Testing Couplings of the Three Vector Bosons Through Hadron Collision at Tevatron Energy

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Abstract

A test for vector boson trilinear couplings through hadron collision at Tevatron energy is investigated. The contributions from the standard model to the polarization cross section for $W^+W^-$ pair production process are given and deviations from the standard model are considered. It is pointed out that some effects may be observable in Tevatron collider experiments.

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I. Introduction

Since the weak boson $W^\pm$ and $Z^0$ were discovered [1], tests of the couplings of $W^\pm$, $Z^0$ and $\gamma$ to see if they are of gauged $SU(2) \times U(1)$ symmetry have become one of the most interesting problems [2-29]. It is important not only for proving the standard model [30], but also for testing models beyond [31].

In order to pose this problem, one can focus on experiments at LEP-II in the production of charged weak boson ($W^\pm$) pair in $e^+e^-$ annihilation, which is related to the couplings of $W^+W^-Z^0$ and $W^+W^-\gamma$. Having relatively higher luminosity ($5 \times 10^{31} \text{cm}^{-2} \text{sec}^{-1}$) and being in the clean environment of $e^+e^-$, undoubtedly LEP-II will give us a lot of useful information on the couplings. However, since the $pp$-collider Tevatron has begun to accumulate data, we think it is worthwhile to entertain the possibilities to test these couplings and to investigate what kinds of deviations from the standard model are comparatively easier to be detected in the Tevatron Collider.

Although a hadron collider is not in a clean environment to test these couplings as in $e^+e^-$, besides having at present a relatively low luminosity to work with, it has some advantages. It was pointed out in ref. [32] that if there are some deviations from the standard model and their effects are proportional to the masses of the fermions in the pair production processes $f\bar{f} \rightarrow W^+W^-$, then it is almost impossible for $e^+e^-$ collider to observe them. However, for hadron collider it is possible due to heavy quark components in the nucleon structure functions. Another advantage is that in hadron collider the invariant mass of the $W$ pair can reach a relatively high value. We know that the higher the invariant mass, the easier it becomes to test the gauge cancellation and to see the deviations from gauge couplings if there are any.

Therefore, under the Tevatron condition, how well one can test the couplings of the three vector bosons deserves a careful analysis. On the other hand, if someone hopes to discover new physics through $W^+W^-$ production, he also needs to know the standard model contributions as well as possible deviations as a background. Thus, in this paper we will investigate these problems, i.e., considering the Tevatron conditions, we will compute the $W^+W^-$ production cross sections systematically.

In ref. [32], the authors pointed out that due to heavy quarks, all of the most general couplings of the three vector bosons should be considered in hadron collision...
processes where there are ten types in total, as opposed to what the authors of refs. [7,28] did in the e⁺e⁻ collider case where they ignored all couplings if the contributions to the amplitude are proportional to the fermion masses, which certainly is a very good approximation for e⁺e⁻ collision. Moreover, in hadron collision, the Higgs exchange mechanism is not ignorable either for the same reason. In order to be consistent, we also should consider the general couplings of Higgs to the bosons. Thus, in W⁺W⁻ pair production processes through annihilation of a pair of q̅q, in general, 23 types of couplings (or, say, form factors) in total will be relevant. The 23 consist of ten for W⁺W⁻Z vertex, seven for W⁺W⁺, three for W⁺W⁻H (here, the Higgs H couples to quarks as a scalar) and three for W⁺W⁻H⁺ (here the “Higgs” H⁺ couples to quarks as a psuedoscalar). In this paper, we will discuss their effects separately in p̅p collision at Tevatron energy.

Being interested in the three vector boson couplings, we will not touch the related mechanism such as the equivalent W boson fusion mechanism and gluon fusion through fermion loop mechanism, etc. [27,33], since their contributions are incoherent to what we will discuss. Thus, we focus on the W⁺W⁻ pair production through Drell–Yan mechanism [34]. Owing to such a large number of possible couplings, if one wants to obtain any information on the individual coupling, one has to perform a polarization analysis [7,8,25,28].

One may imagine that if one keeps all mass terms, the formalism would be complicated, especially with respect to those couplings which would not contribute in the massless limit. However, in fact, we find that the density matrix can be factorized into four terms so the final results are not so complicated.

This paper is organized as follows: In the next section, the most general couplings for W⁺W⁻γ, W⁺W⁻Z⁺, W⁺W⁻H and W⁺W⁻H⁺ will be given; the density matrix of polarization for W⁺ and W⁻ are derived. In Section III, the formulas of W⁺W⁻ pair production cross sections for p̅p collision are presented. Adopting the EHLQ structure functions [26], and setting the couplings to deviate from those of the standard model predictions one by one, we compute the numerical values of the polarization cross sections and plot the cross sections against the angle between the proton beam and the produced W boson in the hadron center of mass system and against the invariant mass of the W⁺W⁻ pair. In order to compare with the standard model, in every figure we also plot the corresponding standard model
curves. In the last section, Section IV, we will discuss our results and draw some conclusions.

II. General Couplings and Polarization Density Matrix

A. General Couplings of the Three Vector Bosons

The strategy of investigating the couplings for the three boson vertex in a model independent way is to start from the most general Lorentz covariant couplings, which are independent of each other, and then to set each term to a non-standard model value to see its corresponding consequences. Therefore, we first discuss the most general couplings.

In this paper, we restrict ourselves to consider only the $W^+W^-$ pair production, hence, we need the most general couplings for $W^-W^+Z$ and $W^+W^-\gamma$. In ref. [28], the authors considered the process $e^+e^- \rightarrow W^+W^-$ in which the masses of the fermions may be ignored so that only fourteen of the most general couplings are effective. As a matter of fact, the most general couplings for the three vector bosons with $W^+W^-$ on shell should involve seventeen couplings. They are

$$\Gamma^\gamma_\mu(q_1, q_2, P) = f_1^\gamma(q_1 - q_2)g^{\alpha\beta} - \frac{f_2^\gamma}{m_w^2} (q_1 - q_2)_{\mu} P_{\alpha} P_{\beta} + f_3^\gamma (P_{\alpha} g^{\mu\beta} - P_{\beta} g^{\mu\alpha})$$

$$+ i f_4^\gamma (P_{\alpha} g^{\mu\beta} | P_{\beta} g^{\mu\alpha}) + i f_5^\gamma \varepsilon^{\alpha\beta\rho\sigma} (q_1 - q_2)_\rho$$

$$+ \frac{f_6^\gamma}{m_w^2} e^{\mu\beta\rho\sigma} P_{\rho} (q_1 - q_2)_\sigma$$

$$+ i f_7^\gamma P_{\mu} g^{\alpha\beta} - i \frac{f_8^\gamma}{m_w^2} P_{\mu} P_{\alpha} P_{\beta} - i \frac{f_9^\gamma}{m_w^2} P_{\mu} e^{\alpha\beta\rho\sigma} P_{\rho} (q_1 - q_2)_\sigma$$

(2.1)

where $\Gamma_{\gamma}^\mu$ is the vertex of the three bosons (Fig. 1a).

$$\langle W^- (q_1 \lambda_1) W^+ (q_2 \lambda_2) \mid J_\gamma^{\mu} \mid 0 \rangle = \epsilon_\alpha (\lambda_1) \epsilon_\beta (\lambda_2) \Gamma_{\gamma}^{\mu\alpha} (q_1, q_2, P),$$

(2.2)

and $\gamma$ presents $\gamma$ or $Z^0$. Note that $f_8^\gamma$ vanishes and $f_7^\gamma, f_9^\gamma$ are not independent of $f_4^\gamma, f_5^\gamma$ due to charge conservation. In fact, even if charge conservation is violated,

3The authors of ref. [7] had done the same, but as pointed out in ref. [28], they included two more couplings which are not independent of the rest when fermion masses are neglected.
$f_f, f_2, f_3$ will not contribute to $f f \rightarrow W^+W^-$ at all due to the vector coupling of $\gamma$ to fermion. Therefore, in eq. (2.1), these three would not be included in our discussions. Considering $WW\gamma$ and $WWZ$, we now have seventeen couplings $f_i^2 (i = 1, 2 \cdots 10)$ and $f_i^7 (i = 1, 2 \cdots 7)$. It is easy to check that these $f_i^2$ and $f_i^7$ are independent of each other and taking into account the on-shell conditions

$$q_1^2 = q_2^2 = m_w^2, \quad (q_1 e^{\lambda_1}) = (q_2 e^{\lambda_2}) = 0$$

there is no more independent couplings.

In addition to what has been shown for the fourteen couplings in ref. [28], we list the C,P,T, properties of the seventeen couplings in Table 1.

**Table 1**

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In the standard model, non-Abelian gauge invariance gives very strong constraints on the couplings:

$$f_i^2 (S) = 1 + 0(\alpha)$$
$$f_i^7 (S) = 0(\alpha)$$
$$f_i^7 (S) = 2 + 0(\alpha)$$

and the others are either $0(\alpha)$ or higher order in $\alpha$. Since we are testing the three vector boson couplings (Fig. 1) through Drell-Yan mechanism [34] (Fig. 2a) and we want to keep the heavy quark mass effects so we also include the related diagram through scalars (Fig. 2c). To test the standard model thoroughly, it is also necessary to test the deviation from the minimum Higgs sector. Therefore, we also generalize the Higgs couplings to the $W^+W^-$ pair. According to the different couplings with or without $\gamma_5$ with fermions, the scalar couplings to $W^+W^-$ are denoted as $\Gamma_{(H)}^{\alpha\beta}$, where the scalar couples to fermions without $\gamma_5$, and $\Gamma_{(H_5)}^{\alpha\beta}$ where the scalar couples
to fermions with $\gamma_5$ through (Fig. 2c). With the definition of the vertex (Fig. 1b)

$$< W^- (\lambda_1 q_1) W^+ (\lambda_2 q_2) | J | 0 > = \epsilon_\alpha (\lambda_1) \epsilon_\rho (\lambda_1) \Gamma_{(H,H^\prime)}^{\alpha \beta}$$

the most general $\Gamma_{(H)}^{\alpha \beta}$ and $\Gamma_{(H^\prime)}^{\alpha \beta}$ are as follows

$$\Gamma_{(H)}^{\alpha \beta} = f_1^H g^{\alpha \beta} + \frac{f_2^H}{m_w^2} p^\alpha p^\beta + \frac{f_3^H}{m_w^2} \epsilon^{\alpha \beta \rho \sigma} P_\rho (q_1 - q_2)_\sigma$$

and

$$\Gamma_{(H^\prime)}^{\alpha \beta} = i f_1^{H^\prime} g^{\alpha \beta} + \frac{i f_2^{H^\prime}}{m_w^2} P^\alpha p^\beta + \frac{i f_3^{H^\prime}}{m_w^2} \epsilon^{\alpha \beta \rho \sigma} P_\rho (q_1 - q_2)_\sigma .$$

For the standard model

$$f_1^H = 1 + O(\alpha)$$
$$f_2^H = 0(\alpha)$$
$$f_3^H = 0(\alpha) .$$

It turns out that the contributions from $\Gamma_{(H,H^\prime)}^{\alpha \beta}$ are relatively small at Tevatron energy except at the peak of resonances in the $W^+W^-$ channel. Therefore, later on we will not discuss them in this paper, but we still keep them in the formulas so that they can be used to analyze the $S$-channel resonances.

**B. The polarization Density Matrix**

In order to determine all the couplings in eq. (2.2), one has to analyze the polarization behavior.

In the hadron collision, the subprocess concerned is $q\bar{q} \rightarrow W^+W^-$. To calculate the cross section, we need to consider all diagrams in Fig. 2.a-c. According to the Feynman rules, we can easily write down the scattering amplitude. Neutral current and low-energy experiments indicate that the couplings for the vector bosons ($\gamma, Z$) to the fermions (quarks and leptons) should be given by those of the standard model. For definiteness, we assume the scalars $H, H^\prime$ couple to the fermions in the same way as the Higgs do in the standard model.

Let $M(k_1 \sigma_1, k_2 \sigma_2; q_1 \lambda_1, q_2 \lambda_2)$ be the scattering amplitude of $q\bar{q} \rightarrow W^+W^-, k_1, k_2$ and $\sigma_1, \sigma_2$ are the momenta and helicities of quark and antiquark respectively, and
$q_1, q_2$ and $\lambda_1, \lambda_2$ are the momenta and polarization respectively (Fig. 3) for the $W$ bosons. In the paper, we will use the rectangular basis $e^\lambda(q)$ for the polarizations of $W$ and use the conventions of ref. [28] so that $\lambda = 1$ labels the transverse polarization in the production plane, $\lambda = 2$ labels the transverse polarization perpendicular to the production plane, and $\lambda = 3$ labels the longitudinal polarization.

The polarization density matrix is defined by

$$\rho_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2} = \sum_{\sigma_1 \sigma_2} M(k_1 \sigma_1, k_2 \sigma_2; q_1 \lambda_1, q_2 \lambda_2) M^*(k_1 \sigma_1, k_2 \sigma_2; q_1' \lambda'_1, q_2' \lambda'_2). \quad (2.9)$$

Through a straightforward calculation, $\rho_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2}$ can be factorized into four terms

$$\rho_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2} \equiv 2e^4 \bar{\rho}_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2} \quad (2.10)$$

$$\bar{\rho}_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2} = R_{\lambda_1 \lambda_2} R^{* \alpha}_{\lambda'_1 \lambda'_2} + R^{* \beta}_{\lambda_1 \lambda_2} R^{\alpha}_{\lambda'_1 \lambda'_2} + S^{\alpha}_{\lambda_1 \lambda_2} S^{\beta*}_{\lambda'_1 \lambda'_2} + S^{\beta}_{\lambda_1 \lambda_2} S^{\alpha*}_{\lambda'_1 \lambda'_2}. \quad (2.11)$$

where $R_{\lambda_1 \lambda_2}$ and $S^{\alpha}_{\lambda_1 \lambda_2}$ are 3x3 matrices. Here, and later on, the star "*" means to take complex conjugate. Because the couplings of $W^+W^-\gamma$ and $W^+W^-Z$ appear in the matrices with certain combinations, we define the following quantities:

$$a_n = Q_i \left( f^+_n - \frac{s}{s - m_i^2} f^+_n \right); \quad b_n = -\left( \frac{L_i - R_i}{2 \sin^2 \theta_w} \frac{s}{s - m_i^2} f^+_n \right), \quad n = 1, \ldots, 10 \quad (2.12)$$

$$b^{\alpha, \beta} m = \frac{1}{4 \sin^2 \theta_w} \sum_t \frac{2 \sin^2 \theta_w}{t - m_i^2} V_{it} \quad (2.12a)$$

$$b^{\alpha, \beta} m = \frac{1}{2 \sin^2 \theta_w} \frac{s}{s - m_H^2 - i\Gamma_H m_H} f^{m, \alpha, \beta} m = 1, 2, 3 \quad (2.12b)$$

where $Q_i$ are the electric charge of the incoming quark, $L_i, R_i$ their left and right weak charge

$$L_i = \tau_i^3 - 2Q_i \sin^2 \theta_w, \quad R_i = -2Q_i \sin^2 \theta_w \quad (2.12c)$$

$m_i$ are the masses of the corresponding $t$-channel quarks, $V_{it}$ are the KMC quark mixing matrix elements, $m_H$ and $\Gamma_H$ are the mass and widths of the neutral scalars.

The matrices $R^{\alpha}, R^{\beta}, S^{\alpha}$ and $S^{\beta}$ have the following properties

$$A_{12} = A_{21}^*, \quad A_{13} = -A_{31}^*, \quad A_{23} = -A_{32}^* \quad (2.13)$$

where $A$ represents any one of $R^{\alpha}, R^{\beta}, S^{\alpha}$, $S^{\beta}$ matrices and $a_n, b_n, b^{\alpha, \beta} m$ are treated as real quantities. The matrix elements of $R^{\alpha}, R^{\beta}, R^{\gamma}$ and $S^a$ for the down type
incoming quarks are the following

\[ R_{11}^v = \left[ \left( a_1 - \frac{1}{2} b_1 \right) \beta_w - b (\beta_w - \beta_i \cos \theta) + \frac{b}{2} \beta_w \right] \sin \theta \]

\[ R_{12}^v = \left[ -2 (2 a_7 - b_7) \beta_w^2 \gamma_w^2 - \left( a_6 - \frac{1}{2} b_6 \right) + i \beta_i^2 \sin \theta \right] \sin \theta \]

\[ R_{13}^v = -\frac{1}{2} \gamma_w \left[ (2 a_5 - b_5) \beta_w \cos \theta + \beta_i \left( b_5 \beta_w^2 + i b_6 \right) + b \beta_i \beta_w (\beta_w - \beta_i \cos \theta) \right] \cos \theta \]

\[ R_{22}^v = \left[ - \left( a_1 - \frac{1}{2} b_1 \right) \beta_w + \frac{b}{2} \beta_w \right] \sin \theta \]

\[ R_{23}^v = -\frac{i}{2} \gamma_w \left\{ b - \beta_i \beta_w + \left[ (2 a_5 - b_5) \beta_w^2 + i (2 a_6 - b_6) \right] \cos \theta + b \beta_i (\beta_w - \beta_i \cos \theta) \right\} \]

\[ R_{33}^v = \left[ - \left( a_1 - \frac{1}{2} b_1 \right) \beta_w \left( 2 \gamma_w^2 - 1 \right) + 2 (2 a_2 - b_2) \beta_w^3 \gamma_w^4 + (2 a_3 - b_3) \beta_w \gamma_w^2 \right] \sin \theta \]

\[ R_{11}^o = \left[ - \frac{1}{2} b_1 \beta_w - b (\beta_w - \beta_i \cos \theta) + \frac{b}{2} \beta_w \right] \beta_i \sin \theta \]

\[ R_{12}^o = \left[ \frac{1}{2} \left( 4 b_7 \beta_w^2 \gamma_w^2 + b_6 \right) + \frac{i b}{2} \right] \beta_i \sin \theta \]

\[ R_{13}^o = \frac{1}{2} \gamma_w \left[ b - \beta_i \beta_w \cos \theta + (2 a_5 - b_5) \beta_w^2 + i (2 a_6 - b_6) - b (\beta_w - \beta_i \cos \theta)^2 + b \beta_i^2 \sin^2 \theta \right] \]

\[ R_{22}^o = \frac{1}{2} \left( b_1 + b \right) \beta_w \beta_i \sin \theta \]

\[ R_{23}^o = \frac{i}{2} \gamma_w \left[ (2 a_5 - b_5) \beta_w + \left( b_5 \beta_w^2 + i b_6 \right) \beta_i \cos \theta - b \beta_i \beta_w (\beta_w - \beta_i \cos \theta) - b (\beta_w - \beta_i \cos \theta) \right] \]

\[ R_{33}^o = \left[ \frac{1}{2} b_1 \beta_w \left( 2 \gamma_w^2 - 1 \right) - 2 b_2 \beta_w^3 \gamma_w^4 - b_3 \beta_w \gamma_w^2 - \frac{1}{2} b \beta_w - b \gamma_w^2 (\beta_w - \beta_i \cos \theta) \right] \beta_i \sin \theta \]
\[ S_{11} = \frac{1}{2\gamma_i} \left[ -\beta_i b_i^2 + (2a_1 - b_1) \beta_w \cos \theta - b (\beta_i - \beta_w \cos \theta) + 2b\beta_i^2 \sin^2 \theta \right] \]

\[ S_{12} = \frac{1}{2\gamma_i} \left\{ \left[ 2a_6 - b_6 + 2(2a_7 - b_7) \beta_w^2 \gamma_w^2 \right] \cos \theta + 4b_i^2 \beta_i \beta_w \gamma_w^2 \right\} \]

\[ S_{13} = \frac{\gamma_w}{2\gamma_i} \left[ (2a_1 - b_1) \beta_w - 2b (\beta_w - \beta_i \cos \theta) + b\beta_w \right] \sin \theta \]

\[ S_{12} = \frac{1}{2\gamma_i} \left[ \beta_i b_i^2 + (2a_1 - b_1) \beta_w \cos \theta + b (\beta_i - \beta_w \cos \theta) \right] = -S_{11} + \frac{b}{\gamma_i} \beta_i^2 \sin^2 \theta \]

\[ S_{13} = -\frac{i\gamma_w}{2\gamma_i} \left[ (2a_5 - b_5) \beta_w^2 + i(2a_6 - b_6) - b\beta_w^2 \right] \sin \theta \]

\[ S_{33} = \frac{1}{2\gamma_i} \left\{ \beta_i \left[ b_i^2 \left( 2\gamma_w^2 - 1 \right) - 4b_i^2 \beta_w^2 \gamma_w^4 \right] + \left[ (2a_1 - b_1) \left( 2\gamma_w^2 - 1 \right) - 4(2a_2 - b_2) \beta_w^2 \gamma_w^4 \right] \beta_w \cos \theta - 2(2a_3 - b_3) \gamma_w^2 \beta_w \cos \theta + b \left( 2\gamma_w^2 - 1 \right) (\beta_i - \beta_w \cos \theta) + 2b\gamma_w^2 (\beta_w - \beta_i \cos \theta) \cos \theta \right\} \]

(2.16)

\[ S_{11}^a = -\frac{1}{2\gamma_i} \left[ (4\gamma_w^2 - 1) b_8 + b_1^P \right] \]

\[ S_{12}^a = \frac{1}{2\gamma_i} \left[ (4\beta_w \gamma_w^2 b_{10} - b_5 \beta_w) \left( 4\gamma_w^2 - 1 \right) + 4\beta_w \gamma_w^2 b_3^P + b (\beta_w - \beta_i \cos \theta) \right] \]

\[ S_{13}^a = -\frac{i\gamma_w}{2\gamma_i} \left[ b_3 \beta_i \beta_w \sin \theta \right] \]

\[ S_{22}^a = -S_{11} \tau_i \]

\[ S_{23}^a = -\frac{\gamma_w}{2\gamma_i} b_3 \beta_i \sin \theta \]

\[ S_{33}^a = \frac{1}{2\gamma_i} \left\{ \left[ b_8 \left( 2\gamma_w^2 - 1 \right) - 4b_9 \beta_w^2 \gamma_w^4 + 2b_4 \beta_w^2 \gamma_w^2 \right] \left( 4\gamma_w^2 - 1 \right) + b_1^P \left( 2\gamma_w^2 - 1 \right) - 4b_2^P \beta_w^2 \gamma_w^4 \right\} \]

(2.17)

where \( \gamma_i, \gamma_w, \gamma_z, \beta_i, \beta_w, \beta_z \) are, as usual,

\[ \gamma_w = \sqrt{\frac{3}{2m_w}} \quad \beta_w = \sqrt{1 - \frac{4m_w^2}{\delta}} \quad \text{etc.} \]

(2.18)
and \( a_+ = a_3 - i a_4, b_- = b_3 - i b_4 \). For up-type quark, the density matrix is again given by equations (2.10) and (2.11) with the following changes. In eqs. (2.12a)(through t), (2.14), (2.15), (2.16) and (2.17), \( \theta \) should be changed into \( \pi - \theta \); the matrices \( R^{\alpha a}, S^{\alpha a} \) should be transposed \( (R, S \rightarrow R, S) \); the signs of \( f_n^V \) for \( n = 1, 2, 3, 6, 7 \) should be flipped; the up-type quark masses in eq. (2.12a) should be substituted with the down-type quark masses and the down-type charges in eq. (2.12) should be substituted with the up-type charges.

Now we are ready to derive the cross sections. In the following section, we shall use \( \hat{s} \) for the parton CM energy squared and \( s \) for the corresponding quantity of the incoming hadrons.

III. Formulas of \( W^+W^- \) Pair Production Cross Sections and Numerical Results

A. Formulas of \( W^+W^- \) Pair Production Cross Sections

From the density matrix, it is straightforward to obtain the corresponding differential cross section for polarized \( W^+ \) and \( W^- \) in the parton subprocess \( q_i \bar{q}_i \rightarrow W^+W^- \):

\[
\left( \frac{d\sigma}{d\cos \theta^*} \right)_{\lambda_1 \lambda_2 \lambda_1' \lambda_2'} = \frac{\pi \alpha^2 \beta_w}{4 \delta \beta_i} \tilde{\rho}_{\lambda_1 \lambda_2 \lambda_1' \lambda_2'} (\theta^*, \sqrt{\hat{s}}),
\]

where \( \theta^* \) is the angle between the \( W^- \) and the incoming hadron \( A \) in the hadron center of mass system. It follows from the factorization theorem for Drell-Yan processes [35] that the \( W^+W^- \) polarization cross section for \( AB \rightarrow W^+W^- \) (where \( A = P, B = \bar{P} \) for Tevatron) can be written as

\[
\left( \frac{d\sigma}{dM d\cos \theta^* dy_+} \right)_{\lambda_1 \lambda_2 \lambda_1' \lambda_2'} = \frac{\pi \alpha^2 \beta_w}{s \delta M N_c} \sum_j \left( \frac{1}{2 \beta_i^2} \frac{\alpha^2}{4 \beta_i} h^2 y_+-s h^2 y_+ \right) \left\{ f_j^A \left( x_a, M^2 \right) f_j^B \left( x_b, M^2 \right) \tilde{\rho}_{\lambda_1 \lambda_2 \lambda_1' \lambda_2'} (\theta^*, M) + f_j^A \left( x_a, M^2 \right) f_j^B \left( x_b, M^2 \right) \tilde{\rho}_{\lambda_1 \lambda_2 \lambda_1' \lambda_2'} (\pi + \theta^*, M) \right\},
\]

where, \( N_c = 3 \) is the color factor; \( j = u, c, t \) and \( d, s, b; M = \sqrt{\hat{s}}; f_j^A(\bar{q}) \) and \( f_j^B(\bar{q}) \) are the parton distribution functions for parton \( j(\bar{j}) \) in \( A \) hadron and \( j(\bar{j}) \) in \( B \) hadron and

\[
y_+ = \frac{1}{2} (y_1 + y_2).
\]
Here $y_1$ and $y_2$ are the rapidities, in the hadron $CM$ frame, of $W^-$ and $W^+$, and they are related to $\theta^*$ as follows

$$\beta_w \cos \theta^* = \tanh \frac{1}{2} (y_1 - y_2) . \tag{3.4}$$

The effects of the polarizations of $W^+W^-$ can be seen through the correlations of their decay products. This has been discussed extensively by Hagiwara, et al. in Ref. 28, in the event when the masses of the final state fermions can be neglected. Although we shall discuss primarily the polarization cross sections as given in eq. (3.2), we give below the decay matrices for completeness.

Let $M_2(q_1, \lambda_1; p_1 \sigma_1, p_2 \sigma_2)$ be the amplitude for the decay of $W^-$ into fermions of momenta $p_1, p_2$ and helicities $\sigma_1, \sigma_2$. Similarly, let $M_3(q_2, \lambda_2; p_3 \sigma_3, p_4 \sigma_4)$ be the corresponding quantity for $W^+$. We use $\theta_1, \phi_1$ to denote the polar and azimuthal angles of the down-type quark (lepton) in $W^-$ decay and in the $W^-$ rest frame. We use $\theta_3, \phi_3$ for the corresponding angles in $W^+$ decay (and in the $W^+$ rest frame). The decay matrix $D_{\lambda\lambda'}$ is defined by

$$D_{\lambda\lambda'} = \sum_{\sigma_1, \sigma_2} M_2(q_1, \lambda_1; p_1 \sigma_1, p_2 \sigma_2) M_3^* (q_1, \lambda_1'; p_1 \sigma_1, p_2 \sigma_2) \tag{3.5}$$

$D_{\lambda\lambda'}$ for $W^+$ decay is defined similarly. One can show that

$$D_{\lambda\lambda'} = \frac{-e^2}{2 \sin^2 \theta_w} V_{t\ell} \left| \frac{\lambda}{2} \right|^2 \sum_{\sigma_1, \sigma_2} (p_1^1 - \sigma_1 | \bar{p}_1 |) (p_2^2 + \sigma_2 | \bar{p}_2 |) \ell_\lambda \ell_{\lambda'} \tag{3.6}$$

where

$$\ell_\lambda = \cos \theta_1 \cos \phi_1 + \frac{i}{2} (\sigma_1 - \sigma_2) \sin \phi_1,$$

$$\cos \theta_1 \sin \phi_1 - \frac{i}{2} (\sigma_1 - \sigma_2) \cos \phi_1, -\sin \theta_1 > \lambda . \tag{3.7}$$

Similarly,

$$D_{\lambda\lambda'} = \frac{e^2}{2 \sin^2 \theta_w} V_{t\ell} \left| \frac{\lambda}{2} \right|^2 \sum_{\sigma_3, \sigma_4} (p_3^3 + \sigma_3 | \bar{p}_3 |) (p_4^4 - \sigma_4 | \bar{p}_4 |) \bar{\ell}_\lambda \bar{\ell}_{\lambda'} \tag{3.8}$$

where

$$\bar{\ell}_\lambda = \cos \theta_3 \cos \phi_3 + \frac{i}{2} (\sigma_3 - \sigma_4) \sin \phi_3 ,$$
In these formulas, $c_-, c_+$ are color factors and equal $1(\sqrt{3})$ for lepton (quark) final state. If we neglect the masses of the final state fermions, it is easily seen that $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ can take only a particular value so that the summations over $\sigma_i$ reduces to one term and we get the result of Ref. 28.

Equations (3.5), (3.8) can be combined with eq. (3.2) to give the complete cross sections for decay correlations. We shall postpone a detailed analysis of such correlations as was done in Ref. 28 for $e^+e^-$ collisions, but rather analyze the polarization channels and the kinematical regions ($\theta^*$ and $M$) where a certain deviation from the Standard Model predictions is most easily found. For this purpose, we shall limit our discussions to the diagonal elements of eq. (3.2).

For given invariant mass $m$ and the rapidity $y_+$ of the $W^+W^-$ pair, the fractions $x_a, x_b$ of hadron momenta carried by the incoming partons in hadron $A$ and hadron $B$ are fixed to be

$$x_a = \sqrt{r}(\beta_j \cosh y_+ + \sinh y_+), \quad x_b = \sqrt{r}(\beta_j \cosh y_+ - \sinh y_+),$$

(3.10)

where $\sqrt{r} = M/\sqrt{s}$.

We shall integrate over $y_+$ with the following cuts. Kinematically, it is constrained by $0 \leq x_a, x_b \leq 1$. We put in the rapidity cuts of $|y_1| \leq 2.5, |y_2| \leq 2.5$ for $W^-$ and $W^+$. Finally, since the mass of top quark is set at 70 GeV, which is comparable to the $W$ mass, $x_a$ and $x_b$ for $t\bar{t}$ parton processes can be smaller than $10^{-4}$, the limit of validity of the EHLQ structure functions which we are using. Moreover, so far as we know, there is still no completely satisfactory way of taking into account the heavy quark masses in the parton distribution functions [26,36]. To avoid the problem of small $x$, we put in the cuts $x_a, x_b \geq r$ which are satisfied automatically if the masses of the partons are neglected. Such cuts reduce the $t$-quark effective luminosity, but has very little effect on the luminosities of other quarks. At Tevatron energy, the differential cross sections are not sensitive to such cuts since the top luminosity is small anyway.

As pointed out above, taking $\lambda_1 = \lambda_1', \lambda_2 = \lambda_2'$, we have the polarized $W^+W^-$ double differential cross section
\[
\left( \frac{d\sigma}{dM d\cos \theta^*} \right)_{\lambda_1 \lambda_2} = 
\frac{\pi \alpha^2 \beta_w}{3 M s} \sum_i \int dy_+ \left( \frac{1}{\beta_+} \cosh^2 y_+ - \sinh^2 y_+ \right) \left\{ f_j^A \left( x_a, M^2 \right) f_j^B \left( x_b, M^2 \right) \tilde{P}_{\lambda_1 \lambda_2}(\theta^*, M) \right\} 
+ f_j^A \left( x_a, M^2 \right) f_j^B \left( x_b, M^2 \right) \tilde{P}_{\lambda_1 \lambda_2}(\pi + \theta^*, M) \}
\]

(3.11)

where the integration over \( y_+ \) is restricted by the kinematic cutoff we chose as explained above.

B. Numerical Results

In our numerical calculations, the EHLQ structure function (set 2, \( \Lambda = 290 \text{ MeV} \)) [26] is used and the parameters are chosen as follow

\[
m_w = 81.8 \text{ GeV}, \quad \sin^2 \theta_w = 0.226, \quad \sqrt{S} = 2 \text{ TeV} 
\]

\[
m_u = 3 \text{ MeV}, \quad m_d = 5 \text{ MeV}, \quad m_s = 150 \text{ MeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_b = 5.5 \text{ GeV} 
\]

\[
m_t = 70 \text{ GeV}, \quad M_H = 400 \text{ GeV}, \quad \Gamma_H = 30 \text{ GeV} \quad (3.12)
\]

The KMC matrix is taken to be

\[
\begin{pmatrix}
V_{ud} V_{us} V_{ub} \\
V_{cd} V_{cs} V_{cb} \\
V_{td} V_{ts} V_{tb}
\end{pmatrix} = 
\begin{pmatrix}
0.975 & 0.222 & 0.00954 \\
0.221 & 0.9644 & 0.1455 \\
0.0231 & -0.144 & 0.9893
\end{pmatrix}
\]

(3.13)

Here we ignore the CP phase, but insist on unitarity.

The mass of the Higgs scalar (and that of \( H' \)) are arbitrarily chosen to be 400 GeV with the corresponding width \( \Gamma_H \) [26] just to show a typical resonance behavior. As we shall find out, they have very little effect at Tevatron energy.

In order to see the various couplings' contributions, we plot our results according to the polarization channels and consider the anomalous \( W^+ W^- \gamma \) and \( W^+ W^- Z^0 \) couplings separately. In order to facilitate comparison with the standard model, we plot our results of the standard model with solid line. One point which we
should note is that in the standard model we should consider the contributions from different flavors $i \neq j$ through the diagram, Fig. 4. However, due to the small KMC matrix mixing and/or the small quark mass difference, this mechanism, in fact, contributes little to the $W^+W^-$ production at Tevatron energy. It is lower than others by at least an order of magnitude over the entire kinematical region we considered so that it can be safely ignored. In the figures, the numbers which accompany the dotted lines denote the corresponding coupling which deviates from the standard model by one unit and which contributes to the cross sections of the given channel as indicated in the Figure. In order to see the angle and energy dependences and to estimate the total events, we put the two graphs together in a Figure. We plot the angular distribution by taking the invariant mass of the $W^+W^-$ pair equal to 0.3 TeV. On the other hand, we plot the dependence on the invariant mass of the $W^+W^-$ production by taking a suitable angle to dramatize the variance with $M$ changing.

In refs. [2-19, 29], the so-called “anomalous” moment $\kappa_\lambda$ of $W$ have been discussed a lot, which corresponds to the $f'_3^2$ deviation. We also plot the $\lambda_\nu$ term contributions, which are marked with $\lambda$ in the Figs. The $\lambda_\nu$ term is quite similar to the anomalous moment term. In space time representation, the anomalous moment term is $W^+ W_{\mu} V^{\mu\nu}$, but the $\lambda_\nu$ term is $W^+ W_{\mu} V^{\mu\nu}$, where $W_{\mu}$ is the vector potential of $W$, $W_{\mu
u}$ and $V_{\mu\nu}$, are the field strength tensors of $W$ and $\gamma$ (or $Z^0$) respectively. They are related to the magnetic moment $\mu_w$ and the electric quadrupole moment $Q_w$ of $W$ by

$$
\begin{align*}
\mu_w &= \frac{e}{2m_w} (1 + \kappa_\gamma + \lambda_\gamma) \\
Q_w &= -\frac{1}{\sqrt{2}} (\kappa_\gamma - \lambda_\gamma) .
\end{align*}
$$

(3.14)

In standard model, $\kappa_\nu = 1$ and $\lambda_\nu = 0$ at tree level. $f_1, \kappa$ and $\lambda$ are the only allowed form factors if one insists on global SU(2) invariance upon switching off electromagnetism [24,31].

Owing to the relation in eq. (2.13), the magnitude of channels (12) and (21), (13) and (31), (23) and (32) are equal, hence, we plot only their sum. Similarly, as one can see from eqs. (2.14)-(2.17), the difference in (2.11) and (3.4) channels is down by a power of $\gamma_w^2$ compared to their sum so that we plot their sum only.

Finally, we note that Figures 5, 7, 9, 11 and 13 are for the form factors $f'_1(i =$
IV. Discussions and Conclusions

By examining the figures, we can easily see the following features. Firstly, in standard model, the production cross section of $W$ pairs in the central region (say $60^\circ \leq \theta^* \leq 120^\circ$) drop rapidly as the invariant mass $M$ increase. The major contribution comes from the (12) transverse channel which originates from t-channel exchange diagrams. This rapid drop in cross section is, of course, the result of gauge cancellation as one expected from the standard model. Next, we see that except for $f_8$ and $f_{10}$, an anomalous form factor of order 1 will increase the high mass $W$ pair production cross section by at least an order of magnitude in the appropriately chosen $\theta^*$ region. This is usually the central region or near a dip in $\theta^*$ distribution. Finally, we notice that an anomalous $f_t^*$ in general produce a larger deviation than an anomalous $f_t$, but otherwise, with very similar $\theta^*$ and $M$ distribution. This may make separating the contributions from $f_t^*$ vs. $f_t$ difficult.

Now let us discuss each anomalous form factors in the order of decreasing contributions to the high mass $W$ pair production. From Figs. 9 and 10, we see that the contribution of an anomalous $f_2$ to (33) channel is greater than the corresponding standard model value by more than three orders of magnitude. Moreover, it keeps on increasing even when $M$ reach 550 GeV. Tree level unitarity is badly broken and the $\gamma_\alpha$ factor associated with $f_2$, as one can see from equations (2.14) and (2.15), overwhelms the decrease in parton luminosity as $M$ is increased. The next in line are the form factors $f_t$ and $\lambda$. $f_t$ contributes to (12) channel only while $\lambda$ contributes to (11), (22), (13) and (23) channels. Their contributions are more than two order of magnitudes above the standard model predictions for the transverse channels. The form factors $f_4, f_5, f_6$ contribute significantly only to the (13) and (23) channels. The deviations from the standard model predictions are large only for high mass ($> 300$ GeV) $W$ pairs and depend very much on the $\theta^*$ one chose. Note that the form factors $f_3$ and $\lambda$, for unit deviation from the standard model value, give the same contribution to (13) and (23) channels. $f_3$ also contributes to (11), (22), (33) channels as does $f_1$. Their contributions are more than one order of

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*By anomalous, we mean unit deviations from the corresponding standard model values.*
magnitude above the standard model values. Finally, among the three form factors $f_6^g, f_8^g, f_{10}^g$, only $f_8^g$ contributes appreciably above the standard model predictions to the high mass $W$ pair production cross section. Indeed its contribution to the $(33)$ channel stays at about the same level for $M$ increases from 200 to 500 GeV.

As for the form factors associated with the neutral scalar–vector boson couplings, we find that an order one deviation from the standard model value for these form factors produce, in most cases, no more than a factor of two increases in differential cross sections as is the case for $f_6^g$ and $f_{10}^g$. This is expected as their contributions are suppressed by a factor of $(m_i/M)^2$, where $m_i$ is the mass of the incoming fermions, while the probability of finding heavy quarks remain small for the values of $M$ we considered at Tevatron energy. The contribution from $f_6^g$ is helped by a factor of $\gamma^4$ which compensates for the decrease in heavy quark luminosity.

As noted before, in this paper, we discuss only the polarization cross sections for $W^+W^-$ pair production. To proceed to the correlations of the decay products of the $W^+W^-$ is straightforward and the numerical results will be presented elsewhere.

Now let us draw some conclusions based on the above discussion. At an integrated luminosity of $10 \text{ pb}^{-1}$, we expect, from standard model prediction, to see no high mass ($> 300 \text{ GeV}$) central ($60^\circ \leq \theta^* \leq 120^\circ$) $W^+W^-$ pairs events. Any such event signals new physics. If the new physics are due to anomalous tri–vector boson couplings or Higgs–vector boson couplings which might arise, for example, from a new neutral gauge boson or neutral scalar, then our formalism can be applied to analyze such events. If the integrated luminosity can reach $100 \text{ pb}^{-1}$, we expect to see a few such events. They should come mainly from the $(12)$ channel if standard model is correct.

It is very difficult to see at Tevatron energy any effects of the form factors whose contributions to the cross sections are proportional to the incoming quark masses. To test these form factors as well as to study in detail the polarization cross sections, one may have to wait for the next generation machines such as the SSC [32] or an upgraded high luminosity Tevatron.
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References


Figure Captions

Fig. 1: The three boson vertex. (a) $VW^+W^-$ (b) $H_{fp}W^+W^-$. 

Fig. 2: The Feynman Diagrams for $q\bar{q} \rightarrow W^+W^-$
(a). The $\gamma$ and $z^0$ exchange diagram.
(b). The t-channel diagram.
(c). The scalar exchange diagram.

Fig. 3: $q\bar{q} \rightarrow W^+W^-$ process and its momentum assignment.

Fig. 4: Feynman diagram for $q_i\bar{q}_j \rightarrow WW (i \neq j)$.

Fig. 5: $\frac{d\sigma}{dm_{WW}d\cos\theta}$ for (1,1) + (2,2) polarization channel; solid line is for standard model; dotted lines numbered by 1 and $\lambda$ are for $f_1^\gamma$ and $\lambda\gamma$ respectively with unit deviation from their standard model values.

Fig. 6: $\frac{d\sigma}{dm_{WW}d\cos\theta}$ for (1,1) + (2,2) polarization channel; solid line is for standard model; dotted lines numbered by 1 and $\lambda$ are for $f_1^\gamma, f_8^\gamma$ and $\lambda\gamma$ respectively with unit deviations from their standard model values.
Fig. 7: $\frac{d\sigma}{d\cos\theta^*}$ for (1,2) + (2,1) polarization channel; solid line is for standard model; dotted lines numbered by 6 and 7 are for $f_6^2$ and $f_7^2$ respectively with unit deviations from their standard model values.

Fig. 8: $\frac{d\sigma}{d\cos\theta^*}$ for (1,2) + (2,1) polarization channel; solid line is for standard model; dotted lines numbered by 6, 7 and 10 are for $f_6^2$, $f_7^2$ and $f_{10}^2$ respectively with unit deviations from their standard model values.

Fig. 9: $\frac{d\sigma}{d\cos\theta^*}$ for (3,3) polarization channel; solid line is for standard model; dotted lines numbered by 1, 2 and 3 are for $f_1^3$, $f_2^3$ and $f_3^3$ respectively with unit deviation from their standard model values.

Fig. 10: $\frac{d\sigma}{d\cos\theta^*}$ for (3,3) polarization channel; solid line is for standard model; dotted lines numbered by 1, 2, 3, 4, 8 and 9 are for $f_1^3$, $f_2^3$, $f_3^3$, $f_4^3$, $f_8^3$ and $f_9^3$ respectively with unit deviations from their standard model values.

Fig. 11: $\frac{d\sigma}{d\cos\theta^*}$ for (1,3) + (3,1) polarization channel; solid line is for standard model; dotted lines numbered by 3, 4, 5, 6 and $\lambda$ are for $f_3^3$, $f_4^3$, $f_5^3$, $f_6^3$ and $f_{\lambda}^3$ respectively with unit deviations from their standard model values.

Fig. 12: $\frac{d\sigma}{d\cos\theta^*}$ for (1,3) + (3,1) polarization channel; solid line is for standard model; dotted lines numbered by 3, 4, 5, 6 and $\lambda$ are for $f_3^3$, $f_4^3$, $f_5^3$, $f_6^3$ and $f_{\lambda}^3$ respectively with unit deviations from their standard model values.

Fig. 13: $\frac{d\sigma}{d\cos\theta^*}$ for (2,3) + (3,2) polarization channel; solid line is for standard model; dotted lines numbered by 3, 4, 5, 6, and $\lambda$ are for $f_3^3$, $f_4^3$, $f_5^3$, $f_6^3$ and $f_{\lambda}^3$ respectively with unit deviation from their standard model values.

Fig. 14: $\frac{d\sigma}{d\cos\theta^*}$ for (2,3) + (3,2) polarization channel; solid line is for standard model; dotted lines numbered by 3, 4, 5, 6 and $\lambda$ are for $f_3^3$, $f_4^3$, $f_5^3$, $f_6^3$ and $f_{\lambda}^3$ respectively with unit deviations from their standard model values.
Fig. 5

\[ \frac{d\sigma}{dM \cos \theta^*} \text{ [pb/GeV]} \]

- \( M = 0.3 \text{ TeV} \)
- \( \theta^* = 90 \)

\( (11) + (22) \)
\[ \Delta \psi \frac{d\psi}{dq} \theta \cos \theta \]
Fig. 9
Fig. 14

$d\theta/\theta dM [\text{GeV}] \theta \cos \theta [\text{eV/GeV}]$