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CAN BULK VISCOSITY DRIVE INFLATION ?

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Abstract

Contrary to other claims, we argue that, bulk viscosity associated with the interactions of nonrelativistic particles with relativistic particles around the time of the grand unified theory (GUT) phase transition cannot lead to inflation. Simply put, the key ingredient for inflation, negative pressure, cannot arise due to the bulk viscosity effects of a weakly-interacting mixture of relativistic and nonrelativistic particles.

Since the original proposal of Guth[1] and the crucial modifications of Linde[2] and Albrecht and Steinhardt[3] various models have been suggested to implement slow-rollover inflation. Inflationary models based upon GUT symmetry breaking, SUSY symmetry breaking, induced gravity, R^2 theories of gravity, compactification of extra dimensions, and 'random', weakly-coupled scalar field exist. This has led some to elevate inflation to an early Universe paradigm. [For a detailed and up to date review of inflation we refer the reader to ref. 4.]

Several authors have raised the possibility that bulk viscosity can also be the driving force of the accelerated expansion associated with inflation[5-7] The proposal relies on the observation that the effect of bulk viscosity in an expanding Universe is to decrease the value of the pressure. Since inflation requires an equation of state where $p = -\rho$ one can ask whether or not irreversible processes can be strong enough to drive the Universe into an inflationary stage. These authors[5-7] have suggested that bulk viscosity arising around the time of a grand unified theory (GUT) phase transition can indeed lead to negative pressure, and thereby drive inflation. Their idea is that during and just after the GUT phase transition the Universe is comprised of a mixture of highly relativistic and nonrelativistic particles, with a mean interaction time of the order of the age of the Universe; as is well known such a mixture has a significant bulk viscosity. The nonrelativistic particles are the so-called leptoquark gauge bosons which acquire mass of order T_c (the critical temperature $\approx 10^{14} - 10^{16}$GeV) during the phase transition.

In this paper we shall show that such an inflationary model cannot work. In the case of weakly-interacting particles the associated bulk viscosity cannot make the pressure negative, excluding any form of inflation, whether exponential or generalized (generalized inflation[8] being a phase where $\rho+3p < 0$, so that $\ddot{R} > 0$, where R is the cosmic scale factor). The physical reason is very simple. For point-like, weakly interacting particles[9] bulk viscosity arises due to the incomplete equilibrium of the relativistic and nonrelativistic components. And due to the non-equilibrium nature of the particle distributions the energy density of the system as a whole decreases more slowly than it would if equilibrium distributions were maintained. Hence, the effect of bulk viscosity can be and is described as a lowering of the pressure from its equilibrium value. (Recall $d(\rho R^3) = -pdR^3$.)

However, it is clear that the pressure can never become negative due to such effects. Recall the definition of pressure p for weakly interacting particles. In terms of the distribution function f

$$p = \int \mathbf{k}^2 f d\Pi \tag{1}$$

where $\mathbf{k}^2$ is the square of the one-particle three-momenta and $d\Pi$ is the invariant integration element, which for the Robertson-Walker line element has the form

$$d\Pi = d^3\mathbf{k}/[(2\pi)^3 E]$$

Here E is the one-particle energy. The positivity of p is manifest. (For a detailed discussion of bulk viscosity see ref. 10.)

To make more definite the general statements discussed above we shall construct a simple model which incorporates all the essential physical features. To begin, let us review the basic equations governing the evolution of the early Universe. For simplicity we will assume that the space-time metric is that of a flat Robertson-Walker model (although in the real spirit of inflation we should work with a general inhomogeneous and anisotropic model):

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) \tag{2}$$

The matter content of the Universe will be an imperfect fluid, but for vanishing shear viscosity and no heat conduction the energy-stress tensor can be written in the form of a perfect fluid with an effective pressure[11,12] P

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} \tag{3}$$

Here ρ is the energy density, u^μ is the four-velocity of the matter (in the comoving frame it is δ_0^μ) and the effective pressure P is given by

$$P = p - 3(\dot{R}/R)\varsigma \tag{4}$$

with p the equilibrium pressure and ς the coefficient of bulk viscosity.

Einstein's equations take the form:

$$H^2 \equiv (\dot{R}/R)^2 = 8\pi\rho/(3m_{Pl}^2) \tag{5}$$

$$\ddot{R}/R = -4\pi(\rho + 3P)/(3m_{Pl}^2) \tag{6}$$

Throughout units are chosen such that $\hbar = c = k_B = 1$ and $G_N^{-1/2} \equiv m_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass.

The key condition for having inflation is that $\ddot{R}$ be greater than or equal to zero: this requires that for generalized or exponential inflation,

$$\rho + 3p < 9H\varsigma \tag{7}$$

$$\rho + p = 3H\varsigma \tag{8}$$

respectively.

Clearly one needs to be more specific about ρ, p and ς to further discuss these conditions. The authors of ref. 6 solve for the coefficient of bulk viscosity to obtain constraints on ς necessary for inflation to occur. We will examine a more or less realistic model to see if sufficient bulk viscosity can actually arise to drive inflation.

Having these remarks in mind we set up the equations which govern the behaviour of matter. Since we have a mixture of relativistic and nonrelativistic gases we write:

$$\rho = NT^4 + \rho_m(n,T)$$
$$p = NT^4/3 + p_m(n,T) \tag{9}$$
$$N = \pi^2(\sum g_b + (7/8)\sum g_f)/30$$

where ρ_m and p_m are the energy density and pressure of the nonrelativistic component, n is the number density of nonrelativistic particles and g_b, g_f count the bosonic and fermionic degrees of freedom of the relativistic particles. These equations are written with the Eckart choice for the temperature[10-12]. The bulk viscosity in this model arises due to the lack of equilibrium between the relativistic particles and the nonrelativistic particles. The viscosity coefficient can be calculated[12] in terms of n, T

$$\varsigma = 4NT^4(1/3 - (\partial p/\partial\rho)_n)^2\tau \tag{10}$$

where τ is the mean time for relativistic particles to scatter off nonrelativistic particles, estimated as

$$\tau \simeq 1/(n\sigma) \tag{11}$$

where σ is the cross section for scattering of the relativistic particles on the nonrelativistic particles. This expression is valid for $H\tau \ll 1$, the collisional limit[12] (i.e., many collisions per expansion time).

In the collisionless limit (i.e., $H\tau \gg 1$), Eqn(10) must be modified. Since the probability for a collision in a Hubble time ($\equiv H^{-1}$) is $(H\tau)^{-1}$, the bulk viscosity must scale as $H^{-1}(H\tau)^{-1}$. Thus the general form of the bulk viscosity coefficient ς should be of the form

$$\varsigma = 4NT^4(1/3 - (\partial p/\partial\rho)_n)^2\beta\tau/(\beta + (H\tau)^2) \tag{12}$$

where β is a numerical constant expected to be of order unity. Note that the viscosity vanishes both in the collisional limit ($H\tau \to 0$) and in the collisionless limit ($H\tau \to \infty$), as it must, and achieves its maximum value for $H\tau \sim \mathcal{O}(1)$. (This same form for the shear viscosity as a function of $H\tau$ has been rigorously derived in ref. 13.)

For a dynamical description we need also an evolution equation for n: in the Robertson-Walker metric it has the form

$$\dot{n} + 3Hn = \Psi \tag{13}$$

where Ψ is the net source term for creation of the nonrelativistic particles and the $3Hn$ term as usual accounts for the dilution of particles due to the expansion of the Universe.

Given ρ_m, p_m, σ and Ψ we have a closed system of equations for H, T and n. The quantities ς and τ can be calculated from Eqns(10,11). As usual, we can avoid the second-order equation for R, Eqn(6), by employing the energy-conservation equation:

$$\dot{\rho} + 3H(\rho + P) = 0 \tag{14}$$

From thermodynamic considerations we can calculate ρ_m and p_m. For a gas of nonrelativistic particles[10]

$$\rho_m = nm + 3nT/2 \tag{15}$$

$$p_m = nT \tag{16}$$

where m is the mass of the nonrelativistic particles. The scattering cross-section is taken to have the form

$$\sigma \simeq \alpha^2/(m^2 + T^2) \tag{17}$$

where α is the unified coupling strength (of order 1/40 or so). This form has the expected behaviour in the $T >> m$ and $T << m$ limits.

Introducing the entropy per nonrelativistic particle s

$$s = 4NT^3/(3n) \tag{18}$$

the expression for the viscosity coefficient can be simplified:

$$\varsigma = 4NT^4\beta\tau/9[(2s+1)^2(\beta + (H\tau)^2)] \tag{19}$$

Then condition (8) for inflation becomes:

$$(2s+1)^2(s + 5/2 + m/T)/s = \beta H\tau/(\beta + (H\tau)^2) \tag{20}$$

We can ask what value of β is required to satisfy this condition. It is straightforward to show that β is bounded from below by $\beta(s, H\tau) \geq 2.2 \times 10^3$: if instead we use condition (7) for generalized inflation, the bound becomes slightly weaker $\beta(s, H\tau) \geq 760$. In any case these are highly unreasonable values, since we would expect $\beta \simeq O(1)$.

Note that for reasonable values of β (i.e., of order unity) negative pressure is not attained because the viscosity does not continue to increase linearly with the scattering time τ: in the collisionless regime ($H\tau \geq 1$) the viscosity must and does decrease because the two components are not interacting. We could stop here saying that in the best case we need an extremely large value for β (which is physically unreasonable) to reach an inflationary stage. However, to make the analysis complete - although purely academic - we shall assume that for some unknown reason the viscosity can be enhanced and ask if an inflationary stage can be achieved.

It is easy to see that if the number of nonrelativistic particles is conserved the system escapes very rapidly from the accelerated expansion. If $\Psi = 0$, then it follows from Eqn(13) that $n \propto R^{-3}$ (just the conservation of the total number of massive particles). From this we see that $\tau \simeq 1/(\sigma n)$ must increase as R^3, and so

Eqn(12) implies that ς must eventually decrease as R^{-3} – thereby shutting off any negative pressure (should it have developed).

Now consider the effect of particle creation: relativistic particle + relativistic particle→2 massive bosons. The rate for this process is $\Psi \propto n_{rel}^2 \sigma$, i.e.,

$$\Psi \simeq \alpha^2 N_*^2 T^6 \exp(-2m/T)/(T^2 + m^2)$$

where the $\exp(-2m/T)$ factor takes into account the Boltzmann suppression for creating particles with $m \gg T$. Here $n_{rel} = N_* T^3$ is the number density of relativistic particles and N_* is given by

$$N_* = \varsigma(3)(\sum g_b + (3/4)\sum g_f)/\pi^2 \tag{21}$$

(Here $\varsigma(3) \simeq 1.202$ denotes the Riemann ς function)

Note that the timescale for the interaction between relativistic and nonrelativistic particles must also set the timescale for massive particle creation, which is what we have assumed. We have ignored massive particle annihilations in our model; however they would only *decrease* the number of massive particles (and the bulk viscosity). To perform the numerical analysis we introduce dimensionless variables:

$$\begin{aligned} \chi &= \sqrt{8\pi/3}(m/m_{Pl})mt \\ x &= T/m \\ y &= n/m^3 \end{aligned} \tag{22}$$

remembering that the only dimensional parameter available is the mass of the nonrelativistic species.

We characterize the enhancement of viscosity by multiplying the expression for ς, Eqn(12) or (19), with a factor Δ and taking β equal to unity. Before discussing the results of our numerical calculations we note that the parameters N, N_*, m and α do not appear independently in the equations: N_* can be written as

$$N_* = \gamma N \tag{23}$$

where γ is between 0.32 and 0.37 depending on the ratio $(\sum g_b)/(\sum g_f)$, while m, α occur only in the combination

$$\lambda = \sqrt{8\pi/3}(m/m_{Pl})/\alpha^2 \tag{24}$$

Physically, $\lambda \sim (H\tau)\,|_{T=m}$ measures whether or not collisions between the two components are occurring rapidly. For $m \approx O(10^{14} - 10^{16}\ \mathrm{GeV})$, λ is of order unity. As discussed earlier, the bulk viscosity coefficient ς achieves its maximum value for $(H\tau) \sim 1$, or $\lambda \sim O(1)$.

We have looked for stationary solutions of our system (H = const., T = const., n = const, etc.), i.e., for inflating models. For a given assumed steady state temperature, we have numerically solved for the relationship between λ and Δ (using $N = 50$ and $\gamma = 0.37$). The results are shown in Fig.1. In order to achieve an inflationary state with $T \leq m$, the bulk viscosity enhancement parameter Δ must be in excess of 10^3, irrespective of the value of λ–which confirms our expectation that to achieve an inflationary stage one would need a much larger viscosity than can be produced by physical processes.

We conclude that the bulk viscosity which arises due to the 'weak' (i.e., perturbative) interactions between a nonrelativistic and a very relativistic component cannot drive inflation. Firstly the bulk viscosity which arises cannot on physical grounds drive the pressure negative. Rather, the nonequlibrium effects associated with bulk viscosity merely cause the energy density to *decrease* less rapidly than in the equilibrium case, which is equivalent to a decrease in pressure (relative to the equilibrium case). By using a formula for bulk viscosity which is only valid in the collisional limit previous authors were misled into believing that bulk viscosity could drive the pressure negative. Secondly, should 'non-perturbative' interactions enhance the bulk viscosity due to the loose coupling between particles, the dilution of the massive particle component, except under the most extreme of assumptions for the bulk viscosity coefficient, drive the bulk viscosity rapidly to zero. Finally, we note that in order to reasonably expect negative pressure we must appeal to 'non-perturbative' effects, which of course is the source of negative pressure in the usual inflationary scenario. Purely perturbative effects do not lead to negative pressure.

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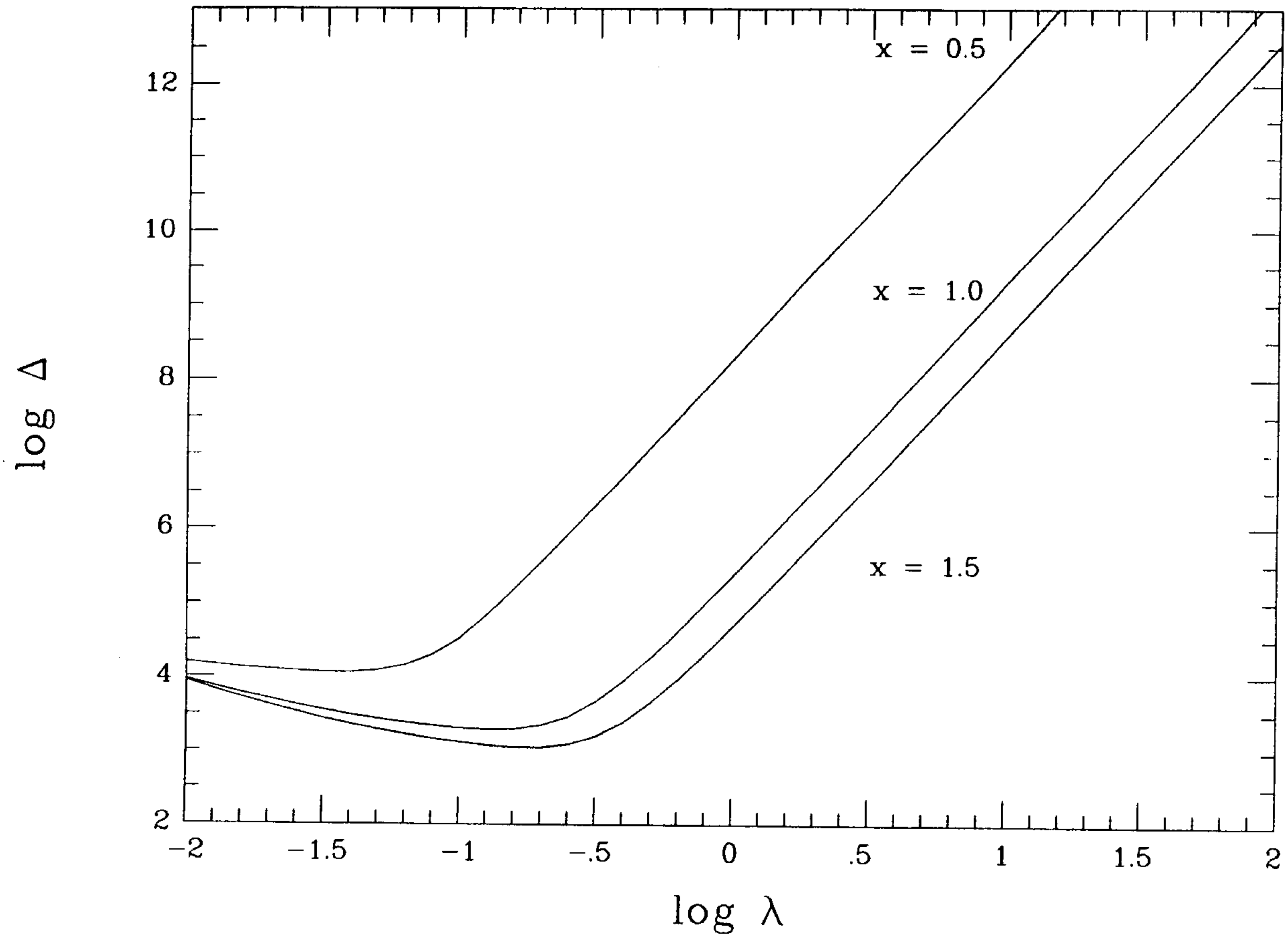
Figure Caption

Figure 1. - The value of the viscosity enhancement Δ require to achieve a steady inflationary state with temperature $T = xm$ as a function of λ. Note, the curves for $x \leq 0.5$ lie above those displayed. The requisite value of Δ is minimized for $\lambda \sim O(1)$; This is what one would expect, as for $\lambda >> 1$ or $\lambda << 1$ the viscosity coefficient $\varsigma \rightarrow 0$.



log Δ
12
10
8
6
4
2
−2
−1.5
−1
−.5
0
.5
1
1.5
2
log λ
x = 0.5
x = 1.0
x = 1.5