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SUSY GUT with Automatic Hierarchy and Low Energy Physics

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Abstract

A mechanism is suggested which resolves in a natural way the doublet–triplet hierarchy problem of the SUSY $SU(5)$ theory. Under this mechanism, which we call GIFT ("Goldstones Instead of Fine Tuning"), the doublets are the pseudogoldstone bosons of the (sponteneously broken) $SU(6)$ symmetry of the super potential and remain massless unless supersymmetry is broken. However, generally they acquire some mass of the order of the scale at which SUSY is broken.

The GIFT mechanism was applied to a specific case of SUSY breaking through SUGRA. In calculation of the effective low energy Lagrangian GIFT allows us to find the otherwise arbitrary parameters for the initial conditions (at the scale of grand

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unification) for the renormalization group equations. As usual, the electroweak breaking is due to the Yukawa coupling of the t quark. It turns out that at fixed high energy parameters (the gravitino mass $m_{3/2}$ and the parameter A), the correct breaking occurs only in a narrow range of the values of the t quark mass. There exists an absolute upper limit for its mass $m_t < 52\ GeV$.

Of the two massless in the limit of the exact SUSY Higgs fields, one acquires the mass $2m_{3/2}$ while the second gets its mass only due to radiative corrections, this mass being calculable and likely to be near $2.2\ GeV$. All other masses are also fixed but unknown numerically since the values of high energy parameters are unknown.

At the end of the paper, we consider a slightly different mechanism of symmetry breaking, a-lá Coleman–Weinberg type.

I. Introduction

Any theory of grand unification encounters the well-known "hierarchy problem," namely, the problem of the existence of at least two mass scales which differ by many orders of magnitude. For the $SU(5)$ theory, the masses of the colorless weak Higgs doublet and the color triplet (weak singlet) being comprised in a single $SU(5)$ pentaplet should differ by 13–14 orders of magnitude since the exchange of the triplet leads to proton decay while the doublet is, in fact, the usual Glashow–Salam–Weinberg weak doublet. The most severe part of this problem is readily resolved in a supersymmetric version of grand unification: the radiative corrections do not spoil hierarchies once built into the tree approximation [1-3]. Nevertheless, one usually achieves the desired hierarchy by a certain fine tuning of the parameters of the free Lagrangian.

Consider, for example, the minimal SUSY $SU(5)$ theory. This theory contains the Higgs fields, $\underline{24}$ - plet Φ, $\underline{5}$ - plet H_1 and $\underline{5}^*$ - plet H_2, which are the scalar components of the chiral superfields $\hat{\Phi}, \hat{H}_1, \hat{H}_2$. The most general form of the superpotential for this theory is

$$W = \frac{1}{2}M \ \ Sp\hat{\Phi}^2 + \frac{\lambda}{3}Sp\hat{\Phi}^3 + f\left(\hat{H}_2\hat{\Phi}\hat{H}_1\right) + m\left(\hat{H}_2\hat{H}_1\right) \ . \tag{1.1}$$

From this, one immediately obtains that the Higgs potential is equal to

$$\begin{aligned} V &= Sp|M\Phi + \lambda\Phi^2 - \frac{1}{5}Sp\Phi^2 + fH_2 x H_1|^2 \\ &+ H_2(m+f\Phi)\left(m+f\Phi^+\right)H_2^+ + H_1^+\left(m+f\Phi^+\right)(m+f\Phi)H_1 \end{aligned} \tag{1.2}$$

The supersymmetric minimum of $V(V=0)$

$$<\Phi>=\frac{M}{\lambda}\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} , <H_1>=<H_2>=0 \tag{1.3}$$

corresponds to the $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ breaking. Then it follows from (1.1) that the masses of H_1 and H_2 are $m + f < \Phi >$, i.e., the triplet mass is

$m + 2fM/\lambda$, while for the doublet it is $m - 3fM/\lambda$. Thus, to have the massless or almost massless doublets, one should impose the relation

$$m = \frac{3fM}{\lambda}. \tag{1.4}$$

As it was already mentioned, this equation would not be destroyed by radiative corrections. However, the relation (1.4) itself appears to be very artificial.

Several attempts have been made to explain the doublet–triplet hierarchy in a natural way. In ref. [4], this hierarchy is caused by the group structure of the Higgs sector ("missing partner mechanism"). The price for this is that a Higgs content of the theory is to be quite complicated. In ref. [5], it is the "sliding singlet mechanism" [6] that provides the mass hierarchy. This mechanism has been criticized, however, for being unstable for SUSY breaking in hidden sector at $\sim 10^{10}\ GeV$[7].

A rather simple way has been suggested in refs. [8–9]. However, it includes either explicit [9] or more implicit [8] fine tuning. In the implicit form, it is, in fact, the assumed equality of the coupling constants of singlet and 24- plet Higgs fields. This equality is not maintained by any symmetry and presents a sort of fine tuning. If not for supersymmetry, it would have been destroyed by radiative corrections.

In the papers[10], the local $SU(6)$ group is considered. This theory again has a very complicated Higgs sector.

At last, in ref. [11], the E_6 group is discussed in connection with the superstring models.

We shall confine ourselves by the simplest $SU(5)$ supersymmetric theory with the minimal Higgs content. A very simple mechanism will be proposed which automatically guarantees the masslessness of the doublets in the scale of grand unification mass M while the triplets would get the mass of the order of M. This is achieved by constructing a model in which the doublets appear to be pseudogoldstone bosons of a certain broken global symmetry of the superpotential, which is not the symmetry of a whole theory. More accurately, this is one of the doublets which turns out to be a genuine pseudogoldstone boson while the second remains massless as a superpartner of the first one. When the explicit breaking of supersymmetry is taken into account, which we do by means of supergravity [12], the first doublet remains massless. The reason is that the additional terms in the Lagrangian are connected with supergravity and they obey the assumed global symmetry. Thus, the spectrum still contains a genuine pseudogoldstone boson. The second Higgs acquires a mass of order of $m_{3/2}$, the gravitino mass. At last, when the gauge and Yukawa couplings are taken into account, the first doublet also acquires some mass since these couplings do not possess the global symmetry in question. Thus, the mass of the lightest Higgs boson, as well as the breakdown of the standard group $SU(3)_c \times SU(2)_L \times U(1)$ is due to radiative corrections.

Obviously the described scenario consists of two different parts: first, it relates to the pure supersymmetric theory and provides a strict masslessness for the doublets. This mechanism seems to be of rather general character, we call it "GIFT," "Goldstones Instead of Fine Tuning." The second part is connected with the explicit form of SUSY breaking; it allows to develop the effective low energy theory and to obtain some predictions for experiment. This consideration is similar to that of refs [13-18]. Actually, with the help of the GIFT, we are able to calculate the values of some constants of the effective low energy Lagrangian which are usually assumed to be the arbitrary phenomenological constants. It appears, as in [13-18], that the existence of the t quark is essential for the breaking of the electroweak group. The mass of the t quark is about 40–50 GeV (the upper bound is 52 GeV), and its exact value depends on the only unknown numerical parameter A depending on the properties of the hidden sector of supergravity. An interesting feature of the solution is that at fixed A, the Yukawa coupling of the t quark, and, consequently, its mass, is almost fixed, but varies in a range of the order of 50–200 MeV. Eventually, we calculate the masses of all the particles in the theory including the most interesting mass of the lightest Higgs boson. That one turns out to be 2.25 GeV.

The paper is organized as follows. In Section II, the GIFT mechanism is described in the context of the simplest supersymmetric $SU(5)$ theory. In Section III, we formulate a consistent low energy model by the assumption that the SUSY breaking takes place through the supergravity. In Section IV, we calculate the renormalization of the parameters of the effective low energy Lagrangian, consider a question of electroweak breaking and estimate the masses of the particles. In Section V, we consider a slightly different mechanism of the electroweak symmetry breaking, a-lá Coleman and Weinberg. Lastly, in Section VI, we summarize the main results of the paper. Some results described in the first part of this paper have already been published [19].

II. Solution of the Hierarchy Problem

The simplest possible content of the Higgs fields for the $SU(5)$ SUSY GUT is two 5's, $H_1 \sim \underline{5}$ and $H_2 \sim \underline{5}^*$, and one 24-plet, $\Phi \sim \underline{24}$. All these fields are the scalar components of the corresponding chiral fields which we denote $\hat{H}_1, \hat{H}_2, \hat{\Phi}$. Let us add to these fields a singlet $SU(5)$ chiral field $\hat{\phi}$. The 35 fields altogether can be comprised in a single adjoint representation of some $SU(6)$ group. If $\hat{\Sigma} \sim \underline{35}$ of the $SU(6)$, then the decomposition of $\hat{\Sigma}$ in $\hat{H}_1, \hat{H}_2, \hat{\Phi}$ and $\hat{\phi}$ is:

$$\hat{\Sigma} = \begin{pmatrix} \frac{-5\hat{\phi}}{\sqrt{30}} & \hat{H}_1 \\ \hat{H}_2, & \hat{\Phi}_{ij} + \delta_{ij}\frac{\hat{\phi}}{\sqrt{30}} \end{pmatrix} \tag{2.1}$$

where the "hypercharge" $SU(6)$ matrix $Y_{35} = \left[\frac{-5}{\sqrt{30}}, \frac{1}{\sqrt{30}} \cdots \frac{1}{\sqrt{30}}\right]_{diag}$ is normalized to unity $SpY_{35}^2 = 1$.

Suppose now that the Higgs sector of the theory possesses a higher symmetry as compared to the gauged $SU(5)$, namely, the $SU(6)$ global symmetry. Then, the superpotential depends only on one field, $\hat{\Sigma}$, and has the simplest form

$$W = \frac{1}{2}MSp\hat{\Sigma}^2 + \frac{1}{3}\lambda Sp\hat{\Sigma}^3 . \tag{2.2}$$

In terms of the $SU(5)$ fields that means, of course, some relations between otherwise independent coefficients in the superpotential. One easily gets from (2.2):

$$\begin{aligned} W &= \frac{1}{2}MSp\hat{\Phi}^2 + M\left(\hat{H}_2\hat{H}_1\right) + \frac{1}{2}M\hat{\phi}^2 + \frac{1}{3}\lambda Sp\hat{\Phi}^3 + \lambda\left(\hat{H}_2\hat{\Phi}H_1\right) \\ &- \frac{2}{3}\sqrt{\frac{2}{15}}\lambda\hat{\phi}^3 + \frac{\lambda}{\sqrt{30}}\left(Sp\hat{\Phi}^2\right)\hat{\phi} - 2\sqrt{\frac{2}{15}}\hat{H}_2\hat{H}_1\hat{\phi} . \end{aligned} \tag{2.3}$$

A superpotential of the type of (2.2) for the general case of the $SU(n)$ theory ($\hat{\Sigma} \sim (n^2-1)$) leads to the potential V:

$$V = Sp|M\Sigma + \lambda\Sigma^2 - \frac{\lambda}{n}Sp\Sigma^2|^2 \tag{2.4}$$

where $|M\Sigma + ...|^2$ implies the product of the matrix $\left(M\Sigma^\alpha_\beta + \lambda\left(\Sigma^2\right)^\alpha_\beta - \delta^\alpha_\beta\frac{\lambda}{n}Sp\Sigma^2\right)$ and its hermition conjugated.

There is a set of degenerated supersymmetric vacua of zero energy corresponding to the symmetry breakdown

$$SU(n) \to SU(m) \mathrm{x} SU(n-m) \mathrm{x} U(1), \tag{2.5}$$

$(m = 1, ... n-1)$. The vacuum expectation values (v.e.v.'s) $< \Sigma >$ for these vacua are

$$< \Sigma >= \begin{pmatrix} x & & & & & & & & & \\ & x & m & & & & & & & \\ & & \cdot & & & & & & & \\ & & & \cdot & & & & & & \\ & & & & \cdot & & & & & \\ & & & & & y & & & & \\ & & & & & & \cdot & n-m & & \\ & & & & & & & \cdot & & \\ & & & & & & & & \cdot & \\ & & & & & & & & & y \end{pmatrix}, \quad x = \frac{M}{\lambda}\ \frac{n-m}{2m-n}, y = \frac{M}{\lambda}\ \frac{-m}{2m-n} \tag{2.6}$$

Any of these zero energy states can be chosen as a real physical ground state.

Suppose that for the $SU(6)$ symmetric potential the ground state preserves the $SU(4)$ x $SU(2)$ x $U(1)$ symmetry, i.e.,

$$< \Sigma >= \frac{M}{\lambda} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}. \tag{2.7}$$

As follows from the decomposition (2.1) in terms of the $SU(5)$ classification, this v.e.v. means that

$$< \phi >= -\frac{M}{\lambda}\sqrt{\frac{6}{5}}, \quad < \Phi >= \frac{M}{\lambda}\begin{pmatrix} 6/5 & & & & \\ & 6/5 & & & \\ & & 6/5 & & \\ & & & -9/5 & \\ & & & & -9/5 \end{pmatrix}. \tag{2.8}$$

The gauged $SU(5)$ symmetry is broken to the $SU(3)$ x $SU(2)$ x $U(1)$, as is necessary. On the other hand, one sees from (2.7) that there remains an additional symmetry between color and the $SU(5)$ singlet ϕ. The number of broken generators is altogether equal to $35 - (15 + 3 + 1) = 16$. This leads to 16 Goldstone bosons 12 of which are eaten up by the Higgs mechanism and provide the masses for X and Y gauge bosons of the $SU(5)$. The remaining four Goldstones are $SU(2)$ weak doublets. One can see this from the structure of $< \Sigma >$. These Goldstones are attached to those broken generators of the $SU(6)$ which connect the $SU(5)$ singlet to the $SU(2)$ doublet. Thus, at least one massless doublet Higgs boson is expected (four real degrees of freedom). However, the supersymmetry ensures that both doublet fields, which are comprised in H_1 and H_2, appear to be massless.

To understand this, one can imagine for a moment that the $SU(6)$ symmetry is gauged. Then, with the real $< \Sigma >$ and the pure imaginary generators in the adjoining representation $\underline{35}$, only the hermition part of Σ, that is $\Sigma + \Sigma^{+}$, could mix with the gauge bosons. The hermition field $\Sigma + \Sigma^{+}$ contains $H_1 + H_2^{+}$, the doublet part of which is, therefore, a genuine Goldstone boson. However, the mass of the doublet part of $H_1 - H_2^{+}$ also vanishes because of supersymmetry. This is, in fact, nothing but the degeneration of two real scalar physical degrees of freedom of any chiral field.

These group theoretical arguments can be varified in a straightforward way. For this, one should make a substitution

$$\hat{\Phi} =< \Phi > +\hat{\Phi}', \quad \hat{\phi} =< \phi > +\hat{\phi}' \tag{2.9}$$

and to rewrite the superpotential (2.3) in terms of $\hat{\Phi}'$ and $\hat{\phi}'$. The validity of such a procedure is based on the fact that supersymmetry is not broken. No F terms have nonvanishing vacuum expectation values, so we can consider $< \Phi > \neq 0$ and

$<\phi>\neq 0$ as a superfield $<\hat{\Phi}>\neq 0$ and $<\hat{\phi}>\neq 0$ with a single nonvanishing A-component. Subtracting $<\hat{\Phi}>$ and $<\hat{\phi}>$ from $\hat{\Phi}$ and $\hat{\phi}$ we get (2.9). Would the supersymmetry not have been preserved, one could not use the superpotential and would be forced to calculate the potential V itself.

From (2.9) and (2.3), we obtain

$$\begin{aligned} W &= \hat{H}_2 m \hat{H}_1 + \frac{3}{2} M Sp\left(\hat{A}^2 - \hat{D}^2\right) + \frac{13}{10} M \hat{\phi}'^2 + \sqrt{\frac{6}{5}} M Sp \hat{A} \hat{\phi}' \\ &+ \frac{1}{3}\lambda Sp \hat{\Phi}'^3 - \frac{2}{3}\sqrt{\frac{2}{15}}\lambda \hat{\phi}'^3 + \lambda\left(\hat{H}_2 \hat{\Phi}' \hat{H}_1\right) + \frac{\lambda}{\sqrt{30}} \hat{\phi}' Sp \hat{\Phi}'^2 - 2\sqrt{\frac{2}{15}}\lambda \hat{H}_2 \hat{H}_1 \hat{\phi}', \\ m &= M \begin{pmatrix} 3,0 \\ 0,0 \end{pmatrix}, \hat{\Phi}' = \begin{pmatrix} \hat{A}, \hat{B} \\ \hat{C}, \hat{D} \end{pmatrix}, Sp\hat{A} = -Sp\hat{D} \ . \end{aligned} \tag{2.10}$$

Here we use partly the notations (for m and $\hat{\Phi}'$) where we explicitly separated triplet color and weak doublet indices. We see that the doublets from $\hat{H}_1$ and $\hat{H}_2$ remain massless. Using (2.10) one can describe the whole spectrum of the particles. Apart from massless doublets one has:

1. Massive color triplet scalar superfields of the mass 3M entering $\hat{H}_1$ and $\hat{H}_2$.

2. Twelve massive gauge bosons of the $SU(5)$ (X and Y bosons), in which the corresponding nondiagonal $(\Phi' + \Phi'^{+})_{nondiag.} = B + C^{+}$ Goldstone bosons are absorbed. The mass of these bosons is $M_X = M_Y = (3/\sqrt{2})g(M/\lambda)$. The scalar bosons $B - C^{+}$ acquire their mass from the D terms:

$$\begin{aligned} D_a^2 &= \frac{g^2}{8}\left(\Phi^{+} t^a \Phi\right)^2 = \frac{1}{4} g^2 Sp\left[\Phi^{+}, \Phi\right]^2 \Rightarrow (\Phi = <\Phi> + \Phi') \Rightarrow \\ & \frac{1}{4} g^2 Sp\left[<\Phi>, \Phi' - \Phi'^{+}\right] = \frac{9}{2} g^2 \left(\frac{M}{\lambda}\right)^2 Sp\left(B - C^{+}\right)\left(B^{+} - C\right) \ . \end{aligned} \tag{2.11}$$

These bosons are degenerated with X and Y gauge bosons and give altogether 12 x 3 + 6 x 2 = 48 boson degrees of freedom. There are also 12 massive Dirac fermions (12 x 4 = 48 fermionic degrees of freedom). The lefthanded components of these fermions are actually the chiral fields $\hat{B}$ and $\hat{C}$ while their righthanded components are the X and Y calibrinos.

3. There remain $\hat{A}$ and $\hat{D}$ superfields. The traceless part of $\hat{A}$ transforms as (8,1) of $SU(3)_c$ x $SU(2)_L$ while the traceless part of $\hat{D}$ is (1,3). Their mass is 3M. There is also a $\hat{\Phi}_{24}$ field in $\hat{A}$ and $\hat{D}$ connected to the weak hypercharge

$$\frac{\lambda_{24}}{\sqrt{2}} = \sqrt{\frac{2}{15}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix} . \tag{2.12}$$

The $\hat{\Phi}_{24}$ field is mixed with the $SU(5)$ singlet $\hat{\phi}$. Two mixtures

$$\hat{X}_1 = \frac{3}{\sqrt{10}}\hat{\phi} + \frac{1}{\sqrt{10}}\hat{\Phi}_{24}, \hat{X}_2 = -\frac{1}{\sqrt{10}}\hat{\phi} + \frac{2}{\sqrt{10}}\hat{\Phi}_{24} \tag{2.13}$$

have the masses 3M and M respectively.

III. Supergravity and the Lagrangian of the Light Fields

To formulate a consistent low-energy theory, one should point out the mechanism of the supersymmetry breaking. We assume that a soft SUSY breaking takes place through the supergravity which is probably the most popular version nowadays. Then, instead of the potential (2.4) (for n=6), one has [20]:

$$\begin{aligned} V &= Sp|M\Sigma + \lambda\Sigma^2 - \frac{\lambda}{6}Sp\Sigma^2 + m_{3/2}\Sigma^+|^2 \\ &+ (A-3)m_{3/2}Sp\left(\frac{1}{2}M\Sigma^2 + \frac{1}{2}M\Sigma^{+2} + \frac{1}{3}\lambda\Sigma^3 + \frac{1}{3}\lambda\Sigma^{+3}\right), \end{aligned} \tag{3.1}$$

where $m_{3/2}$ is the gravitino mass ($m_{3/2} \sim 100GeV$), and A is the numerical constant depending on the hidden sector of the theory (generally $A \sim 1$).

Since $m_{3/2} \ll M$, one can look for the minimum of V in the form of eq. (2.7):

$$< \Sigma >= x \cdot \frac{M}{\lambda} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} , \tag{3.2}$$

and find x as a perturbation series in $m_{3/2}/M$. One gets

$$x = 1 + \frac{m_{3/2}}{M} + (A - 3)\frac{m_{3/2}^2}{M^2} + 0\left(\frac{m_{3/2}^3}{M^3}\right) . \tag{3.3}$$

To obtain the masses of the scalar doublet fields H_1 and H_2, (since we are no more interested in triplets, from now on we shall call doublet fields by the same notations, H_1 and H_2, as we have previously used for 5's) one should turn to the potential arising from the superpotential (2.3). This is because, as it was explained before, in the case when supersymmetry is broken, we cannot use the superpotential any more. We shall not write down a rather cumbersome expression which comes out when the substitution $\Sigma =< \Sigma > +\Sigma'$ $(\Phi =< \Phi > +\Phi', \phi =< \phi > +\phi')$ in a full potential, including H_1 and H_2 is done. Let us only give the final result making a few short comments. There are two sources for the mass of the scalars H_1, H_2: (a) nonequality of x to unity, and (b) the direct "supergravity contribution" of the type of (3.1). The straightforward calculation shows that these contributions cancel each other in the leading order $M^2\left(m_{3/2}/M\right) = M_{3/2}m$ for the (mass)^2 for H_1 and H_2. This is known to be a general property [21]. In the next order, one can obtain the following mass terms:

$$\begin{aligned} V &= 2m_{3/2}^2\left(H_1^+H_1 + H_2^+H_2 - H_1H_2 - H_1^+H_2^+\right) \\ &= 4m_{3/2}^2\left(\frac{H_1^+ - H_2}{\sqrt{2}}\right)\left(\frac{H_1 - H_2^+}{\sqrt{2}}\right) . \end{aligned} \tag{3.4}$$

So the field $(H_1 - H_2^+)/\sqrt{2}$ acquires the mass $2m_{3/2}$ while the field $(H_1 + H_2^+)/\sqrt{2}$ remains massless. This is not surprising since $(H_1 + H_2^+)/\sqrt{2}$ is a genuine Goldstone boson. Curiously, the mass of $(H_1 - H_2^+)/\sqrt{2}$ turns out to be independent on the parameter A.

Another mass term, arising from SUSY breaking, is the mass of spinor components of $\hat{H}_1$ and $\hat{H}_2$, i.e., the Higgsinos masses. One can easily see that this mass appears in the first order in $m_{3/2}/M$ for $x-1$. A simple calculation gives the result for this mass term

$$- m_{3/2} \left(\tilde{H}_1 C \tilde{H}_2 \right) + h.c. \ , \tag{3.5}$$

where $\tilde{H}_1$ and $\tilde{H}_2$ are the Higgsinos lefthanded chiral fields and C is the charge conjugation matrix.

Compare these results with the "phenomenological" low energy Lagrangian of the most general form [21]. Evidently, the superpotential can only be written in the form

$$W = -\mu_0 \left(\hat{H}_1 \hat{H}_2 \right) \ . \tag{3.6}$$

Then with the SUSY breaking through SUGRA, the Higgs potential of the scalar fields is

$$\begin{aligned} V(H_1, H_2) \ &= \ \left(\mu_0^2 + m_{3/2}^2 \right) \left(H_1^+ H_1 + H_2^+ H_2 \right) - B m_{3/2} \mu_0 \left(H_1 H_2 + H_1^+ H_2^+ \right) \\ &+ \ \frac{g'^2}{8} \left(H_1^+ H_1 - H_2^+ H_2 \right)^2 + \frac{g^2}{8} \left(H_1^+ \vec{\tau} H_1 + H_2^+ \left(-\vec{\tau}^* \right) H_2 \right)^2 . \end{aligned} \tag{3.7}$$

The last terms are the D terms, and B is an unknown parameter. Being compared to the eq. (3.4), this gives

$$\mu_0 = m_{3/2} \ \ , B = 2 \ . \tag{3.8}$$

The value $\mu_0 = m_{3/2}$ is also in an agreement with the Higgsinos mass (3.5). Thus, the GIFT mechanism allows us to calculate the phenomenological parameters of the low energy Lagrangian.

IV. Renormalization and Electroweak Symmetry Breaking

We can now analyze the question of the electroweak breaking. Leaving in (3.7) only

$$< H_1^0 >= v_1/\sqrt{2} \ \ , < H_2^0 >= v_2/\sqrt{2} \tag{4.1}$$

we get

$$V \ = \ \frac{1}{2} \ \mu_1^2 v_1^2 + \frac{1}{2} \ \mu_2^2 v_2^2 - \mu_3^2 v_1 v_2 + \frac{\bar{g}^2}{32} \left(v_1^2 - v_2^2 \right)^2 \ \ , \tag{4.2}$$

$$\mu_1^2 = \mu_2^2 = \mu_0^2 + m_{3/2}^2 \ \ , \mu_3^2 = Bm_{3/2}\mu_0 \ \ , \bar{g}^2 = g^2 + g'^2$$

or, with the relations (3.8) taken into account

$$\mu_1^2 = \mu_2^2 = \mu_3^2 = 2m_{3/2}^2 \ . \tag{4.3}$$

In general cases when $\mu_1^2 \neq \mu_2^2 \neq \mu_3^2$, some conditions are necessary to have symmetry breaking. These conditions are

$$\frac{1}{2}(\mu_1^2 + \mu_2^2) \geq \mu_3^2 \geq \sqrt{\mu_1^2\mu_2^2} \ . \tag{4.4}$$

The first of these inequalities is necessary to ensure stability at $v_1 = v_2$. If it is not satisfied, $V \to -\infty$ at $|v_1| = |v_2| \to \infty$. The second condition makes the point $v_1 = v_2 = 0$ to be unstable since in this point $\| \partial^2 V/\partial v_i \partial v_j \| < 0$.

The eq. (4.3) shows that we are at the edge of symmetry breaking. However, in the values (4.3), the effects of renormalization are neglected. Since we get the conditions (4.3) from the grand unification theory, it is reasonable to assume that this form is valid at the scale of order of grand unification mass (or even Plank mass). Then, at the energies $\sim 100\ GeV$ $\mu_1^2 \neq \mu_2^2 \neq \mu_3^2$ because of renormalization. This line of reasoning is quite popular now [13–18]. In calculation of renormalization effects, we shall follow closely refs. [17–18].

Suppose first that we take into account only the gauge couplings of the $SU(3)_c$ x $SU(2)_L$ x $U(1)$ neglecting the Yukawa couplings. Then the relation $\mu_1^2 = \mu_2^2$ is preserved while $\mu_3^2 \neq \mu_1^2 = \mu_2^2$. The conditions (4.4) are not satisfied and the symmetry breaking is impossible. That means that one should turn to the Yukawa couplings which distinguish H_1 and H_2 and, therefore, result in $\mu_1^2 \neq \mu_2^2$. Of these couplings, we leave only the Yukawa coupling of t quark and t quarkino of the form

$$L_Y = h_t \left[\bar{\hat{t}}_R \left(\hat{H}_1 \varepsilon \hat{Q}_L \right) \right]_{F-term} = h_t \bar{\hat{t}} \left(\hat{H}_1^0 \hat{t}_L - \hat{H}_1^+ \hat{b}_L \right)_{F-term} \ . \tag{4.5}$$

The renormalization group equations for this case were first obtained in ref. [13].

In the notations of ref. [18], we get

$$
\begin{aligned}
\mu_1^2(\ell_0) &= \mu_0^2 q^2(\ell_0) + m_{3/2}^2 a_1(\ell_0) = m_{3/2}^2\left[q_1^2(\ell_0) + a_1(\ell_0)\right] \ , \\
\mu_2^2(\ell_0) &= \mu_0^2 q^2(\ell_0) + m_{3/2}^2 a_2(\ell_0) = m_{3/2}^2\left[q_1^2(\ell_0) + a_2(\ell_0)\right] \ , \qquad (4.6)\\
\mu_3^2(\ell_0) &= m_{3/2}\mu_0 B\cdot q(\ell_0)\cdot\left(\frac{\tilde{B}(\ell_o)}{B}\right) = 2m_{3/2}^2\left[q(\ell_0)\cdot\frac{\tilde{B}(\ell_0)}{B}\right] \ , \ell_0 = log\frac{M_{GUT}^2}{M_W^2} \ .
\end{aligned}
$$

Using the eqs. of the Appendix of [18], we can find the functions $q(\ell_0), a_1(\ell_0)$, $a_2(\ell_0), \tilde{B}(\ell_0)$.[2] For simplicity, we assume that the calibrino mass $M_{1/2} = 0(\tilde{\gamma} =$ $= M_{1/2}/m_{3/2} = 0)$ which is, though not literally acceptable phenomenologically, corresponding to "minimal SUGRA coupling". Then we obtain for $q(\ell_0)$:

$$
\begin{aligned}
q(\ell_0) &= \frac{\left[\lambda_2(\ell_0)\right]^{\frac{3}{2b_2}}}{\left[\lambda_1(\ell_0)\right]_{\frac{1}{3b_1}}}\cdot\frac{1}{\left[D(\ell_0)\right]^{1/4}} \ , \\
\lambda_j(\ell_0) &= 1 + b_j\,\frac{g_j^2(0)}{16\pi^2}\,log\,\frac{M_{GUT}^2}{M_W^2} \ , \frac{g_{3(0)}^2}{4\pi} = \frac{g_{2(0)}^2}{4\pi} = \frac{5}{3}\frac{g_1^2(0)}{4\pi} = \frac{1}{24} \ , \qquad (4.7)\\
b_1 &= 11, b_2 = 1, b_3 = -3, M_{GUT} \simeq 6.10^{15} GeV, \ell_0 \simeq 64 \ .
\end{aligned}
$$

The coupling constants $g_3(0), g_2(0), g_1(0)$ of the $SU(3)$ x $SU(2)$ x $U(1)$ in (4.7) are taken at grand unification scale M_{GUT}. The function $D(\ell)$ governs the evaluation of the Yukawa coupling constant $h_t(\ell)$. The connection between $h_t(\ell_0)$, which determines the t quark mass ($m_t = h_t(\ell_0)v_1/\sqrt{2}$ and $h_t(0)$, the coupling at M_{GUT} is given by:

$$
\begin{aligned}
\left[h_t(0)\right]^2 &= \left[h_t(\ell_0)\right]^2\frac{D(\ell_0)}{E(\ell_0)} \ , D(\ell_0) = \frac{1}{1 - \frac{6F(\ell_0)}{E(\ell_0)}\frac{\left[h_t(\ell_0)\right]^2}{16\pi^2}} \ , \\
E(\ell_0) &= exp\ \int_0^{\ell_0}\left[\frac{16}{3}g_3^2(\ell) + 3g_2^2(\ell) + \frac{13}{9}g_1^2(\ell)\right]\frac{d\ell}{16\pi^2} \ , \qquad (4.8)\\
F(\ell_0) &= \int_0^{\ell_0} E(\ell)\,d\ell \ .
\end{aligned}
$$

Since the gauge couplings are known while $h_t(\ell_0)$ is not a-priori fixed (since we do not know the t quark mass) we put in the numerical values of g_1, g_2, g_3 and obtain

---

[2]Note that μ_1 and μ_2 are mixed up with each other in this Appendix.

[18]:

$$E(\ell_0) \simeq 13 \quad , \quad F(\ell_0) \simeq 290 \quad , D(\ell_0) = \frac{1}{1 - 0.848\left[h_t(\ell_0)\right]^2} \quad ,$$

$$\left[h_t(0)\right]^2 = \frac{\left[h_t(\ell_0)\right]^2}{13} \frac{1}{1 - 0.848\left[h_t(\ell_0)\right]^2} \, . \tag{4.9}$$

For the function $q(\ell_0)$, we find:

$$q(\ell_0) = 1.28\left(1 - 0.85h_t^2\right)^{1/4} \simeq 1.28\left(1 - 0.21h_t^2 - 0.068h_t^4\right) \quad , \tag{4.10}$$

where $h_t = h_t(\ell_0) = \sqrt{2}m_t/v_1$. In the last equation (4.10), we made an expansion in h_t^2 since we shall see later that $h_t^2 \ll 1$.

Now we should calculate $a_1(\ell_0), a_2(\ell_0)$ and $\tilde{B}(\ell_0)$ of eq. (4.6). The renormalization of $a_2(\ell_0)$ is determined only by the Yukawa couplings of the bottom quark which has been neglected. Hence:

$$a_2(\ell_0) = 1 \, . \tag{4.11}$$

As to the functions $a_1(\ell_0)$ and $\tilde{B}(\ell_0)$, they unfortunately depend also on the parameter A introduced in the previous section. A is determined by the v.e.v.'s of the hidden sector and is the coefficient in the cubic term of the potential: $Am_{3/2}[scalar fields]^3$. One can find for $a_1(\ell_0)$:

$$a_1(\ell_0) = 1 - 0.424(A^2 + 3)h_t^2 + 0.360A^2h_t^4 \quad . \tag{4.12}$$

At last for $\tilde{B}(\ell_0)/B(B = 2)$, one has

$$\tilde{B}(\ell_0)/B = 1 + 0.212Ah_t^2 \quad . \tag{4.13}$$

From the eqs. (4.10)-(4.13), we get the renormalized values of μ_1^2, μ_2^2 and μ_3^2:

$$\begin{aligned} \mu_1^2 &= m_{3/2}^2\left[2.64 - \left(1.97 + 0.42A^2\right)h_t^2 + \left(0.36A^2 - 0.15\right)h_t^4\right] \quad , \\ \mu_2^2 &= m_{3/2}^2\left[2.64 - 0.70h_t^2 - 0.15h_t^4\right] \quad , \\ \mu_3^2 &= m_{3/2}^2\left[2.56 + 0.54\left(A - 1\right)h_t^2 - \left(0.17 + 0.115A\right)h_t^4\right] \, . \end{aligned} \tag{4.14}$$

As it was already mentioned, without the Yukawa coupling $(h_t = 0)\mu_1^2 = \mu_2^2 \neq \mu_3^2$. An instructive feature of the solution (4.14) is that μ_3^2 is almost equal to $\mu_1^2 = \mu_2^2$, though the overall renormalization is not so small. That means that the terms $\sim h_t^2, h_t^4$ should also be small, $h_t^2 \ll 1$.

In order to satisfy the conditions (4.4), μ_3^2 should lay between two very close values $(\mu_1^2 + \mu_2^2)/2$ and $\sqrt{\mu_1^2\mu_2^2}$. Indeed, though μ_1^2 differs from μ_2^2 in order $0(h_t^2)$ the difference between $(\mu_1^2 + \mu_2^2)/2$ and $\sqrt{\mu_1^2\mu_2^2}$ is of the next order, i.e., $0(h_t^4)$. This means that there exists a narrow range of h_t^2, with the width $\sim h_t^4$, in which the condition (4.4) is satisfied. The maximum value of h_t^2 is determined by the equation

$$\mu_3^2 = \frac{\mu_1^2 + \mu_2^2}{2} \ . \tag{4.15}$$

This gives approximately

$$h_{max}^2 = \frac{1}{10}\ \frac{1}{1 + 0.675A + 0.263A^2} + \frac{1}{1000}\ \frac{0.25 + 1.44A + 2.25A^2}{(1 + 0.675A + 0.263A^2)^3} \ . \tag{4.16}$$

The minimal value h_{min}^2 is given by the condition

$$\mu_3^2 = \sqrt{\mu_1^2\mu_2^2} \ . \tag{4.17}$$

We can expand the difference between h_{max}^2 and h_{min}^2 in h_t^2 and obtain:

$$h_{max}^2 - h_{min}^2 = \frac{(0.31 + 0.10A^2)^2}{1 + 0.675A + 0.263A^2} h_{max}^4 \ . \tag{4.18}$$

These limits for h_t determine the maximum and minimum value of the t quark mass. Since $\mu_1^2 \simeq \mu_2^2$, the symmetry breaking takes place at $v_1 \simeq v_2$. Hence:

$$m_t = \frac{h_t v_1}{\sqrt{2}} = \frac{h_t v}{2} \ , v = (C_F\sqrt{2})^{-1/2} = 246 \ \ GeV \ . \tag{4.19}$$

Consider two numerical examples. At $A = 0, h_{max}^2 \simeq 0.10, h_{max}^2 - h_{min}^2 \simeq 0.001, m_t^{max} = 39.36GeV, m_t^{max} - m_t^{min} \simeq 0.2GeV$; at $A = 3, h_{max}^2 \simeq 0.02, m_t^{max} = 16.82GeV, m_t^{max} - m_t^{min} = 0.04GeV$. It is interesting that there exists an absolute upper limit for the t quark mass (corresponding to $A = -1.28$) This limit is

$$m_t < 51.7GeV \ . \tag{4.20}$$

However, if A is treated as a free parameter, there is no lower limit for m_t.

The general solution of the equation $dV/dv_i = 0$ with V determined by (4.2) is (for $v_1 < v_2$):

$$\frac{v_1}{v_2} = \left[\frac{\mu_1^2 + \mu_2^2 - \sqrt{(\mu_1^2 + \mu_2^2)^2 - 4\mu_3^4}}{\mu_1^2 + \mu_2^2 + \sqrt{(\mu_1^2 + \mu_2^2)^2 - 4\mu_3^4}}\right]^{1/2} ,$$

$$m_Z^2 = \frac{\bar{g}^2(v_1^2 + v_2^2)}{4} = (\mu_1^2 + \mu_2^2)\left[\frac{|\mu_1^2 - \mu_2^2|}{\sqrt{(\mu_1^2 + \mu_2^2)^2 - 4\mu_3^4}} - 1\right] . \tag{4.21}$$

We see that for our case $v_1 \approx v_2$, while for $v(v^2 = v_1^2 + v_2^2)$ one can obtain from (4.21) the expression in which h_{max} and h_{min} as defined by eqs. (4.15)-(4.18) are used:

$$m_Z^2 = \frac{\bar{g}^2 v^2}{4} = \left(\mu_1^2 + \mu_2^2\right)\left[\sqrt{\frac{h_{max}^2 - h_{min}^2}{h_{max}^2 - h_t^2}} - 1\right] . \tag{4.22}$$

When h_t varies from h_{min} to h_{max}, v runs from $v = 0$ to $v = \infty$.

The mass of the pseudoscalar neutral particle which is, in fact $(v_1 ImH_2^0 - v_2 ImH_1^0)$, generally is

$$m_P^2 = \mu_1^2 + \mu_2^2 . \tag{4.23}$$

We have from the eq. (4.14):

$$m_P^2 \simeq 5.28 m_{3/2}^2 . \tag{4.24}$$

The mass of the charged Higgs equals $m_{H^+}^2 = m_P^2 + m_W^2$.

For the neutral scalar particles, the general expression is

$$m_{s_{H,L}}^2 = \frac{1}{2}\left(m_P^2 + m_Z^2\right) \pm \frac{1}{2}\sqrt{\left(m_P^2 + m_Z^2\right)^2 - 4m_Z^2 m_P^2 cos^2 2\theta} , \tag{4.25}$$

where $sin2\theta = 2\mu_3^2/(\mu_1^2 + \mu_2^2)$.

Since $cos2\theta \approx 0$, the heavier scalar is almost degenerated with the charged Higgs, while for the light scalar, one has:

$$m_{s_L}^2 = \frac{m_P^2 m_Z^2}{m_P^2 + m_Z^2} cos^2 2\theta . \tag{4.26}$$

This is the mass of the Higgs particle, $(H_1 + H_2^+)$, which is exactly massless in the limit $\mu_1^2 = \mu_2^2 = \mu_3^2$. The $cos^2 2\theta$ is proportional to $\mu_1^2 + \mu_2^2 - 2\mu_3^2$; this, in turn, is proportional to $h^2_{max} - h^2_t$. The latter quantity could be expressed from eq. (4.22) as

$$\begin{aligned} h^2_{max} - h^2_t &= \left(\frac{m_P^2}{m_Z^2 + m_P^2}\right)^2 \left(h^2_{max} - h^2_{min}\right) \\ &= \left(\frac{m_P^2}{m_Z^2 + m_P^2}\right)^2 \frac{0.31 + 0.1A^2}{1 + 0.675A + 0.263A} h^4_{max} \end{aligned} \tag{4.27}$$

so that eventually one obtains

$$m_{s_L} = \frac{m_P^3 m_Z m_t^2}{(m_P^2 + m_Z^2)^{3/2} v^2} \left(0.96 + 0.31A^2\right) \quad . \tag{4.28}$$

Since $[m_P^3/(m_P^2 + m_Z^2)^{3/2}] < 1$ we have, using (4.16),

$$m_{s_L} \leq m_Z \frac{1}{40} \frac{0.96 + 0.31A^2}{1 + 0.675A + 0.263A^2} \approx \frac{m_Z}{40} = 2.25GeV \quad . \tag{4.29}$$

If $m_P \gg m_Z$ then $m_{s_L} = 2.25GeV$. Of course, actually this limit is almost saturated at $m_P^2 \gtrsim m_Z^2$. So $m_{s_L} \simeq 2.25GeV$ if $m_{3/2} \gtrsim 40 - 45GeV$. (See eq. (4.24)).

Now consider the masses of the superpartners of the W and Z mixed with the Higgsinos. For the charged Higgsino-Winos mass, one has at $v_1 = v_2 = v/\sqrt{2}$ two massive Dirac spinors with the masses:

$$m^2_{H,L}\left(\tilde{W}\right) = \frac{1}{2}\mu'^2 + \frac{g^2 v^2}{4} \pm \frac{1}{2}\mu' \sqrt{\mu'^2 + g^2 v^2} \quad , \tag{4.30}$$

where $\mu' = m_{3/2} q(\ell_0) = 1.28\ m_{3/2}$ (see eq. (4.10)). If $\mu' \gg m_W$ (i.e., $m_{3/2} > 70 - 80GeV$) the lighter $\tilde{W}_L$ has the mass $m(\tilde{W}_L) = m_W^2/\mu'$. Since there is the experimental limit $m(\tilde{W}_L) > 23GeV$, one has $m_{3/2} < 220GeV$.

For the neutral Higgsino-zinos and photino, there are altogether four Majarano spinors. Their masses are

$$\begin{aligned} m^2_{1,2}\left(\tilde{Z}\right) &= \frac{1}{2}\mu'^2 + \frac{\bar{g}^2 v^2}{4} \pm \frac{1}{2}\mu' \sqrt{\mu'^2 + \bar{g}^2 v^2} \quad , \\ m_3\left(\tilde{Z}\right) &= \mu' \quad , m_4\left(\tilde{\gamma}\right) = 0 \ . \end{aligned} \tag{4.31}$$

The masslessness of the photino is, of course, a result of the simplification made by the assumption of the "minimal" character of SUGRA.

At last, it is interesting to estimate the masses of the superpartners for quarks and leptons. Within the accuracy $0(h_t^2)$, all these masses, but the $\tilde{t}$-quarkino, are equal to $m_{3/2}$ (the renormalization corrections are of the order of $\sim h_t^2 = 4m_t^2/v^2$). For $\tilde{t}$-quarkino, it is easy to get:

$$m^2_{\tilde{t}_h, \tilde{t}_l} = m_{3/2}^2 \pm (A+1)\, m_{3/2} m_t. \tag{4.32}$$

This comes out as a result of the mixing of lefthanded and righthanded quarkinos which are the superpartners of t_L and t_R quarks.

V. The Coleman-Weinberg Type of Symmetry Breaking

The renormalization effects destroy the symmetry $\mu_1^2 = \mu_2^2 = \mu_3^2$ which is due to the GIFT mechanism. The "large-scale" renormalization, however necessary in a real world it may be, seems to lie somewhat outside of the scope of the model considered. From a purely theoretical point of view one can ask a question: What would happen to the theory with the Higgs potential if no large-scale renormalization takes place? That question seems to be at least self-consistent for the model described.

The answer, of course, is that at first place, the Coleman-Weinberg corrections to the potential should be calculated. Indeed, at the tree level, the potential (4.2) in the limit $\mu_1^2 = \mu_2^2 = \mu_3^2 = 2m^2$ has the form:

$$\begin{aligned} V_0 = m^2 (v_1 - v_2)^2 + \frac{\bar{g}^2}{32}\left(v_1^2 - v_2^2\right)^2 &= 2\eta^2 \left(m^2 + \frac{\bar{g}^2}{16}\xi^2\right) , \\ \xi = \frac{v_1 + v_2}{\sqrt{2}} \ , \eta &= \frac{v_1 - v_2}{\sqrt{2}} \end{aligned} \tag{5.1}$$

(We change the notation $m_{3/2} \to m$). The minimum of V_0 corresponds to $< \eta >= 0 (v_1 = v_2)$, but does not determine $< \xi >$. This quantity should be found from one loop Coleman-Weinberg potential. As is well known, this one is given by:

$$64\pi^2 V_1 (\xi_1 \eta) = \sum_i (-1)^F m_i^4 \, log\, m_i^2 \ , \tag{5.2}$$

where m_i are the masses of all the particles in the external field (ξ, η) and (-1) stands for all the fermions.

It is easy to see that the minimum at $<\eta>=0$ remains valid for $V_0 \to V_0+V_1$. (That is because V_1 depends actually on η^2). Thus, one has to calculate V_1 at $\eta=0$, i.e., at $v_1=v_2, \xi=v$.

Let us start by neglecting the Yukawa coupling so that only the contribution of H_1, H_2, W, Z and their fermion superpartners are taken into account in (5.2). One obtains

$$
\begin{aligned}
64\pi^2 V_1(v) &= 2\left(\frac{g^2v^2}{4}+4m^2\right)^2 log\frac{\frac{g^2v^2}{4}+4m^2}{\Lambda^2} + 6\left(\frac{g^2v^2}{4}\right)^2 log\frac{\left(\frac{g^2v^2}{4}\right)}{\Lambda^2} \\
&- 4\left(\frac{g^2v^2}{4}+\frac{1}{2}m^2+\frac{1}{2}m\sqrt{m^2+g^2v^2}\right) log\,\frac{\frac{g^2v^2}{4}+\frac{1}{2}m^2+\frac{1}{2}m\sqrt{m^2+g^2v^2}}{\Lambda^2} \\
&- 4\left(\frac{g^2v^2}{4}+\frac{1}{2}m^2-\frac{1}{2}m\sqrt{m^2+g^2v^2}\right) log\,\frac{\frac{g^2v^2}{4}+\frac{1}{2}m^2-\frac{1}{2}m\sqrt{m^2+g^2v^2}}{\Lambda^2} \\
&+ \frac{1}{2}(g\to\bar{g})\ . \qquad (5.3)
\end{aligned}
$$

Here first terms are the contributions from H^+ and W, while the negative terms correspond to Higgsino-wino (Cf. eq. (4.30). The terms with $g\to\bar{g}$ are the contributions of the neutral particles.

The potential (5.3) has no cutoff dependence apart from an unessential additive constant $(42m^4log\Lambda^2)$. This partly comes as a surprise. The cancellation of the $v^4log\Lambda^2$ terms is understandable. At $m=0$ and $v_1=v_2$, the supersymmetry is actually not violated so that the masses of the bosons in an external fixed v is still equal to the masses of the fermions. As to the cancellation of the $m^2v^2log\Lambda^2$ terms, it looks rather "accidental". For example, we shall see below that when t quark and $\tilde{t}$ quarkino contribution is included, there is no such cancellation.

Anyhow, the expression (5.3) can be rewritten in a form not containing Λ. Omitting an additive constant, one gets

$$
\frac{64\pi^2 V_1}{m^4} = f(x)+\frac{1}{2}f(\bar{x})\ ,x=\frac{g^2v^2}{4m^2}\ ,\bar{x}=\frac{\bar{g}^2v^2}{4m^2}\ , \qquad (5.4)
$$

$$
f(x)=28logx + 2(x+4)^2 log\frac{x+4}{x} - 4(2x+1)\ \sqrt{4x+1}\,log\,\frac{1+2x+\sqrt{4x+1}}{2x}\ .
$$

This x dependence gives the curvature of the potential along the valley $v_1=v_2$. Unfortunately $f(x)$ has no minimum, but maximum at $x=2.2$ while at

$x \to \infty f(x) \simeq -8x \to -\infty$. So instead of self-consistent symmetry breaking we have an instability at large v. The situation is immediately improved if the contribution of t-quark-quarkino is added to (5.3). This one is

$$\begin{aligned} 64\pi^2 V_1^{(t)} &= 6\left[m^2 + m(A+1)\left(\frac{h_t v}{2}\right) + \left(\frac{h_t v}{2}\right)^2\right]^2 log\frac{m^2 + m(A+1)\frac{h_t v}{2} + \left(\frac{h_t v}{2}\right)^2}{\Lambda^2} \\ &+ 6\left[m^2 - m(A+1)\left(\frac{h_t v}{2}\right) + \left(\frac{h_t v}{2}\right)^2\right]^2 log\frac{m^2 - m(A+1)\frac{h_t v}{2} + \left(\frac{h_t v}{2}\right)^2}{\Lambda^2} \\ &- 12\left(\frac{h_t v}{4}\right)^4 log\frac{\left(\frac{h_t v}{2}\right)^2}{\Lambda^2} \quad , \end{aligned} \tag{5.5}$$

where h_t is the Yukawa constant and A is the same parameter as before. The asymptotics of this expression at large v is

$$64\pi^2 V_1^{(t)} \sim 3m^2 h_t^2 v^2 \left[(A+1)^2 + 2\right] log\ v^2 \quad , \tag{5.6}$$

while the asymptotics of V_1 (5.4) is

$$64\pi^2 V_1 \sim -2m^2\left(g^2 + \frac{1}{2}\bar{g}^2\right) v^2 \ , \tag{5.7}$$

which means that at v large enough, the sum $V_1^{(t)} + V_1$ is positive. There is a minimum in the function $V_1^{(t)} + V_1$, and if one put $v = (G_F\sqrt{2})^{-1/2}$ in this minimum, as required, one can calculate the Higgs mass which is independent on Λ. This is, of course, the well known Coleman-Weinberg mechanism. At $h_t v/2 \gg m$ one finds

$$m^2_{S_L} = \left(\frac{d^2\left(V_1^{(t)} + V_1\right)}{dv^2}\right)_{v=v_{min}} = \frac{3}{4\pi^2}\left(\frac{m^2 m_t^2}{v^2}\right)\left[(A+1)^2 + 2\right] . \tag{5.8}$$

This is different from the expression (4.28) for this mass of the previous section.

VI. Conclusion

Let us summarize briefly the main results of the paper. First, we suggested mechanism which explains naturally the masslessness of the doublets comprised in 5's of the $SU(5)$ SUSY theory. The reason for this is that the doublets are the pseudogoldstone bosons of the global broken $SU(6)$ symmetry of the superpotential. Though this symmetry is not a symmetry of the whole theory, doublets nevertheless remain massless until supersymmetry is broken. When it is broken, doublets acquire, in general, some mass of order of the scale of SUSY breaking.

This general "GIFT" mechanism was applied to a specific case of SUSY breaking through supergravity. For this case, even after SUSY breaking, one of the doublets remains massless at the tree level since it remains to be the pseudogoldstone boson for the tree potential. However, with the supersymmetry broken, it acquires some mass due to radiative corrections. We considered first in details when this happens owing to the renormalization of the parameters of low energy potential. We showed that at fixed values of the parameters given by SUGRA ($m_{3/2}$, the gravitino mass, and A, which is the parameter connected to the v.e.v.'s of the hidden sector of the theory) only a narrow interval in masses of t quark gives the correct electroweak symmetry breaking. There is an absolute upper limit for the mass of t quark which is $m_t < 52GeV$. The mass of the lightest scaler particle is likely to be near 2.2 GeV. All other masses are also fixed, but unknown numerically since the values of the parameters $m_{3/2}, A$ are unknown.

In the last section of the paper, we considered a slightly different mechanism of symmetry breaking, of the Coleman-Weinberg type. If "large-scale" renormalization of the parameters of the Higgs potential takes place, then the Coleman-Weinberg corrections are only of few percents and can safely be neglected. However, if the symmetry, which is due to the GIFT mechanism, persists at rather low energies, then the Coleman-Weinberg type treatment is necessary. It turns out that here again the Yukawa coupling of t quark is essential to provide the correct symmetry breaking. The mass of the lightest scalar boson is calculatable, but this time it is dependent on $m_{3/2}$ and A.

In conclusion, we notice that the GIFT mechanism itself seems to be of much more general character than the specific consideration connected to SUGRA SUSY breaking.

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