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STr M^2 Sum Rule for Superstring Motivated Kähler Potentials [1]

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Abstract

We study the sum rule for the mass squared of the N=1 four dimensional effective supersymmetric theory whose Kähler potential is of the general form that appears in compactified superstrings. We take full account of the non-minimal nature of the models.

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Superstring theories which are tachyon and ghost free can be constructed in non-compact ten dimensional space time. When one imposes the extra condition that the resulting theory be a fully consistent quantum theory, i.e. that it be anomaly free, one finds the extraordinary result that the only allowed gauge groups are either[1] $SO(32)$ or $E(8) \otimes E(8)'$. This opens up the possibility for a consistent unification of gravity with the remaining three forces in a gauge theory where the gauge group is no longer arbitrary and one can ask questions that range from the physics of quantum gravity to the physics of, say, CP violation.

To obtain phenomenology from this type of theories one follows a two-step scenario[2]. First we truncate the infinite superstring spectrum and consider only the zero mass modes with spin less than 5/2, so that we have a form of N=1 supergravity in ten dimensions. Secondly, we compactify on an appropriately chosen six-dimensional manifold, consistent with the equations of motion. Because of the requirement of N=1 supersymmetry in four dimensions and the necessity for chiral fermions in the theory, the gauge group $E(8) \otimes E(8)'$ is reduced to the group $E(6) \otimes E(8)'$.

In this way, we are led to an N=1 supergravity theory in four dimensions, described by a Kähler potential of the form

$$G = \alpha \ln(S + S^*) + \beta \ln(T + T^* - \gamma|\phi|^2/M) - \ln|W|^2 \tag{1}$$

where $\alpha = 1$, $\beta = 3$, γ is unspecified, M is Planck's mass and W is the superpotential which in principle can be a function of S, T, and the matter fields, ϕ. Here, S and T are gauge singlets, while ϕ transform according to the 27 and 27* of $E(6)$. Matter multiplets are singlets under the hidden $E(8)'$group, and the only nonsinglet fields under $E(8)'$ are its gauge supermultiplet in the adjoint representation. In addition, the kinetic energy term for both the visible and hidden gauge sectors is modulated by the factor

$$Q(S) = S \tag{2}$$

Supersymmetry breaking is supposed[3] to take place in the hidden $(E(8)')$ sector via gaugino condensation. This is assumed to occur at a scale much larger than 10^{10} GeV where the corresponding coupling constant goes through its critical value.

The gravitino mass, $m_{3/2}$, that is so generated remains, however, undetermined at the tree level.

Of paramount importance for the study of a system with broken supersymmetry is an analysis of the supersymmetric sum rule[4,5] for STr M^2. The sum rule gives an indication of how the supersymmetry breaking manifests itself in the physics of the components and gives us a handle on the possible phenomenology of the model.

Recently there has been a flurry of activity[6] on this subject, pointing out the heretofore unsuspected quadratic divergence in the theory. We should remark that all the previous calculations in ref.[6] neglected the <u>curved</u> character of the field manifold and more or less resorted to the conventional strategy of expanding about the vacuum. This procedure does not respect the original global symmetries of the Lagrangian[7].

In this paper, we explore in detail the supersymmetric sum rule for the scenarios of the type described by eqn(1) and eqn(2), taking into account the full curved and non-minimal field theoretic structure of the model. This class of theories is not power-counting renormalisable, even though the component Lagrangian has at most two space-time derivatives of the bosonic fields and one space-time derivative of the fermionic components. With a kinetic energy for the boson fields of the form

$$g_{i\bar{j}}\partial_\mu\phi^i\partial_\mu\overline{\phi}^{\bar{j}}$$

where $g_{i\bar{j}}$ is the Kähler metric, there is no "free particle" propagation and standard perturbation theory is not valid. However, by exploiting the Kähler nature of the field manifold and its associated gauge invariance, one can find a (normal) gauge where the metric is locally Kronecker and therefore do standard perturbation theory in that gauge. At the end of the calculation one may re-write the local geometrical quantities in terms of their tensor counterparts such as the curvature tensor, the Ricci tensor, etc, so that the final result is valid for any parametrisation of the field manifold.

Following these methods[5], one finds the following supertrace sum rule for a general matter system coupled to N=1 supergravity in four dimensions

$$\sum_{J=0}^{3/2}(-1)^{2J}(2J+1)M_J^2 =$$

$$-2\{d^a(T_a)^i_i + R^i_j\overline{f}_i f^j - (N-1)[m^2_{3/2} - \frac{1}{2}\kappa^2(Q+\overline{Q})_{ab}d^a d^b]$$
$$+d^a(T_a)^j_i\overline{a}_j\Gamma^i + Tr[\overline{Q}^i(Q+\overline{Q})^{-1}Q_j(Q+\overline{Q})^{-1}]\overline{f}_i f^j$$
$$- Tr[\overline{Q}^j(Q+\overline{Q})^{-1}](T_a)^i_j\overline{a}_i d^a\} \tag{3}$$

where R^j_i is the Ricci tensor of the field manifold, Γ^i is the contracted connection, κ is the gravitational constant and d^a are the appropriate D-terms. The Ricci tensor and the contracted connection are respectively given in an arbitrary Kähler manifold with metric g by

$$R_{\alpha\overline{\beta}} = \partial_\alpha\partial_{\overline{\beta}}\ln\det g$$

and

$$\Gamma_\alpha = \partial_\alpha \ln\det g$$

We are interested in evaluating eqn(3) for the theory described by eqn(1) and eqn(2). With respect to the ground state of the theory, which is where we need to evaluate the STrM^2, all the D-term contributions vanish. To obtain the contribution due to the other terms we calculate the metric g of the field manifold and find ($\hat{S} \equiv S + S^*; \hat{T} \equiv T + T^*; \hat{Z} \equiv \hat{T} - \gamma|\phi|^2/M$)

$$g \equiv M^2 G_{\alpha\overline{\beta}} = \begin{pmatrix} -\frac{\alpha M^2}{\hat{S}^2} & 0 & 0 \\ 0 & -\frac{\beta M^2}{\hat{Z}^2} & \frac{\beta\gamma\phi^A M}{\hat{Z}^2} \\ 0 & \frac{\beta\gamma\overline{\phi}_A M}{\hat{Z}^2} & \frac{-\beta\gamma M}{\hat{Z}}(\delta^A_B + \frac{\gamma\overline{\phi}_B\phi^A}{M\hat{Z}}) \end{pmatrix} \tag{4}$$

which is non-singular and has the inverse

$$g^{-1} \equiv M^{-2}G^{\alpha\overline{\beta}} = \begin{pmatrix} -\frac{\hat{S}^2}{\alpha M^2} & 0 & 0 \\ 0 & -\frac{\hat{Z}\hat{T}}{\beta M^2} & -\frac{\hat{Z}\phi^A}{\beta M^2} \\ 0 & -\frac{\hat{Z}\overline{\phi}_A}{\beta M^2} & -\frac{\hat{Z}}{\beta\gamma M}\delta^A_B \end{pmatrix} \tag{5}$$

It follows from this that

$$\det g^{-1} = \frac{\hat{S}^2}{\alpha\beta M^2}\left(\frac{\hat{Z}}{M}\right)^{n+2}(-\beta\gamma)^{-n}$$

where n indicates the total number of chiral fields in ϕ. For example, if one had three 27's of $E(6)$, then $n = 81$.

The components of the Ricci tensor turn out to be given by

$$R_{S\overline{S}} = \frac{2}{\hat{S}^2} \tag{6}$$

$$R_{T\overline{T}} = \frac{n+2}{\hat{Z}^2} \tag{7}$$

$$R_{T\overline{\phi}_A} = -\gamma\frac{n+2}{M\hat{Z}^2}\phi^A \tag{8}$$

$$R_{\phi^A\overline{\phi}_B} = \gamma\frac{n+2}{M\hat{Z}}\left(\delta^B_A + \frac{\gamma}{M\hat{Z}}\overline{\phi}_A\phi^B\right) \tag{9}$$

and are zero otherwise. This, when combined with eqn(4), shows that the field manifold is the direct product of two Einstein manifolds: one describing the S sector and the other the $(T-\phi)$ sector.

Although unnecessary for our present purposes, we also display here the six components of the contracted connection which are given by

$$\Gamma^S = 2\frac{\hat{S}}{\alpha M^2} \tag{10}$$

$$\Gamma^T = (n+2)\frac{\hat{Z}}{\beta M^2} \tag{11}$$

and

$$\Gamma^{\phi^A} \equiv 0 \tag{12}$$

The last equation is a consequence of the cancellation between the derivatives of $\det(g^{-1})$ and is due to the geometric properties of this particular field manifold. Eqn (10-12) imply that for the class of Kähler potentials considered here, all the contributions to $\mathrm{STr}M^2$ coming from the contracted connection vanish identically even if some of the d^a were not zero.

We also need the auxiliary fields which are calculated from[5]

$$G_{\alpha\overline{\beta}}\overline{f}^{\overline{\beta}} + m_{3/2}G_{\alpha} = 0 \tag{13}$$

although for our purposes, it is only necessary to calculate explicitly f^S, which turns out to be

$$f^S = m_{3/2}\hat{S}\left[1 - \frac{\hat{S}}{\alpha}\frac{\overline{W}_{\overline{S}}}{\overline{W}}\right]$$

Here $m_{3/2}$ is the gravitino mass given by

$$m_{3/2} = M\langle e^{-G/2}\rangle$$

To obtain the total contribution of the Ricci tensor to Eq.(3), it is convenient to take advantage of the "doubly Einstein" character of the manifold and compute $R_{\alpha\overline{\beta}}f^{\alpha}\overline{f}^{\overline{\beta}}$ separating the contributions as those due to the S and $(T-\phi)$ submanifolds. One gets respectively

$$R_{\alpha\overline{\beta}}f^{\alpha}\overline{f}^{\overline{\beta}}\Big|_S = +2m_{3/2}^2\left|1 - \frac{\hat{S}}{\alpha}\frac{W_S}{W}\right|^2 \tag{14}$$

and

$$R_{\alpha\overline{\beta}}f^{\alpha}\overline{f}^{\overline{\beta}}\Big|_{T,\phi} = (n+2)\left\{(1-\gamma|\phi|^2/M^2)\left|1 - \frac{\hat{T}}{\beta}\frac{W_T}{W}\right|^2 + \gamma|\phi|^2/M^2\right.$$
$$\left. + \frac{\hat{Z}}{\beta^2}\left[\frac{M}{\gamma}\frac{W_{\phi}\cdot\overline{W}_{\overline{\phi}}}{|W|^2} + \left(\frac{W_T}{W}\frac{\overline{\phi}\cdot\overline{W}_{\overline{\phi}}}{\overline{W}} + \text{h.c.}\right)\right]\right\} m_{3/2}^2 \tag{15}$$

When we take vacuum expectation values, and set VEV of ϕ equal to zero, eq.(15) becomes

$$R_{\alpha\bar{\beta}} f^{\alpha} \bar{f}^{\bar{\beta}} \Big|_{T,\phi} = (n+2) \left| 1 - \frac{\hat{T}}{\beta} \frac{W_T}{W} \right|^2 m_{3/2}^2 \tag{16}$$

The final contribution to be computed is the one due to the non-minimal gauge kinetic modulator, Eq.(2). This gives

$$Tr \left[\overline{Q}^{i} (Q+\overline{Q})^{-1} Q_j (Q+\overline{Q})^{-1} \right] \bar{f}_i f^j =$$

$$+ m_{3/2}^2 \left| 1 - \frac{\hat{S}}{\alpha} \frac{W_S}{W} \right|^2 \mathtt{dim(Adj)} \tag{17}$$

where $\mathtt{dim(Adj)}$ is to be understood as the sum over the dimension of the adjoint representation of each of the gauge factor groups whose gauge kinetic energy term is modulated by eq (2), i.e. over both the visible and the hidden gauge sectors.

Putting all this together and evaluating at the vacuum, we finally get

$$\mathrm{STr} M^2 = +2 m_{3/2}^2 \left[N - 1 - (n+2) \left| 1 - \frac{\hat{T}}{\beta} \frac{W_T}{W} \right|^2 \right.$$

$$\left. -(2 + \mathtt{dim(Adj)}) \left| 1 - \frac{\hat{S}}{\alpha} \frac{W_S}{W} \right|^2 \right] \tag{18}$$

Here N is the total number of chiral multiplets in the model and is equal to $n+2$.

It is well known that in the standard superstring scenario where $\alpha = 1$, and $\beta = 3$, the $\mathrm{STr} M^2$ sum rule does not vanish but reduces to the standard result[6]

$$\mathrm{STr} M^2 = -2 m_{3/2}^2.$$

Here we have as usual chosen the VEV of S so that G_S vanishes, resulting in a a zero cosmological constant. The standard theory thus has a quadratic divergence at the one loop level that cannot be avoided. In this context our calculation does not offer anything new except that it confirms the previous result, obtained with different methods.

It is interesting to remark however that if one generalizes the superpotential W to include a dependence on both S and T, then our sum rule (eq.18) suggests a possibility of determining the VEV of S and T such that both $\mathrm{STr}M^2$ as well as the cosmological constant vanish. With this generalized W, the cosmological constant is given by the VEV of

$$e^{-G}\left[\alpha\left|1-\frac{\hat{S}}{\alpha}\frac{W_S}{W}\right|^2+\beta\left|1-\frac{\hat{T}}{\beta}\frac{W_T}{W}\right|^2-3\right]$$

For the superstring values of α and β, the resulting simultaneous equations that follow from our conditions possess a solution only if

$$n > 3(2+\mathtt{dim(Adj)})-1$$

which constrains the number of matter multiplets present in the theory. For the standard scenario of $E(6)\otimes E(8)'$, this implies that there be at least thirty-seven 27's in the theory. By relaxing the values of α and β away from 1 and 3, one may entertain scenarios where the number of 27's is smaller and still have a vanishing $\mathrm{STr}M^2$ and cosmological constant, so long as

$$\alpha < 3\frac{2+\mathtt{dim(Adj)}}{n+1}$$

$$\beta > 3\frac{n+2}{n+1}.$$

Lastly we remark that conventional scenarios have resulted in a negative value for $\mathrm{STr}M^2$, which not only has led to a quadratic divergence of the theory, but has the wrong sign for the effective potential and has a destabilizing influence on the theory[8]. Our result (eq.18) offers the alternative of, if not a vanishing quadratic divergence as discussed above, at least a positive definite (though divergent) effective potential with at the same time a zero cosmological constant. The above constraint on the least number of matter multiplets continues to hold true in this case.

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