqrt{3}\sigma_g \sim \sqrt{3}\left[\frac{M_{galaxy}}{M_c}\right]^{1/3} \sigma_c \sim 50\mathrm{kms}^{-1}\left[\frac{\sigma_c}{700}\right]^2 h_{50}^2 \xi_4 \left[\frac{11 d_g}{d_c}\right]^2 \quad (11)$$

using $M_c = 10^{15} M_\odot h_{50}^{-1}\left[\frac{\sigma_c}{700}\right]^2 = 3\sigma^2 R/G$ where $R = 3h_{50}^{-1}$ Mpc is the Abell radius. If $\Omega_B = 1/8$ and the baryons contract by a factor of 8 then the optical rotation velocity would be similar.

Our galaxies therefore look rather small, but the results include considerable observational uncertainties. If $\sigma_c = 1000$ then $M_{galaxy} \sim 10^{12} M_\odot$. Increasing $\xi$ and $h_{50}$ further increases the result. This increases the string tension required, since[5,12]

$$G\mu = 2.10^{-6} h_{50}^{-1} \beta_{10}^{-1} \nu_{.01}^{-2/3} \xi_4 \left[\frac{\sigma_c}{700}\right]^{8/3} \quad (12)$$

$\beta_{10}$ and $\nu_{.01}$ give the values of the string parameters in units of their "standard" values[5]. Increasing $G\mu$ in turn boosts the magnitude of the expected streaming velocity[13].

With hot dark matter galaxies look very different than with CDM. Phase space arguments[7] show that 30eV neutrinos cannot cluster on scales smaller than about 10Kpc. Hence the inner regions of galaxies would be almost entirely baryonic. The halo would be comprised of neutrinos. The density profile for hot dark matter is $\rho(r) \sim r^{-2}$ which gives a flat halo rotation curve. This result follows from the analysis of Fillmore and Goldreich[14] which shows that an initial spherical perturbation with $\delta M/M \sim r^{-\gamma}$ with $\gamma < 2$ always collapses to give flat rotation curves.

The mass function of objects expected with HDM is also different than with CDM. From (8) we see that to a first approximation the nonlinear mass M scales as $x^3 \sim M_1^3$, since $f_B$ depends only very weakly on the decay time. Let $n(M)dM$ denote the number density of objects with masses in the range $[M, M + dM]$. For strings and CDM $n(M) \sim M^{-5/2}$ on the scale of galaxies which is uncomfortably steep[15]. For HDM we find using $n(M_1)dM_1 \sim M_1^{-5/2}dM_1$ that $n(M) \sim M^{-3/2}$, in better agreement with the Schechter luminosity function. This is valid for masses $M \gg M_{cu}$, where $M_{cu}$ is given by the mass accreted by a loop which decays at $t_{rec}$: $M_{cu} \sim 4.10^{-4} M_{galaxy}$. Objects with $M < M_{cu}$ are seeded by loops which decay at $t_d(R) \sim t_{rec}$. $t_d(R)$ is given by $t_d(R) = (\gamma G\mu)^{-1}R$ with $\gamma \sim 5$ [16]. The mass accreted by such a loop can be determined from (6) by expanding in $R - (\gamma G\mu)t_{rec}$. We find that in the limit $M \to 0$ $n(M) \sim M^{-1/2}$. For clusters $(M > M_J)$ we have the same $n(M)$ as with CDM, which has been shown to fit the data rather well[17].

We conclude that the cosmic string theory with HDM is a viable cosmological model which deserves further study. There are testable differences compared to a model with CDM. Flat halo rotation curves, a characteristic mass function and smaller galaxy masses are the main predictions.

After completing this work we received a preprint by Bertschinger and Watts reporting on similar calculations[18]. Neutrino clustering in a more general context has also recently been considered in Ref. 19.


## Acknowledgements

We wish to thank R. Durrer, G. Efstathiou, P. J. E. Peebles, M. Rees, N. Straumann and in particular J. R. Bond for helpful discussions. DNS was supported in part by the NSF at U. of Chicago and by NASA at FNAL.

References


1. J Yang, M Turner, G Steigman, D Schramm and K Olive, Ap. J. 281, 493 (1984).

2. J Bond, G Efstathiou and J Silk, Phys. Rev. Lett. 45, 1980 (1980); G Bisnovatyi-Kogan and I Novikov, Sov. Astron. 24, 516 (1980).

3. J Bond and A Szalay, Ap. J. 274, 443 (1984).

4. Ya Zel'dovich, Mon. Not. R. Astron. Soc. 192, 663 (1980); A Vilenkin, Phys. Rev. Lett. 46, 1169 (1981).

5. N Turok and R Brandenberger, Phys. Rev. D33, 2075 (1986).

6. N Turok, Phys. Rev. Lett. 55, 1801 (1985).

7. S Tremaine and J Gunn, Phys. Rev. Lett. 42, 407 (1979).

8. R Brandenberger, N Kaiser and N Turok, DAMTP preprint (1987).

9. In deriving (3) we approximated $x^2/(e^x+1)$ with $nx^2e^{-x}$, n is a constant chosen to obtain the correct normalization (see Ref. 8).

10. P J E Peebles, "The large-scale structure of the universe" (Princeton Univ. Press, Princeton 1980).

11. P J E Peebles, in "Physical cosmology", 1979 Les Houches lectures, ed. by R Balian, J Audouze and D Schramm (North-Holland, Amsterdam, 1980).

12. N Turok, in proceedings of the First CERN-ESO School on Particle Physics and Cosmology, Erice Jan 6-25 1987, Imperial College preprint (1987).

13. E P S Shellard, R Brandenberger, N Kaiser and N Turok, Nature 326, 672 (1987);
R Brandenberger, N Kaiser, E P S Shellard and N Turok, Phys. Rev. D, July 15 1987.

14. J Fillmore and P Goldreich, Ap. J. 281, 1 (1984).

15. P J E Peebles, private communication.

16. N Turok, Nucl. Phys. B242, 520 (1984);
T Vachaspati and A Vilenkin, Phys. Rev. D31, 3052 (1985).

17. W Zurek, "Richness distribution of string seeded Abell clusters", Los Alamos preprint (1987).

18. E Bertschinger and P Watts, MIT preprint (1987).

19. R Cowsik and P Ghosh, Ap. J. 317, 26 (1987);
R Paganini, N Straumann and D Wyler, Astron. Astrophys. 177, 84 (1987).

Figure Captions

Figure 1 The net growth of the neutrino perturbation at late times t. $\alpha$ is the wave number in units of $(v_0 \tau_*)^{-1}$ . In particular. $\alpha = 0$ corresponds to CDM growth. $\delta_\nu(\alpha)$ is obtained from C by multiplying by the seed perturbation $M_1/\rho_{\nu,eq}$ and by the scale factor $a(t)/a(t_{eq})$. The solid curve represents the results starting with zero perturbations at a redshift $a(t) \ll a(t_{eq})$, the dashed line starting at $a(t)/a(t_{eq}) = 0.44$.

Figure 2 Loss in growth due to loop decay for galaxy loops. $f_\nu$ is the ratio of the density perturbation with and without loop decay as a function of wave length $\lambda$. $a_J$ is the scale factor when $\lambda$ equals the Jeans length. $a_d$ when the loop decays.

Fig. 1

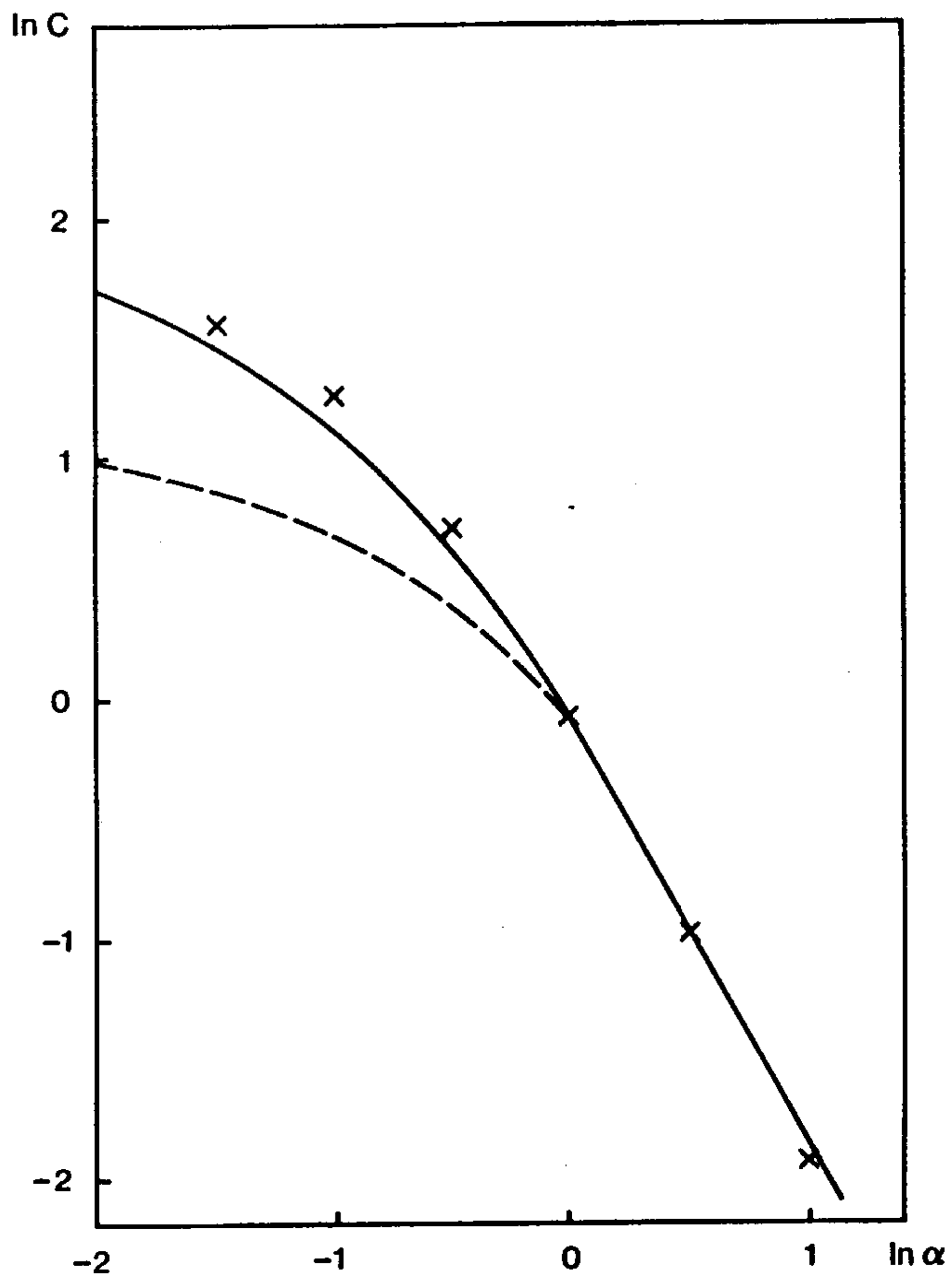

Fig. 2