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Quantum fluctuations and eternal inflation in the R^2 model

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Abstract

We show that in the R^2 inflationary model, as in the scalar field case, quantum fluctuations at early times can be sufficiently large that the universe evolves like a *random walk*. Within this picture we describe the resulting global structure of the universe: the so-called "eternal inflation" scenario. Such behaviour can naturally fit into a picture of "quantum creation of the universe". Inflating domains are present today and in fact are growing in number. Approximately every $10^{-31}s$ new hot radiation dominated domains are created which occupy a volume larger than all the previously existing Friedmann universes; during the first $\sim 10^{-9}$ fraction of this period the power law expanding volume exceeds the inflating volume. Regularly, numerous domains occur where inflation proceeds purely classicaly and sufficiently to solve the problems of standard cosmology.

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Inflationary universe models have in recent years had great success in solving a number of important cosmological problems [1-9]. There have been two main types of inflationary models:

i)those involving the domination of the vacuum energy by a scalar field which has culminated in the chaotic universe scenario proposed by Linde [5].

ii)higher order corrections to Einstein's field equation. These were first constructed by Starobinsky using the trace anomaly [6] of the vacuum energy momentum tensor of conformally coupled quantum fields. However typical numbers and types of fields in the early universe would not produce enough inflation. Therefore a term of the form ϵR^2 was added to the lagrangian [7-9, 19] which results in a (quasi) de Sitter expansion of sufficient size. This model also naturally accepts a chaotic inflation interpretation [9-10].

It has been realized, first in the context of the new inflationary model [2-3] that due to the nature of quantum fluctuations in the field driving inflation, parts of the universe might still be inflating [11][1]. In the chaotic universe model the fields are much larger and the effect of fluctuations is more dramatic: they can result in an eternally expanding universe with domains of stability (those not expanding exponentially) being infinitesimal compared to the total universe [12-14]. In this paper we are interested in the existence and magnitude of fluctuations in R^2 type inflation and whether they can dominate to produce a similar type eternal universe. We first briefly review the eternal chaotic universe scenario.

Consider a sufficiently large scalar field $10^5\mu > \phi > \mu$ (where $\mu^2 = 3(2\pi^2)/8\pi G$, $M_p^2 = G^{-1}$, and M_p is the Planck mass $\sim 10^{19}$GeV) that comes to dominate the energy density of the universe. One assumes that the initial configuration is such that spatial gradients can be neglected, that is, the characteristic scale l on which inhomogeneities show up is much larger than the horizon size H^{-1}. The field then rolls towards its minimum during which time the universe expands exponentially. The roll down time depends on both classical and quantum effects: for a massive scalar field the classical change in ϕ for a Hubble time (t=H^{-1}) is $\Delta\phi \sim M_p^2/\phi$ [12].

[1] A similar thing can occur in the original inflationary model if the probability of "bubble formation" is sufficiently small.

Computing quantum effects while treating the scalar field as effectively homogeneous ($l >> H^{-1}$), one finds that quantum fluctuations can either decrease or increase ϕ by an amount ~H during an Hubble time. This quantum fluctuation $\delta\phi$ has the magnitude $\sim m\phi/M_p$ for a field of mass m [12]. Therefore $|\delta\phi| > \Delta\phi$ if $\phi > \phi_* \equiv M_p(M_p/m)^{1/2}$, which is $\sim 100M_p$ for a realistic model [12], if the mass is restricted to be $m < 10^{-5}M_p$, as required by the limits on the anisotropy of the microwave background. Consider a single domain with an homogeneous field $\phi > \phi_*$ of size H^{-1}; the number of horizon size regions grows like $e^{3Ht} \sim e^3$ (for the Hubble time). In typically half of these regions the fluctuations will increase ϕ: these domains in turn will expand independently and can create further domains with fields $\phi > \phi_*$. The other domains where fluctuations have decreased ϕ below ϕ_* expand exponentially to some (large) finite size and evolve eventually into a universe like our own. However, because the Hubble parameter increases with ϕ the main part of the total universe is made up of the expansion of domains with the largest fields [12-14].

Following closely the analysis of [9] we now quickly review the R^2 type inflation and show that quantum fluctuations lead to a similar behaviour. The evolution equation for the scalar curvature R has the form [6]

$$\Box R + \frac{R}{6\epsilon} = 0 \tag{1}$$

R therefore behaves like a damped harmonic oscillator; when ϵ is large the "potential" is flat so that there is sufficient time for inflation to occur. The curvature and Hubble parameters are related by $R = 12H^2 + 6\dot{H}$. If $\epsilon R >> 1$ there is a (quasi) de Sitter expansion ($\dot{H} << H^2$) with the Hubble parameter being a linearly decreasing function of time

$$H(t) = H_i(1 - \frac{t}{36\epsilon H_i}) \tag{2}$$

The total inflation is $18\epsilon H_i^2$ e-folds (throughout the subscript i denotes values at the start of the de Sitter expansion, e refers to those at the end): to solve the various cosmological problems [1] requires ~70 e-folds. When the de Sitter stage finishes reheating occurs due to an oscillating scale factor-for details see [9, 16-17]. From considerations of scalar field perturbations epsilon is constrained such that: $10^{11} < \epsilon^{-1/2} < 10^{13}$ GeV. [9]. Therefore sufficient inflation requires $H_i \geq H_h \sim 10^{-5}M_p$.

For our purposes it is convenient to work in the so called conformal picture so we perform a transformation of the form [18,19] (conformal quantities are denoted by a $\sim$)

$$\tilde{g}_{\mu\nu} = (1 + 2\epsilon R) g_{\mu\nu} \tag{3}$$

where for a Friedmann Robertson-Walker(FRW) metric the conformal time is related to physical time by

$$d\tilde{t} = (1 + 2\epsilon R)^{1/2} dt \tag{4}$$

There is again a self consistent solution for $\epsilon R >> 1$ which is a constant conformal Hubble parameter and a linearly decreasing R [9]

$$\tilde{H} = \frac{1}{\sqrt{24\epsilon}} \tag{5}$$

$$R(\tilde{t}) = R_i - \frac{\tilde{t}}{3\epsilon\sqrt{6\epsilon}} \tag{6}$$

The Lagrangian $\mathcal{L} = -(16\pi G)^{-1}(R + \epsilon R^2)$ becomes in the conformal picture

$$\mathcal{L} = \frac{6\epsilon^2}{16\pi G(1 + 2\epsilon R)^2} \left(-\dot{R}^2 + \frac{R^2}{6\epsilon} \right) - \frac{\tilde{R}}{16\pi G} \tag{7}$$

In this Lagrangian the physical curvature behaves like a massive scalar field of mass $m \simeq \epsilon^{-1/2}$ coupled to the conformally transformed metric. This is not surprising since $R + R^2$ gravity does have a graviton and a massive scalar in its spectrum [20]. Using the substitution $x = \frac{1}{2} ln(1 + 2\epsilon R)$ we can rewrite this as

$$\mathcal{L} = -\frac{1}{2}(\mu\sqrt{2}\dot{x})^2 + \frac{\mu^2}{96\epsilon}\left(1 - e^{-2x}\right)^2, \tag{8}$$

This Lagrangian now has a regular kinetic term with an unusual potential. However, for $\epsilon R >> 1$ the potential term contributes only a cosmological constant and we can therefore use the results of [21] to determine the quantum fluctuations. The fluctuations are described by $\langle x^2 \rangle$, given by

$$\langle x^2 \rangle = \frac{\tilde{H}^3 \tilde{t}}{8\pi^2 \mu^2} \tag{9}$$

We wish to calculate the fluctuation δR in curvature: since $\delta R = 2R\delta$x, where $\delta x = \sqrt{\langle x^2 \rangle}$

$$\delta R = 2R \left(\frac{\tilde{H}^3 \tilde{t}}{8\pi^2 \mu^2} \right)^{\frac{1}{2}} \tag{10}$$

In analogy with the scalar field case we wish to compare the size of the quantum fluctuation with the classical decrease in R over the same timescale. For a Hubble time ($\tilde{H}^{-1}$) the quantum fluctuation has the magnitude

$$\delta R = \frac{2R\tilde{H}}{\pi \mu \sqrt{8}} \tag{11}$$

This agrees (up to a *logk* factor) with the amplitude of fluctuations of any wavelength at the time of horizon crossing [9], as it should. The classical change in curvature ΔR for the same conformal time is (from eq.6)

$$\Delta R = \frac{2}{3\epsilon} \tag{12}$$

Therefore quantum fluctuations are dominant when $\delta R > \Delta R$ that is for

$$H > \left(\frac{\mu}{\sqrt{\epsilon}} \right)^{\frac{1}{2}} \simeq 10^{-3} M_p, \tag{13}$$

where $\epsilon^{-1/2} \sim 10^{12}$ GeV. has been used, as required by the analysis of the scalar perturbations [9]. Observe that this is a rather natural value for the Hubble parameter, being the geometric mean of the two mass scales present in the model. In physical variables the dispersion is given by,

$$\langle \delta R^2 \rangle = \frac{8G}{\pi \epsilon} H^5 \delta t \tag{14}$$

It is modified with respect to the scalar field case Eq.(9), since the curvature has dimension two. One could compute corrections for smaller ϵR but we are not interested in these here. A similar formula to (14) has been conjectured by Pollock [23].

Our value for the quantum fluctuations in curvature also coincides with those obtained by Vilenkin [16], who considered quantum tunnelling to the conformal anomaly driven Starobinsky inflation. However the physical context is somewhat different: in Vilenkin's case the early phase is determined by the trace anomaly term $\sim R^2 log R/M$, (where M is some (high) subtraction mass), which dominates the R^2 term for $R > M$. The probability for quantum creation of the universe is peaked around a value of the Hubble parameter (H_o) (fixed by the particle content and is typically of Planck size) which is the only self-consistent classical inflationary solution. The spread of the probability distribution for quantum creation around H_o is (eq.(5.19) in ref. [16])-cf. eq.(11)

$$\frac{\delta R}{R} = \frac{\sqrt{G} H_o}{\sqrt{2\pi}} \tag{15}$$

In the R^2 model however, there is a self-consistent inflationary solution (quasi-de Sitter) for every $R >> \epsilon^{-1}$: so the probability distribution for the quantum creation of the universe allows different curvatures at the beginning of the classical evolution [10]. The spread of this distribution is separate to the effects considered in this paper which are the fluctuations around any possible classical inflationary trajectory, regardless of the boundary conditions in Quantum Cosmology. Vilenkin's results and ours have to agree purely for dimensional reasons.

The evolution of the Hubble parameter in the R^2 model is shown on Fig.1. Quantum fluctuations modify the early phase for $H_i > H_* \sim 10^{-3} M_p$. We shall call this phase the *random walk* inflationary phase. For smaller Hubble parameters the effect of quantum fluctuations can be neglected and the evolution proceeds purely classically [9]. As long as $H_i > 10^{-5} M_p$, this classical inflationary phase is sufficient to solve all the problems inflation is called for.

The reason that such fluctuations in the gravitational field show up so prominently below the Planck scale is due not only to the higher derivative term in the gravitational lagrangian, but also because we made the coefficient of the R^2 term large in order to have acceptably low anisotropy of the microwave background. It is this smallness of G/ϵ that makes possible the use of the result [21], to produce a picture of a quantum field in a fixed space-time background. At the same time, this is the weakness of the R^2 or any other inflationary model since we cannot compute such a characteristically small parameter from first principles; or to relate it to some other, physically relevant sector of the theory.

There is however another way to describe the dynamics of de Sitter universe by utilising the method of Starobinsky [15]. Rederivation of our result in this context offers a clearer physical picture of the influence that quantum fluctuations have, and it allows a simple and transparent proof of the eternal inflation. Although the method is applicable to any inflationary model, our analysis will be for the R^2 model only. This is because for the scalar field case, the effective cosmological constant depends on the value of the inflaton field, resulting in a random walk with variable step length. In the R^2 case there is a random walk with constant step length which simplifies the analysis. Due to the classical change this random walk is taking place on a down-hill slope. Thus, the probability distribution is centered on $\phi(t)$, or $R(t)$ which evolve according to the classical equations, while the spread grows as the square root of the elapsed time measured in Hubble units.

Define $\Phi = \mu\sqrt{2}x$, in accordance with Eq.(8). The main idea of the Starobinsky method is to split Φ, such that:

$$\Phi = \Phi_L + \Phi_S \tag{16}$$

where Φ_S contains only modes with physical wavelength less than $\epsilon_s{}^{-1}H^{-1}$, and Φ_L contains all modes with wavelengths larger than $\epsilon_s{}^{-1}H^{-1}$: ϵ_s is some small number which will be specified as follows. We want Φ_L to be our inflaton field. Thus, it has to be homogeneous for the massive scalar field on scales $l \sim m^{-1}$ and larger; for the R^2 model this corresponds to $l \sim \sqrt{2\epsilon R}\sqrt{log(2\epsilon R)}H^{-1}$. Therefore, we shall choose $\epsilon_s \sim \mu/\phi_i$ for the massive scalar field case, and $\epsilon_s \sim 1/\sqrt{2\epsilon R}$ for the R^2 case. This is the weakest point of the construction. As one may conclude from the discussion below, the most natural coarse-graining scale is presumably H^{-1}, that is $\epsilon_s \sim 1$.

Here, we find that Φ_L is defined on domains which contain at least 10^3 and up to 10^{18} horizon volumes! This is because the limit on anisotropy of the microwave background makes ϵ large. The method works regardless of the value of ϵ_s, which makes far more precise the picture outlined before, which used a "homogeneous" inflaton field. A straightforward calculation [15] shows that the equation of motion for the inflaton field Φ_L is the Langevin equation,

$$\dot{\Phi}_L = -\frac{V'(\Phi_L)}{3\tilde{H}} - \xi \tag{17}$$

where the last term describes the effect of the short wavelength modes i.e., Φ_S, on the coarse-grained field. One finds, [15],

$$\langle \xi(\tilde{t}_1, \vec{x}_1)\xi(\tilde{t}_2, \vec{x}_2)\rangle = \frac{\tilde{H}^3}{4\pi^2}\delta(\tilde{t}_1 - \tilde{t}_2)\frac{sin[\epsilon_s \tilde{a}\tilde{H}|\vec{x}_1 - \vec{x}_2|]}{\epsilon_s \tilde{a}\tilde{H}|\vec{x}_1 - \vec{x}_2|} \tag{18}$$

The noise term becomes uncorrelated on *physical* distances greater than $l \sim \epsilon_s{}^{-1}\tilde{H}^{-1}$. As a result, domains l^3 over which Φ_L is smooth, are stochastically independent of each other and Φ_L can jump to new values, as given by the noise term. This has a simple interpretation.

Consider a mode with wavelength just below l. In one Hubble time the expansion stretches it beyond l for an amount $\lambda(H^{-1}) - \lambda(0) \simeq \lambda(0)$, and it becomes a new contribution to Φ_L. Its amplitude is picked at random, so one can think of it as a random signal propagating from short distances up to distances $\sim l$. The expansion of the domain $\sim l^3$ results in about twenty new domains, over which Φ_L will have slightly different values, due to the superposition of all inflowing modes. If the magnitude of the noise term is sufficiently large compared to the classical force, one has the evolution of the universe being a constant fragmentation of the l^3 domains into new ones, where each of them is stochastically independently evolving. We come back to the picture already outlined but the size and meaning of domains are clarified now. From the correlations Eq.(18) one obtains dispersions as given in Eqs. (9) and (14).

We shall proceed to describe this early phase by using simple properties of random walks. Another technique is to introduce a probability distribution for Φ_L and to straightforwardly [25] write down its appropriate Fokker-Planck equation [11,13,15]. Let us only observe that in evaluating the kinetic coefficient for the diffusion term one again finds that H_* is the boundary above which the diffusion term is dominated by quantum fluctuations. Further, this Fokker-Planck equation can be transformed into a Schrodinger-like equation which, for our purpose, can then be solved in the semiclassical approximation [22]. It is straightforward to check that what is being said here could use either of these methods.

The analysis of the random walk inflationary phase proceeds by counting domains assuming the universe started with at least one domain of size l. This is reasonable, since from Quantum Cosmology we know that the Lorentzian evolution begins when the size of the universe is greater than the horizon size. After some period when the initial anisotropy, inhomogeneity and kinetic term are smoothed out, the size of the universe could be sufficiently large as to encompass at least one l-sized domain.

We shall use the conformal picture: one step takes place after each $\delta\tilde{t} \sim \tilde{H}^{-1}$. In what follows, we shall write for simplicity ϕ instead of Φ_L, which is the true inflaton field. For the classical slide-down from ϕ_i to ϕ_* it takes $\tilde{n}_{i*} \sim exp[\sqrt{2}(\phi_i/\mu)]$ steps, which is somewhere between $O(1)$ and 10^{12}. The width of the distribution at $\phi(t) \sim \phi_*$ is

$$\tilde{\sigma}_* \sim \frac{\sqrt{3}}{4\pi} H_i \tag{19}$$

Compared to $\phi \in [1, 30]\mu/\sqrt{2}$, this width is small, and the 1σ bulge which contains $\sim 68\%$ of the total inflating volume is rather narrow. One might be tempted to neglect domains with $\phi = \phi_i$ as the Gaussian factor

$$p_{\tilde{t}}[\phi_i] \sim exp\left[-\left(\frac{\phi_i - \phi_{cl}(\tilde{t})}{\tilde{\sigma}_*}\right)^2\right] \tag{20}$$

is in general very small, within $[10^{-9}, 10^{-4}]$, depending on ϕ_i. Observe however that it *increases* with H_i. This is because a larger H_i means a larger number of steps which broadens the distribution.

Further, the number of created domains is enormous, reaching at $\phi \sim \phi_*$,

$$\tilde{N}_* \sim \tilde{N}_e \simeq e^{3\tilde{n}_{i*}} \in (O(1), e^{3*10^{12}}) \tag{21}$$

If $\tilde{N}_*(\phi_i)$, the number of horizon volumes with $\phi \sim \phi_i$, at the moment $\phi_{cl} = \phi_*$, is $\tilde{N}_*(\phi_i) = 1$, the random walk inflationary phase is regenerative, as the initial configuration is realized; if $\tilde{N}_*(\phi_i) > 1$, the process amplifies itself. We have,

$$\tilde{N}_*(\phi_i) \sim exp\left[-\left(\frac{\mu}{H_i} log(\frac{H_i}{\tilde{H}})\right)^2 + 54\epsilon H_i^2\right] \tag{22}$$

and the exponent is positive as long as $H_i > H_*$. Typically, $\tilde{N}_*(\phi_i) \gg 1$, and the inflationary universe is self-reproducing. Further evolution, $\phi_* \to \phi_e$, changes very little, as the width of the distribution is practically frozen and the number of steps is negligible compared to the preceding phase. When the mainstream of the distribution enters the reheating phase, domains with $\phi \sim \phi_i$ are abundantly present but they occupy only

$$r = \frac{1}{(2\epsilon R_i)^{3/2}} \frac{\tilde{N}_e(\phi_i)}{\tilde{N}_e} \sim p_*[\phi_i], \tag{23}$$

which is a tiny fraction of the total volume of the universe. The R-dependent scaling between physical and conformal volumes makes no numerical difference.

In terms of the physical curvature the width of the distribution is given by

$$R_n^{rms} \sim R(n\tilde{H}^{-1})\, exp\left[\pm\left(\frac{G}{36\pi^3\epsilon}\right)^{1/2}\sqrt{n}\right]. \tag{24}$$

For allowed ranges, $n_{i*} \in [10^{12}, 10^{16}]$, and $\epsilon \in [10^{11}, 10^{15}]G$, it is a very narrow distribution. At $R \sim R_*$, the exponent is bounded by $4 * 10^{-2}$, thus, R_*^{rms} and $R_{classical}$ are always within the same order of magnitude. The probability distribution however,

$$dp_n[R] = \frac{dR}{R}\frac{1}{\sqrt{2\pi n}}\, exp\left[-\left(\frac{18\pi^3\epsilon}{nG}\right) ln^2(\frac{R}{R_{classical}})\right] \tag{25}$$

is log-Gaussian, and with respect to the Gaussian one can take values for R further away from the "mean". In this sense, the R^2 model has a broader distribution in terms of its natural variable.

It is interesting to observe that the width depends only on the initial conditions, and is independent of the parameter ϵ which actually controls fluctuations.

Fairly much the same holds true for the scalar field models as well, but there is one formal difference. In the R^2 case the scalar curvature itself performs the random walk and the peak of the distribution is on the classical trajectory, $\langle R \rangle = R_{cl}$. Whereas for the massive scalar field case the scalar curvature is $\sim \phi^2$, and as a consequence, the expectation value of the physical curvature in these models diffuses away from the classical trajectory:

$$\langle R \rangle \simeq R(t_n)[1 + \frac{1}{8\pi^2}(\frac{m}{\mu})^2 n]. \tag{26}$$

The difference is not very large: when the mainstream reaches R_e, one finds that R_{cl} is only $\frac{1}{2}\sigma_*$ away.

After $\langle R \rangle$ drops below R_* all domains at the peak would follow a classical trajectory that is displaced with respect to one that matches R_i at $t = t_i$. As a result, the relevant number of steps is larger than the classical value n_{ie}.

After the mainstream of the distribution enters the reheating phase, a dramatic change in the global structure of the universe takes place. Up to this point, domains with high values of the curvature occupy a small fraction of the total volume of the universe, as dictated by the Gaussian suppression. But from now on, the mainstream of the distribution expands only as a power law, while the right hand tail of the distribution continues to inflate.

In fact, the Gaussian shape of the distribution will be modified since the random walk takes place within a finite interval $R \in [\epsilon^{-1}, M_p^2]$. The upper limit can be considered a totally reflecting boundary: if the curvature within a domain jumps to a value $\sim M_p^2$, it is reasonable to expect it back with a smaller curvature within Planck time scales since the expansion rate is positive. Because the random-walk time scale is $\sim \sqrt{\epsilon}$, it is an instant reflection.

The lower boundary at $R_e = \epsilon^{-1}$ can be considered an absorbing boundary since every domain which reaches it leaves the inflationary phase. Since domains with $R < \epsilon^{-1}$ have zero diffusion constant, the probability distribution for $R > \epsilon^{-1}$ is depressed with respect to the Gaussian value: the effect is fairly localized though. One can find the distribution explicitly by solving a Fokker-Planck equation with an absorbing boundary, but we need only the classical slide-down velocity,

$$|\dot{\phi}| = \frac{2}{3}\frac{\mu}{\sqrt{12\epsilon}}e^{-\frac{\phi}{\mu}\sqrt{2}} \tag{27}$$

with which domains leave the inflationary phase. The ϕ dependence which this velocity has, causes some further deformation from the Gaussian shape, but it is small: one finds that for one step,

$$\frac{|\Delta\phi(\phi_{cl}+\sigma)|}{|\Delta\phi(\phi_{cl}-\sigma)|} \simeq (0.7)^{\frac{H_i}{\mu}} \tag{28}$$

The 1σ bulge takes only $\Delta\bar{n} \simeq 0.8H_i/\mu$ steps to travels through the left-hand boundary and leave the inflationary phase. During the next step the leftover inflating volume will expand to a larger value than the volume in FRW phase, which expands at a slower rate. However most of the inflating phase is again within 1σ of the lower boundary and at the same time leaves the inflationary phase. As a result, in a few Hubble times a rapid transfer of the volume to the Friedmann phase takes place. The only domains that still inflate are far away at the tail where the drift velocity is small. They expand rapidly, but their initial volume is suppressed by the Gaussian profile, so it takes some time before they catch up in size with the slower expanding Friedmann domains which started with a large initial volume. In this sense, the quantum broadening does not make much difference and the transition is almost the same as if the random walk phase did not exist at all: there will be a period when the power law expanding domains occupy most of the volume in the Universe.

This is the moment when the self-regenerative nature of the evolution can be seen most clearly: the configuration is just like the initial one, only the number of domains with $\phi = \phi_i$ is much larger, Eq.(22). Moreover, regardless of ϕ_i, the random walk generates a nonzero occupation number for ϕ_{pl}. Thus, every subsequent inflationary phase can be considered as starting at the Planck boundary. The number of starting domains grows in geometric progression.

Ignoring the leftover Gaussian tail for the values below ϕ_i one finds that after the M^{th} repetition the new contribution to the FRW volume is,

$$\tilde{V}_{FRW}(t) = \tilde{N}_e \tilde{l}^3 \sqrt{\frac{t}{Mt_e}} \tilde{N}_i^{M-1} \tag{29}$$

where $Mt_e < t < (M+1)t_e$. $t_e \simeq 36\epsilon H_i$ is the duration of the inflationary phase. $\tilde{N}_e$ is the total number of domains generated by the expansion, and $\tilde{N}_i$ is the number of domains with $\phi = \phi_i$ when $t = t_e$.

Since $\tilde{N}_i >> 1$, the total volume in the FRW phase is dominated (instantly) by the volume delivered by the latest inflationary phase. Under the same assumptions, the volume that inflates is

$$\tilde{V}_{INF} \simeq \tilde{N}_i^M \tilde{l}^3\, exp[3\tilde{n}] \tag{30}$$

where in the exponent we have the number of steps since the beginning of the $(M+1)^{th}$ inflationary phase. The inflating domains catch up with the volume of the FRW phase after $\tilde{n}_e$ steps, given by

$$\tilde{n}_e \simeq \frac{\phi_i^2}{6\tilde{\sigma}_*^2} \tag{31}$$

which decreases with the growth of H_i. As explained before, we are interested in $H_i \to \mu$, for which $\tilde{n}_e \sim O(10^3)$. The true value is smaller as we neglected the tail left of ϕ_i. The R-dependent scaling to the physical volume does increases this number by a few percent.

Because the number of e-foldings during the inflationary phase from Planck-scale values to the end is about 10^{12}, we have that the Friedmann domains dominate the volume of the universe for about 10^{-9} of the inflationary phase, which repeatedly occurs every $10^{-31}s$ or so.

We can get some information about the scale of domains today in the following way. The amplitude of the scalar perturbations generated during the R^2 inflation depends on the scale as $\delta_k \sim \epsilon H_{hc}^2(k)$, and exceeds unity precisely for scales that cross the horizon at $H_{hc}(k) \geq H_*$ [9]. We can interpret the scale λ_* which crossed the horizon at $H = H_*$ as the size of the FRW domain. We find,

$$\lambda_* = \frac{H_h}{H_*} e^{18\epsilon H_*^2} H_0^{-1} \quad (H_0 \ is\ todays\ Hubble\ parameter) \tag{32}$$

which is $\sim e^{10^6} H_0^{-1}$, for $H_h \simeq 10^{-5} M_p$. What this means in effect is that only purely classical inflation after H_* inflated our local universe. This size is enormous, out of any observational reach. For a short period ($\sim t_\epsilon$) the size of our local universe refers to the largest FRW domain surrounded by inflating domains. After this, it is the boundary with respect to another, larger FRW domain, with a slightly larger temperature. This temperature change is $T_2/T_1 = \sqrt{1+1/M}$, where M counts inflation (since the creation of the Universe) that caused our local universe, (see Fig.2.).

The FRW domains ought to have $\Omega = 1$ due to inflation. However, if for whatever reason a single FRW domain contracts and collapses, it does not affect the large scale structure since it is always the latest FRW domain among them that dominates. The universe is a two-component mixture, with both phases being eternal.

In conclusion, we have shown that provided there exists a domain with curvature $R \simeq M_p \epsilon^{-1/2}$ fluctuations can cause stochastic behavior resulting in an eternal inflating universe. This does not contradict energy considerations [2] since the rapidly increasing scale factor (whose potential energy is negative) is compensated for by the in turn rapid increase in curvature, whose energy is like "mass" positive, cf.[14]. This confirms the eternal universe picture in the inflationary model based on the higher derivative gravity, and due to the simplicity of the R^2 inflation in the conformal picture, provides a fairly explicit insight into the dynamical role of the quantum fluctuations. In particular, the standard issue of a "field homogeneous over scales larger than the horizon" is resolved by a proper treatment of quantum fluctuations [15].

The growth in the number of the coarse-grained volumes is a built-in arrow of time, and the random walk inflationary phase naturally fits into the concept of quantum creation of the universe: the inflation is eternal in that it has no end, but it is compatible with the existence of a beginning. One task of Quantum Cosmology is to find the distribution for the initial conditions of the universe. Since during the random walk inflationary phase many domains with curvature on the Planck scale are created regardless of the initial curvature, (so long as it permits a random walk phase), we only need to know how likely it is that the initial curvature will exceed R_*.

The Wheeler-DeWitt equation has two independent solutions which can

[2] this is rather heuristic since gravitational energy is not well defined in non asymptotically flat space-times:see for example [24].

be organized into solutions that obey Hawking's [26] ("no boundary"), and Vilenkin's [27] ("tunneling from nothing") boundary conditions. We are interested in the probability measures for the initial conditions of these two cases for $R \in [\epsilon^{-1}, M_p^2]$. They are [10],

$$dP_H = \frac{dR}{R} exp\left(\frac{32\mu^2}{R}\right)\,, \quad dP_V = \frac{dR}{R} exp\left(-\frac{32\mu^2}{R}\right). \tag{33}$$

We find,

$$P_V[R_i > R_*] \simeq 1\,, \qquad P_H[R_i > R_*] \simeq e^{-10^{12}}! \tag{34}$$

For our interpretation, this is a dramatic difference: in Hawking's case it is far more likely that the universe will start with a curvature below R_* and proceed on a purely classical trajectory, while in Vilenkin's case the random walk inflationary phase is unavoidable. When the curvature within a domain falls below this bound, new universe generation by such a domain becomes inefficient: however there is still enough classical inflation (> 70 e-foldings) that the resulting mini-universe is homogeneous and isotropic.

Note added: While writing up this work a paper by Pollock [28] appeared, in which the bound, Eq.(13), is derived by a similar method.

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Figure Captions

Fig. 1) The evolution of the Hubble parameter. The solid line represents the classical trajectory $H_i \rightarrow H_e$. The dashed lines are two examples of a random walk for $H > H_*$. Scales within the current horizon are affected by $H < H_h$ inflation only.

Fig. 2) The global structure of the universe. I and F represent inflating and Friedmann domains respectively. N stands for " nothing": a state with no Lorentzian space-time.

The areas of the boxes represent the volumes of the given phases. Relative sizes are not to scale but are correctly ordered within each column. It takes $\sim 10^{-31}s$ to travel down a column and $\sim 10^{-40}s$ to go to the next column. The size of "our" F domain today is about $exp[10^6]$ times the present horizon length.

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fig 1

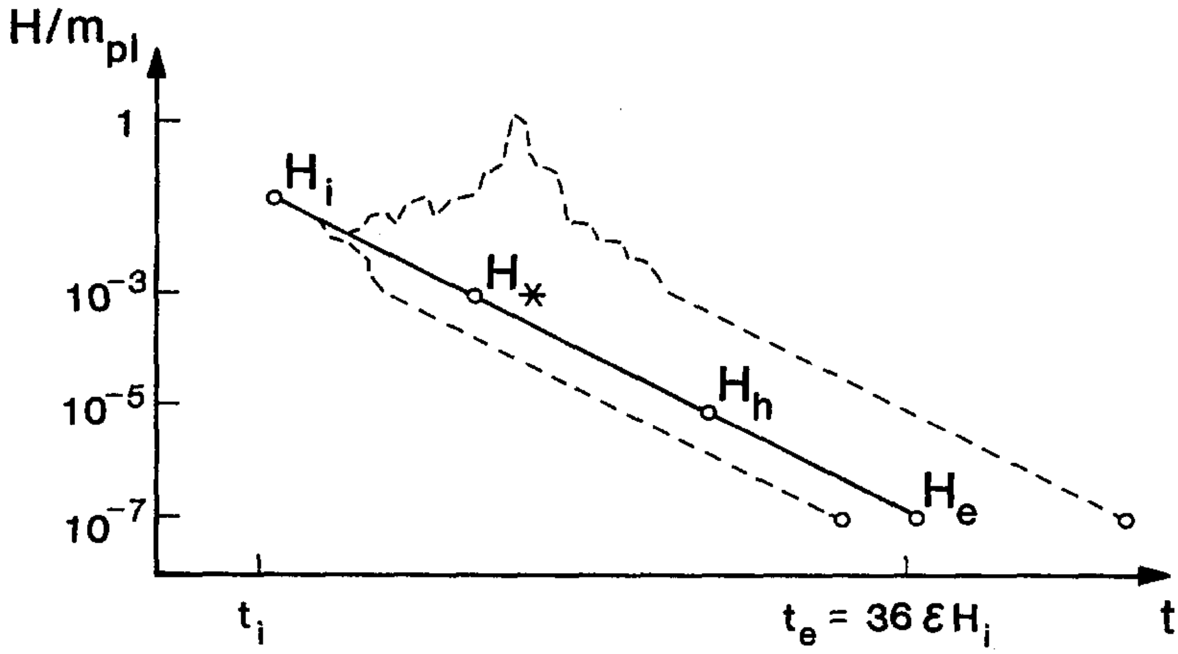

FIG 2