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Sphalerons, Small Fluctuations, and Baryon Number Violation in Electroweak Theory

PETER ARNOLD AND LARRY MCLERRAN
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510 USA

ABSTRACT

We study the formalism of the sphaleron approximation to baryon-number violation in the standard model at temperatures near 1 TeV. We investigate small fluctuations of the sphaleron, the competition of large-scale sphalerons with thermal fluctuations, and the damping of the transition rate in the plasma. We find a suppression of the rate due to Landau damping and due to factors arising from zero modes. Our approximations are valid in the regime $2M_w(T) \ll T \ll 2M_w(T)/\alpha_w$ for models where $\lambda \sim g^2$. We find that the rate of baryon-number violation is still significantly larger than the expansion rate of the universe.

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1. Introduction

Grand-unified models contain interactions which violate baryon number. These violations have spurred the search for proton decay and perhaps given us an explanation of the baryon asymmetry of the universe. The standard model *also* contains baryon-number violation. Baryon number, though a classical symmetry, has an anomaly involving the weak SU(2) gauge group.[1] Nonetheless, such a violation will never appear in the perturbative calculation of an S-matrix element. Baryon number is violated in the S-matrix only through *non-perturbative* effects and arises from transitions between different vacuum states. Each transition violates baryon number by n_f units, where n_f is the number of families. At zero temperature, these transitions are mediated by the instanton of the weak SU(2) group. Instantons correspond to quantum tunnelling between the vacuum states, and so are exponentially suppressed. The suppression is:

$$\left(e^{-8\pi^2/g_w^2}\right)^2 \qquad \sim \qquad 10^{-173} \tag{1.1}$$

which is to say that it never happens.

Instanton tunnelling has also been analyzed at finite temperature.[2] The infrared divergences which plague the analysis at zero temperature, arising from large-scale instantons, are cured by Debye screening at finite temperature. Moreover, to calculate the rate in the semi-classical approximation, one should use the temperature-dependent running coupling constant in (1.1). The prefactors are reliably calculated by analyzing small fluctuations about the instanton. The conclusion of this analysis is that baryon-number violation due to instanton tunnelling is still so small that it is effectively zero.

In a clever analysis, Kuzmin, Rubakov, and Shaposhnikov[3] have argued that baryon-number violation in the standard model is unsuppressed at high temperature, specifically $T \gtrsim 1$ TeV. There is no suppression because the transition arises from *classical thermal fluctuations* rather than quantum tunneling. For example, consider the quantum mechanics of a particle in the one-dimensional potential shown in Figure 1. At zero temperature, the only connection between the two vacua is quantum tunnelling, which is exponentially suppressed. At temperatures high compared to the potential barrier V_0, the thermal distribution favors states with energy $E \gg V_0$ and the particle can move over the barrier *classically*; there is no suppression. At intermediate temperatures, the particle has a certain probability of being thermally excited over the barrier given by the Boltzmann distribution and proportional to $\exp(-\beta V_0)$.

We would like to adapt this picture to the field theory of the standard model. Manton, and Klinkhamer and Manton,[4] have identified an unstable, time-independent solution to the equations of motion of SU(2) Higgs gauge theory. This solution is called the sphaleron and corresponds to the barrier V_0 between vacua. The sphaleron effectively has baryon number $n_f/2$ which is half of the violation caused by a transition. Being unstable, the sphaleron can only correspond to a stationary phase of the Euclidean action, not a minimum. We will show in section 1.1, however, that it is nonetheless appropriate to expand the path-integral about a sphaleron background. At zero temperature, a static solution has infinite Euclidean action and cannot contribute to a semi-classical approximation. At finite temperature, the action is integrated only over the region of $0 \leq \tau \leq \beta$ of imaginary time. The contribution of the classical action of the sphaleron is then just a Boltzmann factor $\exp(-\beta E_{sp})$. So transitions which involve the sphaleron, while suppressed at zero temperature, might become increasingly important as the temperature increases.

In analogy with the simple quantum mechanical example discussed above, the rate at which gauge-field configurations pass over this barrier gives a measure of the rate of transition from the region of one vacuum to another. But each such transition will violate baryon number through the anomaly. Kuzmin, *et. al.* therefore write[3]

$$\frac{dN_B}{N_B dt} \sim -Te^{-\beta E_{sp}} \tag{1.2}$$

where the factor of T is included on dimensional grounds and the energy E_{sp} of the sphaleron is a few times M_w/α_w.[4] Implicit to this analysis is the assumption that each baryon-number sector has thermalized so that the Boltzmann factor is a relevant measure of the rate. Note that the rate becomes very large at the critical temperature T_c where symmetry is restored in the Weinberg-Salam theory[5] because M_w approaches zero there. If eq. (1.2) is appropriate near T_c, then the rate becomes order one in units of T.

If this rate is large enough, then it constrains baryogenesis. A process that violates baryon number will, in equilibrium, equalize the number of baryons and anti-baryons. Thus, any baryon excess created in the early universe may be wiped out. These processes, however, only violate B+L; B-L does not have an anomaly and is exactly conserved in the standard model. If one produces a B-L excess in the early universe, it will not be washed away.[3,6] There may be other possibilities as well. Perhaps one might even imagine *generating* baryon number by non-equilibrium processes involving the sphaleron.[7] Despite these uncertainties, a proper analysis of the basic rate of baryon-number violation in the standard model is an important step in understanding baryogenesis in cosmology. The relation to cosmology will be studied more deeply in a sequel.[8] In this paper, we will investigate the formalism of the sphaleron approximation to baryon-number violation at finite temperature.

The sphaleron solution only exists when the SU(2) symmetry is broken. As T approaches T_c from below, the energy of the sphaleron approaches zero and its spatial extent grows to infinity. Above T_c, there is no energy barrier between vacua. That is, one can find paths through configuration space (*not* solutions to the equations of motion) which connect two vacua and such that the maximum potential energy along the path is arbitrarily small. But there is no path which assumes the smallest such barrier since there is no path along which the potential energy is everywhere zero. Thus, there is no saddle-point to the potential energy, like the sphaleron, about which one can expand. The approximation (1.2) is therefore only sensible for temperatures below the critical temperature $T_c \sim 100$ to 360 GeV. Above that temperature, baryon number may be substantially violated, but this violation cannot be seen in an analysis based upon the sphaleron. However, in the region where the analysis *is* valid, the rate computed in Eqn. (1.2) is sufficiently large to ensure that any baryon (B+L) excess would be washed out.

It will be our purpose to tighten the approximation in Eqn. (1.2) by more rigorously examining the prefactors that multiply the exponential and by analyzing how damping in the plasma affects the rate. We will analyze the model

$\lambda \sim g^2$, where λ is the Higgs self-coupling, at temperatures between $2M_w(T)$ and $2M_w(T)/\alpha_w$. We find that, in the temperature range where our analysis is valid, the rate is indeed high enough to easily wash out any initial baryon excess.

The expression that we shall derive for the rate vanishes as the the critical temperature is approached from below. This occurs because the size of the sphaleron becomes infinite in this limit. We then expect that the sphaleron configuration becomes unlikely since it cannot compete with ordinary thermal fluctuations, which have much smaller spatial extent. However, we shall find this drop in the rate precisely where the approximations we use in our analysis break down. Specifically, the assumption that one can analyze the problem in small fluctuations about the sphaleron will no longer be valid. At the end of the paper, we shall give a rough argument that shorter-range configurations, which can compete with thermal fluctuations, take over the role of the sphaleron near the critical temperature and above.

In the remainder of this section, we shall discuss the formalism that underlies the estimate (1.2) and our computation of the prefactors. Then we will briefly review the sphaleron solution of Klinkhamer and Manton. In section 2, we will compute the dependence of the prefactors on M_w, T, and α_w and carefully discuss the nature of our approximations and the regime in which they are valid. In section 3, we incorporate the damping of the transition rate due to interactions with the plasma. We shall see that Landau damping is the most significant effect. In section 4, we will discuss what may happen at temperatures above the region where the previous analysis is valid. Left for appendicies are (a) a discussion of the formalism of the dilute sphaleron-gas approximation, (b) the calculation of the zero-mode integrations for the sphaleron, (c) a derivation of the pseudo-particle formalism used in section 3, and (d) a more detailed look at the argument of section 4.

1.1 Basic Formalism

The basic idea is to write a path-integral expression for the rate of baryon-number violation and then to calculate this path-integral in a Gaussian expansion about a static sphaleron background. The sphaleron background alone gives

$$\exp\left\{-\int_0^\beta d\tau L_E[\phi_{sp}]\right\} \quad \sim \quad e^{-\beta E_{sp}}. \tag{1.3}$$

The integration over Gaussian fluctuations then gives the prefactor for this exponential.

At first glance, expanding about a static solution may not seem to make sense since we are interested in a non-static process. The purpose of this section is to explain why one should expand about the sphaleron and to tie down the exact path-integral expression that one need calculate. This problem has been investigated for false-vacuum decay at finite temperature by Affleck,[9] Linde,[10] and Mottola.[11]

Our approach is to apply the analysis of Affleck. Let us follow this analysis in the case of the potential in Figure 1. We will work at a temperature large enough to justify a classical treatment, but small compared to the barrier potential. We want to know the rate at which particles go over the barrier when the particles start in approximate equilibrium in the left well. This is the probability of finding a particle at the barrier, heading in the right direction, times the rate at which it crosses the barrier. So

$$\begin{aligned}\Gamma &= \langle \delta(x) p \theta(p) \rangle \\ &= \frac{\int dp\ dx\ \exp\left\{-\beta[\frac{1}{2}p^2 + V(x)]\right\} \delta(x) p \theta(p)}{\int dp\ dx\ \exp\left\{-\beta[\frac{1}{2}p^2 + V(x)]\right\}} \\ &\approx \frac{\omega_0}{2\pi} e^{-\beta V_0}.\end{aligned} \tag{1.4}$$

where the denominator was approximated by a Gaussian integral. This is related to the imaginary part of the free energy evaluated as small fluctuations around the sphaleron and around the left vacuum:[12]

$$\begin{aligned}\mathrm{Im}F &= T\mathrm{Im}(\ln Z) \approx T\frac{\mathrm{Im}Z}{Z} \\ &\approx T\frac{\mathrm{Im}\int dp\ dx\ \exp\left\{-\beta[\frac{1}{2}p^2 + V_0 - \frac{1}{2}\omega_- x^2]\right\}}{\int dp\ dx\ \exp\left\{-\beta[\frac{1}{2}p^2 + \frac{1}{2}\omega_0 x^2]\right\}} \\ &= \frac{\omega_0}{2\omega_- \beta} e^{-\beta V_0}.\end{aligned} \tag{1.5}$$

(The factor of $1/2$ in the last line of the equation arises from the analytic continuation,[13] but here may be considered as mere convention for what we mean by $\mathrm{Im}Z$. It will not be very important to our calculation.) We now have a relation between Γ and path-integrals for cases with a single degree of freedom:[14]

$$\Gamma \approx \frac{\omega_- \beta}{\pi} \mathrm{Im}F \approx \frac{\omega_-}{\pi} \frac{\mathrm{Im}Z_{barrier}}{Z_0}. \tag{1.6}$$

Adding another degree of freedom, so that the barrier is now a saddle instead of a maxima, is easy since both Γ and $\mathrm{Im}F$ are modified by the same factor in the Gaussian approximation:

$$\frac{\int dp_y\ dy \exp\left\{-\beta[\frac{1}{2}p_y^2 + \frac{1}{2}\omega_y^2 y^2]\right\}}{\int dp_y\ dy \exp\left\{-\beta[\frac{1}{2}p_y^2 + \frac{1}{2}\omega_{y0}^2 y^2\right\}} = \frac{\omega_{y0}}{\omega_y}. \tag{1.7}$$

The relation (1.6) is not affected. Affleck shows that, for $T > \omega$, the relation is also not affected by quantum corrections.

For systems with an infinite number of degrees of freedom, the generalization of Eqns. (1.6) and (1.7) is easy:

$$\begin{aligned}\Gamma &\approx \frac{\omega_-}{\pi}\frac{\mathrm{Im}Z_{barrier}}{Z_0} \approx \frac{\omega_-}{2\pi}\,\mathrm{Im}\left(\frac{\det_\beta \omega_0^2}{\det_\beta \omega^2}\right)^{\frac{1}{2}} e^{-\beta V_0} \\ &= \frac{\omega_-}{2\pi}\,\mathrm{Im}\left(\prod \frac{\sinh(\beta\omega_0^i/2)}{\sinh(\beta\omega^i/2)}\right) e^{-\beta V_0} \to \frac{\omega_-}{2\pi}\,\mathrm{Im}\left(\prod \frac{\omega_0^i}{\omega^i}\right) e^{-\beta V_0}\end{aligned} \tag{1.8}$$

where we have calculated the partition functions in Gaussian approximation about the saddle point and then taken the classical limit $\beta \to 0$ in the last line.

Note on notation. The unstable mode has an imaginary frequency $\omega = i\omega_-$. We will often refer to it by the real quantity ω_-, but the reader should keep in mind that ω and ω_- differ by a factor of i.

1.2 The Sphaleron Solution

Manton and Klinkhamer[4] found their static solution for a pure SU(2) gauge theory and then incorporated electromagnetism by perturbing in $\sin^2\theta_w$. In this paper, we shall work in the approximation that $\sin^2\theta_w = 0$. So we shall focus on the pure SU(2) solution.

The solution may be written in $A_0 = A_r = 0$ gauge in the following form. (Recall that after fixing $A_0 = 0$, one may still make another, time-independent gauge fixing. For the static sphaleron solution, it is convenient to work in $A_r = 0$.)

$$\vec{A} = v\frac{f(\xi)}{\xi}\hat{r} \times \vec{\sigma} \qquad \phi = \frac{v}{\sqrt{2}}h(\xi)\hat{r}\cdot\vec{\sigma}\phi_0 \tag{1.9}$$

$$\xi \equiv gvr \qquad \phi_0 \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{1.10}$$

where f and h are numerically-determined functions with

$$f(0) = h(0) = 0 \qquad f(\infty) = h(\infty) = 1. \tag{1.11}$$

The solution approaches the pure gauge $f(\infty) \equiv h(\infty) \equiv 1$ exponentially quickly at spatial infinity. Graphs of f and h appear in Figure 2 for the case we shall study: $\lambda = g^2$.

Manton and Klinkhamer[4] show that this solution corresponds to baryon number

$$Q_B = n_f\frac{g^2}{32\pi^2}\int d^3x\ K^0 = \frac{1}{2}n_f \tag{1.12}$$

where n_f is the number of families and

$$K^\mu = \epsilon^{\mu\nu\rho\sigma}(F^a_{\nu\rho}W^a_\sigma - \frac{2}{3}g\epsilon_{abc}W^a_\nu W^b_\rho W^c_\sigma) \tag{1.13}$$

is the object whose divergence is $\mathrm{tr}F\tilde{F}$. (Some care must be taken to evaluate (1.12) in the right gauge. See ref. [4] for details.) Similarly, the lepton number is also effectively $Q_L = n_f/2$.

The energy of the sphaleron is $E = (2M_w/\alpha_w)\bar{E}$ where $\bar{E}$ varies between 1.56 for $\lambda = 0$ and 2.72 for $\lambda = \infty$. The radius of the sphaleron is roughly $(2M_w)^{-1}$.

2. The Prefactors

2.1 The Rate of Baryon Number Changing Transitions

On the basis of our previous discussion, we want to calculate

$$\Gamma \approx \frac{\omega_-}{\pi} \frac{\mathrm{Im} Z_{sp}}{Z_0}, \tag{2.1}$$

where Z_{sp} is calculated in Gaussian approximation about a sphaleron background. (In calculating the full partition function, it is important to sum multiple-sphaleron configurations. In Appendix A, we discuss this calculation in the dilute sphaleron gas approximation. We show that (1.13) is still valid, where Z_{sp} is expanded about a single-sphaleron background. We also discuss the validity of the dilute gas approximation.)

Let us begin by rescaling fields and coordinates as follows:

$$(r,\tau) \to (\xi,\bar{\tau}) = gv(r,\tau) \qquad A(r,\tau) \to v\bar{A}(\xi,\bar{\tau}) \qquad \phi(r,\tau) \to v\bar{\phi}(\xi,\bar{\tau}) \tag{2.2}$$

The action then becomes

$$S_E = \frac{1}{\hbar g^2} \int_0^{gv\hbar\beta} d\bar{\tau} \int d^3\xi \; \bar{\mathcal{L}}_E[\bar{A}(\xi,\bar{\tau}), \bar{\phi}(\xi,\bar{\tau}), \lambda/g^2] \tag{2.3}$$

We would like to claim that, in the high-temperature limit $gv\beta \ll 1$, only the time-independent, zero-frequency modes of the fields are relevant and we may replace this by the effective 3-dimensional theory:

$$S_3 = \frac{v\beta}{g} \int d^3\xi \; \bar{\mathcal{L}}_3[\bar{A}(\xi), \bar{\phi}(\xi), \lambda/g^2]. \tag{2.4}$$

This is a classical limit as it does not depend on $\hbar$. The coupling constant of this

3-dimensional theory is given by

$$\alpha_3 \equiv \frac{g_3^2}{4\pi} \equiv \frac{g}{4\pi v\beta} = \alpha_w \frac{T}{2M_w}. \tag{2.5}$$

If this coupling is small, then a Gaussian (*i.e.* one loop) approximation to Z_{sp} is justified. We shall work with temperatures high enough that we may use the 3-dimensional theory but low enough that α_3 is small. That is, we assume that the temperature falls in the narrow range:

$$2M_w \ll T \ll 2M_w/\alpha_w. \tag{2.6}$$

We shall find that a significant contribution to baryon number violation occurs in this range. Outside of this narrow range of temperatures, our computational techniques are no longer valid. In particular, if $T \geq M_W/\alpha_W$, a weak coupling analysis is invalid.

Before proceeding, we must discuss a subtlety of the transition to the 3-dimensional theory. The non-zero frequency modes have masses $\sim \nu_n = 2\pi/\beta$ in the 3-dimensional theory. As $\beta \to 0$, they decouple in UV convergent diagrams and can be ignored. They do not, however, decouple in UV divergent diagrams. Thus, they affect the renormalization of the theory. To leading order, their effect can be absorbed into a redefinition of the coupling constants and masses of the theory. Thus, we should work with the effective finite-temperature potential for the theory.[15,16] For the case of Weinberg-Salam theory, the Higgs potential becomes:[17]

$$\lambda\left(\phi^\dagger\phi - \frac{1}{2}v^2(T)\right)^2, \qquad v^2(T) = v^2(0) - (\frac{1}{2} + \frac{3g^2}{16\lambda})T^2. \tag{2.7}$$

The critical temperature and the effective W mass may be written

$$T_c = v(0)\sqrt{\frac{2}{1+\frac{3}{8}g^2/\lambda}}, \qquad M_w(T) = M_w(0)\sqrt{1-\left(\frac{T}{T_c}\right)^2}. \tag{2.8}$$

If we express the critical temperature in terms of $M_W = gv(0)/2$, we find $T_c \sim M_W/(5\alpha_W) \sim 10M_W$ for $g^2/\lambda \sim 1$. Our prescription will be to work to leading order in the 3-dimensional theory (2.4) of the zero frequency modes, but to use the above effective potential.

Let us study the case $\lambda/g^2 \sim 1$. Then the only parameter in our 3-dimensional theory is α_3. Now treat $\mathcal{L}_3$ in Gaussian approximation about the sphaleron background.

$$\mathcal{L}_3 \approx \mathcal{L}_{3,sp} + (\delta\phi)^\dagger \Omega^2_{sp}(\delta\phi) \tag{2.9}$$

where the operator Ω^2 is order unity. The expansion about the vacuum for Z_0 can be treated the same way:

$$\mathcal{L}_3 \approx \phi^\dagger \Omega_0^2 \phi.$$

If we ignore, for the moment, the existence of spatial zero modes of the sphaleron, we can now do the required integrations:

$$\Gamma \approx \frac{\omega_-}{\pi}\frac{\mathrm{Im}Z_{sp}}{Z_0} \approx \frac{\omega_-}{2\pi}\,\mathrm{Im}\left(\frac{\det\Omega_0^2}{\det\Omega_{sp}^2}\right)^{\frac{1}{2}} \sim \frac{\omega_-}{2\pi}e^{-\beta E_{sp}} \times \mathrm{O}(1). \tag{2.10}$$

The dependence on α_3 has cancelled between numerator and denominator. One may now make the following argument (which we shall modify in section 3) for the magnitude of ω_-. Since all of the dimensionful scales have been removed from the problem by rescalings, therefore all of the eigenvalues of Ω^2 will be order unity, including the negative eigenvalue corresponding to the instability of the sphaleron. In terms of the original spatial coordinate r, we must then have $\omega_- \sim 2M_w$, so

$$\Gamma \approx \frac{1}{\pi} M_w(T) e^{-\beta E_{sp}} \times O(1). \tag{2.11}$$

This rate vanishes at the critical temperature where $M_w(T) \to 0$. We shall see later that this drop in the rate occurs in a region where our approximations break down. First, however, we must correctly include the spatial zero-modes of the sphaleron.

The modification which we shall show in Section 3 for this estimate of ω_- involves the observation that for the low frequency region corresponding to the decay of the sphaleron, higher order loop corrections to the vector propagator become important. We shall argue that these corrections may be systematically computed in perturbation theory.

2.2 The Sphaleron Zero Modes

Symmetries of the theory can give important modifications to eq. (2.10). For instance, translational invariance implies that ImF must be proportional to the volume V. In our dimensionless coordinate ξ, this implies a factor of $(gv)^3V$. Note that, since $v(T) \to 0$ as $T \to T_c$, this factor vanishes near the critical temperature.

The sphaleron has zero modes corresponding to its transformation under symmetries of the theory. These do not give Gaussian integrals, but must be integrated separately using the method of collective coordinates.[18] Eq. (2.10) must be modified to:

$$\Gamma \approx \frac{\omega_-}{2\pi}(\mathcal{N}\mathcal{V})\mathrm{Im}\left(\frac{\det g_3^{-2}\Omega_0^2}{\det' g_3^{-2}\Omega_{sp}^2}\right)^{\frac{1}{2}} e^{-\beta E_{sp}} \tag{2.12}$$

where det′ indicates that zero-modes should be excluded from the determinant. The factor $\mathcal{N}\mathcal{V}$ comes from the zero-mode integration: $\mathcal{N}$ is a normalization factor and $\mathcal{V}$ is the volume of the symmetry groups responsible for the zero mode.

If there are N_0 zero-modes, then the numerator in (2.12) will have N_0 more eigenvalues and therefore N_0 many more factors of g_3^{-2}. So,

$$\Gamma \approx \frac{\omega_-}{2\pi}(\mathcal{N}\mathcal{V})g_3^{-N_0}\mathrm{Im}\left(\frac{\det \Omega_0^2}{\det' \Omega_{sp}^2}\right)^{\frac{1}{2}} e^{-\beta E_{sp}} \approx \frac{\omega_-}{2\pi}(\mathcal{N}\mathcal{V})g_3^{-N_0}e^{-\beta E_{sp}} \times \kappa. \tag{2.13}$$

In this equation, the factor κ is of order one. It is the square root of the product of all frequencies of oscillation around the vacuum divided by the product of all non-zero frequencies of oscillation around the sphaleron. Note that each factor of $g_3^{-1} \equiv (2M_w/g^2T)^{1/2}$ causes the rate to vanish more rapidly as $T \to T_c$. One should be mindful, however, that the collective coordinate procedure may break down if the non-zero modes are not well approximated by Gaussians — that is, when $\alpha_3 > 1$.

To use (2.12), we need to count the zero modes of the sphaleron. There are four symmetries of the pure SU(2) theory to consider: translations; rotations; the $\mathrm{SU(2)_L}$ of the weak gauge group; and the global, custodial $\mathrm{SU(2)_R}$ of the Higgs sector. Translations give a factor of $\mathcal{N}_{\mathrm{tr}}V_\xi$ where $V_\xi = (gv)^3V$ is the volume of ξ

space, and V is the ordinary volume of three dimensional space. Rotations give a factor $(\mathcal{N}\mathcal{V})_{\text{rot}}$. Most of the gauge symmetry $SU(2)_L$ is not relevant if we fix the gauge to $A_r = 0$. The only parts which survive are the global gauge rotations. These can be removed by fixing the boundary conditions of the path integral (see Appendix B for a complete argument). Finally, the action of $SU(2)_R$ on the sphaleron turns out to be a linear combination of the others and so gives no new zero modes. We conclude that the relevant zero-modes arise from 3 translations and 3 rotations; so $N_0 = 6$ in (2.13), giving

$$\begin{aligned}\frac{\Gamma}{V} &\approx \frac{\omega_-}{2\pi}\mathcal{N}_{\text{tr}}(\mathcal{N}\mathcal{V})_{\text{rot}}(gv)^3 g_3^{-6} e^{-\beta E_{sp}} \times \kappa \\ &= \frac{\omega_-}{2\pi}\mathcal{N}_{\text{tr}}(\mathcal{N}\mathcal{V})_{\text{rot}}\left(\frac{\alpha_w T}{4\pi}\right)^3 \alpha_3^{-6} e^{-\beta E_{sp}} \times \kappa\end{aligned} \tag{2.14}$$

The normalization and volume factors $\mathcal{N}_{\text{tr}}$ and $(\mathcal{N}\mathcal{V})_{\text{rot}}$ are calculated in Appendix B where we also include a more thorough discussion of the global gauge rotations. In the case $\lambda = g^2$ we find:

$$\mathcal{N}_{\text{tr}} = 26, \qquad (\mathcal{N}\mathcal{V})_{\text{rot}} = 5.3 \times 10^3. \tag{2.15}$$

2.3 Baryon Number Dissipation

So far, the analysis has not distinguished between transitions which increase baryon number and those which decrease it. Both appear to proceed at the rate (2.14). Consider a situation where we start with some baryon excess, say produced very early in the universe. We then expect entropy to favor reactions which dissipate this excess. To see this in our calculation, we must include the chemical potential reflecting the initial density of baryons and leptons.

Baryon number, however, is not a conserved quantity. Instead,

$$Q_B = B + n_f \frac{g^2}{32\pi^2} \int d^3x\ K^0$$

is conserved, with a similar expression for lepton number. If the initial baryon-number sector has thermalized, then we should work with the charges Q above. Let us then add a chemical potential term $-\mu_B Q_B - \mu_L Q_L$ to the Lagrangian. Now reconsider (2.1) in the small μ limit. The baryon decreasing and baryon increasing rates pick up a factor of

$$\exp[\pm\beta(\mu_B Q_B + \mu_L Q_L)] = \exp[\pm\beta(n_f/2)(\mu_B + \mu_L)] \tag{2.16}$$

The difference of the rates picks up a factor of $\approx \beta n_f(\mu_B + \mu_L)$, and each transition changes B by n_f units. So

$$\frac{dN_B}{V dt} \approx -\beta n_f^2(\mu_B + \mu_L)\frac{\omega_-}{2\pi}\mathcal{N}_{\rm tr}(\mathcal{N}\mathcal{V})_{\rm rot}\left(\frac{\alpha_w T}{4\pi}\right)^3 \alpha_3^{-6} e^{-\beta E_{sp}} \times \kappa \tag{2.17}$$

where N_B is the baryon excess. Standard thermodynamics relates μ to N by

$$\mu_B \approx \frac{9}{2n_f}\beta^2\frac{N_B}{V} \qquad \mu_L \approx \frac{2}{n_f}\beta^2\frac{N_L}{V}. \tag{2.18}$$

(We are indebted to M. Shaposhnikov for correcting an error here. See ref. [19].) In the case of a $B - L = 0$ universe, we then have

$$\frac{dN_B}{N_B dt} \approx -13 n_f T\left(\frac{\alpha_w}{4\pi}\right)^4 \frac{\omega_-}{2M_w}\mathcal{N}_{\rm tr}(\mathcal{N}\mathcal{V})_{\rm rot}\left[\alpha_3(T)\right]^{-7} e^{-\bar{E}_{sp}/\alpha_3(T)} \times \kappa \tag{2.19}$$

where we have noted that βE_{sp} is order 1 in units of α_3^{-1} and have written $\bar{E}_{sp}/\alpha_3 \equiv \beta E_{sp}$.

The reader may wonder why we have associated μ with N_B in eq. (2.18) when we previously argued that μ should be associated with Q. The question is one of time-scale. Our underlying assumption is that a given baryon-number sector is already thermalized, but that there has not yet been thermalization between *different* baryon-number sectors.

Now, examining (2.19), note that

$$\alpha_3^{-7} e^{-\bar{E}_{sp}/\alpha_3} \tag{2.20}$$

is a peaked function of α_3 and therefore of temperature. In Figure 3, we have converted time to temperature using

$$t \approx 2.42 \times 10^{-6} g_*^{-1/2} [kT(GeV)]^{-2} s, \qquad g_* \sim 100, \tag{2.21}$$

and we have plotted $dN_B/N_B dT$ as a function of temperature. We have again set $\omega_- \approx M_w(T)$, ignoring the damping effects to be discussed in the next section. At the peak, α_3 is 0.29. But our analysis assumes $\alpha_3 \ll 1$. Thus, the turnover of the rate may be an artifact of our approximations. We shall return to this point in section 4. For comparison, we also plot our result logarithmically in Figure 4 along with the estimate (1.2).

A simple measure of whether the baryon excess will be wiped out is given by comparing the rate to the expansion rate of the universe. This ratio can be read off Figure 3 as $T(dN_B/N_B dT)$. At the peak, the process proceeds roughly 10^{12} times the expansion rate of the universe. Even if, more realistically, we trust our results only for $\alpha_3 \leq 0.1$, we find 10^{10}. In any case, any baryon excess could easily be dissipated. In the next section, we shall discuss additional suppressions due to the fact that ω_- is not precisely equal to $2M_W$. These suppressions are several orders of magnitude, but the basic conclusion about the dissipation of the baryon excess remains unchanged.

In the sequel to this paper,[8] we shall examine the dissipation in detail and consider possible evasions.

3. Damping in the Plasma

Our analysis has been based on the assumption that we can reliably work in Gaussian approximations — that is, that interactions are not important. We have justified this approximation for the calculation of $\mathrm{Im}F$ in the limit $\alpha_3 \ll 1$, but we have not yet justified its use in the derivation of the relation (2.1) between Γ and $\mathrm{Im}F$. Indeed, some care must be taken on this point. The correct interpretation of this relation, and its consequences, are the subjects of this section. We shall find that ω_- in (2.1) must be interpreted as the real-time frequency response of the sphaleron, rather than as the negative eigenvalue of the potential energy expanded about the sphaleron (which is $\sim 2M_w$ by our previous scaling analysis). Thus, ω_- will be damped by effects which damp oscillations in the plasma.

To see that the Gaussian approximation can break down, consider the propagation of gauge fields in the plasma. The oscillations are damped by the interaction of the gauge fields with fermions (and with themselves). This interaction introduces modifications to the propagator of order $gT/\omega \sim gT/2M_W$ as we shall see when we study the propagator in section 3.2. But at the temperatures of interest, $gT/2M_W$ is not $\ll 1$. (For instance, it is 6 at the peak of Figure 2.) So one is not justified in ignoring these interactions.

3.1 FORMALISM

One can reestablish the result (2.1), by working in the *quasi-particle* picture.[20] Consider the quantity Z_{sp}/Z_0. This can be evaluated in the imaginary-time path-integral formalism, as we have successfully done in the preceeding section, or in the quasi-particle formalism. In the latter case, we find that the partition function can be expressed as that of harmonic modes about the sphaleron (see Appendix C):

$$\frac{Z_{sp}}{Z_0} \approx e^{-\beta E_{sp}} \prod \frac{\sum_n \exp\left(-\beta(n+\frac{1}{2})\omega_{sp}\right)}{\sum_n \exp\left(-\beta(n+\frac{1}{2})\omega_0\right)}. \tag{3.1}$$

where the frequencies ω are the *real-time* frequencies associated with the poles of propagators. This is not a convenient formalism to approximate ImF because these frequencies depend on more than just one parameter; as we saw above, they depend on $gT/2M_w$ as well as α_3. It is, however, convenient for determining the relation between Γ and ImF.

Since passage over the sphaleron is a real-time motion of the system, Affleck's analysis should be carried out in the real-time expansion (3.1). Consider the factor in the numerator of (3.1) corresponding to the negative mode of the sphaleron. We can rewrite it in the form

$$\int d(\Delta E)\ \rho(\Delta E)e^{-\beta\Delta E} \equiv \frac{1}{2}\sum_n \exp\left(-\beta(n+\frac{1}{2})i\omega_-\right) = \frac{i}{4\sin(\beta\omega_-/2)} \tag{3.2}$$

where $\rho(\Delta E)$ is the thermal density of states. (As in section 1.1, the additional factor of $1/2$ comes from analytic continuation to imaginary ω.) In the high temperature limit, the free energy then becomes

$$\mathrm{Im}F \approx \frac{1}{2}\omega_- \; \times \; \text{(the other factors)}. \tag{3.3}$$

To calculate the transition rate, we want to take the thermal expectation of that rate; so we should replace this factor by:

$$\int d(\Delta E)\; \rho(\Delta E)\Gamma(\Delta E)e^{-\beta\Delta E} \tag{3.4}$$

where $\Gamma(\Delta E)$ is the real-time transition rate for a given state.

In the classical limit, any wave which has enough energy passes over the barrier and $\rho(\Delta E)\Gamma(\Delta E)$ is just $(2\pi\hbar)^{-1}\theta(\Delta E)$. This can be seen as follows. We are interested in the expectation of the rate $\dot{x}$ at a given configuration 'x' — the sphaleron. Phase space gives a measure of $(2\pi\hbar)^{-1}dp$. But $(2\pi\hbar)^{-1}dp\ \dot{x}$ is the same as $(2\pi\hbar)^{-1}dE$ by the Heisenberg equations of motion.

(3.4) then gives us a factor of $(T/2\pi)\exp(-\beta E_{sp})$ in Γ whereas (3.3) gave us $\omega_-/2$ in $\mathrm{Im}F$. Thus, we find

$$\Gamma \approx \frac{\beta\omega_-}{\pi}\ \mathrm{Im}F$$

as claimed.

To illustrate that this relation works, let us consider a toy scenario in 0+1 dimensions analogous to electromagnetism in a plasma of charged particles. We have a field A which corresponds to $\vec{A}$ in $A_0 = 0$ gauge. Suppose interactions with the charged particles introduce a screening term $\frac{1}{2}\kappa^2\dot{A}^2$ for the electric field in the effective Lagrangian. That is, if we integrate out the charged particles, we get:

$$\mathcal{L}_{\text{eff}} \sim \frac{1}{2}(1+\kappa^2)\dot{A}^2 - V(A). \tag{3.5}$$

Let us now follow Affleck's analysis for this effective system as we did in Section 1.1. The effective Hamiltonian is:

$$\mathcal{H}_{\text{eff}} \sim \frac{p^2}{2(1+\kappa^2)} + V(A), \qquad p = (1+\kappa^2)\dot{A}. \tag{3.6}$$

Now suppose that $V(A)$ has some barrier at A_b, and define $-\bar{\omega}_-^2 \equiv V''(A_b)$. Then the transition rate is given by

$$\begin{aligned}\Gamma &= \langle \delta(A-A_b)\dot{A}\theta(\dot{A})\rangle \\ &= \frac{\int dp\; dA\;\; \exp\{-\beta[\frac{p^2}{2(1+\kappa^2)} + V(A)]\}\delta(A-A_b)\frac{p}{(1+\kappa^2)}\theta(p)}{\int dp\; dA\;\; \exp\{-\beta[\frac{p^2}{2(1+\kappa^2)} + V(A)]\}} \\ &\approx (1+\kappa^2)^{-\frac{1}{2}}\frac{\omega_0}{2\pi}e^{-\beta V_0}.\end{aligned} \tag{3.7}$$

ImF, on the other hand, is given by

$$\begin{aligned}\text{Im}F &\approx T\frac{\text{Im}\int dp\; dA\;\; \exp\{-\beta[\frac{p^2}{2(1+\kappa^2)} + V_0 - \frac{1}{2}\bar{\omega}_- A^2]\}}{\int dp\; dA\;\; \exp\{-\beta[\frac{p^2}{2(1+\kappa^2)} + \frac{1}{2}\omega_0 A^2]\}} \\ &= \frac{\omega_0}{2\omega_-\beta}e^{-\beta V_0}.\end{aligned} \tag{3.8}$$

The relation between the two is then

$$\Gamma \approx (1+\kappa^2)^{-\frac{1}{2}}\frac{\bar{\omega}_-\beta}{\pi}\text{Im}F. \tag{3.9}$$

But $(1+\kappa^2)^{-\frac{1}{2}}\bar{\omega}_-$ is precisely the real-time response frequency obtained by solving the dispersion relation obtained from (3.5) near the sphaleron:

$$(1+\kappa^2)\omega^2 = -\bar{\omega}_-^2. \tag{3.10}$$

This example supports our contention that the relation should use the real-time frequency. As in section 1.1, adding additional degrees of freedom does not change this relation.

3.2 Estimate of Damping

In this section, we will estimate the effects of damping on ω_-; we will not calculate exact numerical factors. We are interested in temperatures much larger than the effective W mass. Also, in the case $\lambda \sim g^2$ that we have analyzed, the temperature is much greater than the neutral Higgs mass $m_H \sim \sqrt{\lambda} v \sim M_w$.

To find ω_-, we must investigate the effective equations of motion for fluctuations in the plasma about the sphaleron background. First consider the gauge fields. At the tree level, the equations of motion for the classical fields A and ϕ are just:

$$D_\mu F^{\mu\nu} = J^\nu. \tag{3.11}$$

Let us consider quadratic fluctuations about the sphaleron background and rewrite (3.11) in the form:

$$k^2 \delta A_\mu = (\Omega^2)_\mu(\delta A, \delta\phi) \quad + \quad \text{higher order.} \tag{3.12}$$

The small-fluctuations operator Ω^2 is linear in δA and $\delta\phi$. From our previous analysis, the operator Ω^2 is of order $(2M_w)^2$.

Now let us consider one-loop corrections. These arise from the temperature-dependent contributions of the diagrams in Figure 5 and result from interactions with the thermal bath. The corrections yield different behavior for the longitudinal and transverse components:[21]

$$k^2 \delta A_\mu = (\Omega^2)_\mu(\delta A, \delta\phi) + \left[M_L^2 Q_{\mu\nu} + M_T^2 P_{\mu\nu}\right](\delta A^\nu + A^\nu_{ext}) \tag{3.13}$$

In this equation, M_L and M_T are longitudinal and transverse masses which will be defined below. The tensors $Q^{\mu\nu}$ and $P^{\mu\nu}$ are longitudinal and transverse projection operators,

$$Q^{\mu\nu} = -\frac{k^2}{|\vec{k}|^2}\left(g^{\mu\lambda} - \frac{k^\mu k^\lambda}{k^2}\right) u_\lambda u_\kappa \left(g^{\kappa\nu} - \frac{k^\kappa k^u}{k^2}\right) \tag{3.14}$$

and P has only spatial vector components as

$$P^{ij} = g^{ij} - \frac{k^i k^j}{|\vec{k}|^2}. \tag{3.15}$$

u is the unit timelike vector, with only a nonzero zeroth component. We shall work in the high-temperature limit where all masses are negligible. We shall also work in the kinematic limit $k^0/|\vec{k}| << 1$ which we shall justify *a posteriori* as appropriate to the calculation of ω_-. The transverse and longitudinal masses are then

$$M_L^2 = (gT)^2 A \tag{3.16}$$

and

$$M_T^2 = \frac{i\pi B}{2}(gT)^2 \frac{k^0}{|\vec{k}|} \tag{3.17}$$

In this equation, B is a number; $B \sim 2$ for the Weinberg-Salam model. M_L

comes from electric screening in the plasma. M_T, in this limit, is due to Landau damping — the absorption of the wave's energy by charged particles in the plasma.

Note that, in units of our natural frequency $2M_w$, these masses are not mere $\alpha_3 \ll 1$ corrections to the propagator, but enter as $gT/2M_w$ as we previously claimed.

The magnitude of the sphaleron field is $\sim 2M_w/g$ (see (1.9) or (2.2)). The reader may therefore worry that we have incorrectly ignored the graphs of Figure 6, which are the same order in g. For each insertion of the external field, however, we pick up a factor of $2M_w/gp_{int}$ where p_{int} is the internal momentum of the loop. Since the loops in Figure 5 have $p_{int} \sim T$, we get a reduction by at least $2M_w/gT$. Thus, (3.13) is valid to leading order in $2M_w/gT$.

We must also consider diagrams with more loops. The potentially most dangerous contribution is the generation of a magnetic mass at the next higher order in perturbation theory.[22] This could in principle modify the dispersion relation for transverse oscillations. Such a transverse mass is at most of magnitude $\mu_T^2 \sim \alpha^2 T^2$. In terms of our natural scale $2M_w$, these modifications are then of order α_3^2. We may then, to good approximation, ignore this effect so long as $\alpha_3 \ll 1$. For the temperatures of interest, this condition is only marginally satisfied. Nevertheless, we expect our analysis to be within an order of magnitude of the full result.

Because the sphaleron field is static and purely transverse, the A_{ext} term on the right of (3.13) is annihilated and may be ignored.

The longitudinal part of A may be ignored in these equations because of its large mass. Specifically, consider the longitudinal piece of (3.13):

$$(\Omega^2)_L(\delta A, \delta\phi) = (k^2 - M_L^2)\delta A_L \sim (gT)^2 \delta A_L. \tag{3.18}$$

So δA_L is small compared to δA_T and $\delta\phi$ by $\sim (2M_w/gT)^2$. Taking δA to be approximately transverse, (3.13) then becomes

$$(k^2 - M_T^2)\delta A \approx (\Omega^2)(\delta A, \delta\phi). \tag{3.19}$$

Ignoring $\delta\phi$ for the moment, let us examine the consequences of (3.19) for ω_- if the negative mode were a fluctuation purely in δA. Taking the Fourier transform of (3.19) and considering the negative eigenmode of Ω^2 gives the dispersion relation:

$$\omega^2 - |\vec{k}|^2 + \frac{i\pi g^2 T^2}{2} B\left(\frac{\omega}{|\vec{k}|}\right) \sim -(2M_w)^2 \tag{3.20}$$

where $|\vec{k}| \sim 2M_w$. The relevant solution is

$$\omega \sim \frac{1}{B\pi}\left(\frac{2M_w}{gT}\right)^2 \times i(2M_w). \tag{3.21}$$

Note that $|\omega| << |\vec{k}|$ as we assumed.

In this analysis, we have ignored the real fluctuations which oscillate with real frequencies. Such fluctuations can only occur if they are on the conventional branch of the plasmon dispersion relation, that is $\omega > gT$. In this case, the transverse and longitudinal masses take a different form than in equations (3.16) and (3.17) and are in fact real. The peculiar feature about the decay of the sphaleron is that it occurs for a range of frequencies which is disallowed for undamped propagation of plasma oscillations. The situation we are describing is the generation of a wave by the decay of the sphaleron in a region which is Landau damped by the media.

Now consider the motion of $\delta\phi$. At tree level, the equations of motion for the small fluctuations may be written in the form:

$$k^2\delta\phi = (\Omega^2)_\phi(\delta A, \delta\phi) \quad + \quad \text{higher order.} \tag{3.22}$$

In the high-temperature limit, one-loop corrections do not modify this equation (beyond the change (2.7) in the potential, which we have already accounted for). Let us then ignore δA for the moment and examine this equation supposing the negative mode were purely $\delta\phi$. Then

$$\omega \sim i(2M_w). \tag{3.23}$$

We shall now argue that the actual ω_- lies within the range of the values (3.21) and (3.23). Qualitatively, we shall argue the following: if there exists a pure $\delta\phi$ fluctuation (*i.e.* $\delta A \equiv 0$) which lowers the energy of the sphaleron, then the system will decay in this undamped direction with ω given by (3.23); if any fluctuation which lowers the energy *must* involve δA, then the decay will be damped as given by (3.21).

To argue these claims, we will work with a simplified model of the equations of motion. Rather than treating the full infinite-dimensional problem, let us pretend that δA and $\delta\phi$ each have one degree of freedom which we shall call x and y respectively. The potential energy near the sphaleron, which gives us Ω^2, will be an unstable, quadratic potential in x and y. So let us consider the problem of a particle moving in a 2-dimensional potential:

$$V(x,y) = -\frac{a}{2}(x-y)^2 + \frac{b}{2}(x+y)^2$$

where there is strong damping in the x direction. This potential has two qualitatively different limits. If $a < b$, then the potential has the form of Figure 7a and there is no pure y fluctuation which lowers the energy. If $a > b$, as shown in Figure 7b, then a pure y fluctuation will lower the energy.

The equations of motion are

$$\begin{aligned} \ddot{x} - \kappa\dot{x} &= -a(x-y) + b(x+y) \\ \ddot{y} &= a(x-y) + b(x+y) \end{aligned} \quad . \tag{3.24}$$

Here, a and b are of order $(2M_w)^2$ and the damping κ is of order $(gT)^2$. Let us rescale to dimensionless variables, and henceforth take $a, b \sim 1$ and $\kappa \equiv 1/\epsilon^2$ where $\epsilon \sim 2M_w/gT$. We will also assume that $|a-b|$ is order 1. We now wish to find the solutions to leading order in ϵ.

Finding the leading-order behavior of the four solutions is straightforward. We find exactly one exponentially-growing mode. For $a < b$ (cannot decay in y-direction), it is

$$\omega \sim i\epsilon^2 \frac{4ab}{b-a} \qquad \vec{x} \sim \begin{bmatrix} b-a \\ b+a \end{bmatrix} . \tag{3.25}$$

For $a > b$ (can decay in the y-direction), it is

$$\omega \sim i(a-b)^{\frac{1}{2}} \qquad \vec{x} \sim \begin{bmatrix} 0 \\ 1 \end{bmatrix} . \tag{3.26}$$

We indeed find the behavior we claimed. If it can, the system decays in the undamped y-direction; otherwise, it decays at the slower, damped rate.

In any case, the modification to ω_- is not large enough to prevent the dissipation of the baryon excess. In Figure 8, we have plotted the rate dN_B/N_BdT using ω_- as given by (3.21) when (3.21) is smaller than $2M_w$. The rate exceeds the expansion rate of the universe by roughly 10^{10} at the peak and 10^8 at $\alpha_3 \sim 0.1$.

4. Near T_c and Above

In this section, we shall present a rough argument that the rate may not vanish at temperatures near T_c and above. The turnover in our expression (2.19) would then be an artifact of the breakdown of our approximations. Specifically, we shall address the size of the sphaleron and whether such transitions can compete with ordinary thermal fluctuations.

Recall that the sphaleron is of interest because it is the *minimum* energy barrier between vacua. One can pass over the barrier through another configuration, say a short-scale one, but it costs more energy to do so. The situation is analogous to a particle on a saddle. One need not pass near the stationary point to get from one side to the other, but it takes the least energy to do so. On the other hand, when the sphaleron's size is much bigger than T^{-1}, it cannot compete with thermal fluctuations — a short-scale configuration would do much better. So there is a trade-off between energy and entropy.

Let us then consider the possibility of passing through short-scale configurations rather than the sphaleron. For definiteness (though the particular choice will not matter), let us consider shrunken sphalerons given by

$$A_{(\lambda)}(\vec{x}) \equiv \lambda^{-1} A_{sp}(\vec{x}/\lambda) \qquad \phi_{(\lambda)}(\vec{x}) \equiv \phi(\vec{x}/\lambda). \tag{4.1}$$

The size of the configuration is $R \sim \lambda/M_w$. The A field has been scaled by a normal scale transformation. We have treated the ϕ field differently to keep $|\phi| \to v/\sqrt{2}$ at spatial infinity so that the energy will be finite.

How does the energy depend on λ? The F^2, $(D\phi)^2$, and $V(\phi)$ contributions scale as λ, λ^{-1}, and λ^{-3} respectively. We therefore see that the Higgs field is unimportant for configurations much smaller than M_w^{-1} and

$$\beta E_\lambda \sim \frac{1}{\alpha_w(RT)}, \qquad R \ll M_w^{-1}. \tag{4.2}$$

Note that the effective baryon number Q_B (1.12) does not depend on λ.

Let us now consider the case of very large T (specifically, $T >> 2M_w(T)/\alpha_w$). A configuration of the same size as thermal fluctuations, $R \sim T^{-1}$, would be very suppressed due to the Boltzmann factor $e^{-\beta E} \sim e^{-1/\alpha_w}$. To avoid this suppression, we must consider configurations of size $R \sim (\alpha_w T)^{-1}$. These do not compete favorably with thermal fluctuations, but previously we saw that this suppression occurred in the prefactors and was algebraic rather than exponential. So the suppression should be some power of $(RT)^{-1} \sim \alpha_w$. (In Appendix D, we put some more flesh on this argument by attempting estimates similar to those presented earlier in this paper.) We therefore expect the rate to have the form

$$\frac{dN_B}{N_B dt} \sim \alpha_w^n T. \tag{4.3}$$

Clearly, this does not tend to zero in the high temperature limit.

Note that these smaller configurations take over the job of the sphaleron when $R \sim (\alpha_w T)^{-1} \ll R_{sp} \sim (2M_w)^{-1}$. So the transition occurs when $\alpha_3 \sim 1$, which is where our previous analysis failed us and we found the turnover in our expression for the rate. Note also that the discussion on damping in section 3 is relevant to these configurations since $R^{-1} \ll gT$.

Recall that the Higgs fields were not relevant to this discussion. If these arguments are correct, they may then also have implications for QCD at finite temperature.

5. Summary

The primary conclusion of our analysis is that, for a certain range of temperature, there exists a well-defined perturbation expansion which allows for the systematic computation of the magnitude of sphaleron decay. Our computations do include the region where the sphaleron rate eventually becomes insignificant as far as cosmological effects are concerned, $T << T_c$. At temperatures very near T_c, we cannot do a computation due to uncontrollable infrared divergences. If we naively extrapolate our results to T_c, we find a vanishing rate for sphaleron-induced processes, although this vanishing may be an artifact of the extrapolation.

The number of sphalerons per unit entropy may be estimated from our analysis as (see Appendix A)

$$N/S \sim \left(\frac{\alpha_w}{4\pi}\right)^3 \alpha_3^{-6} \, \exp\left(-\lambda/\alpha_3\right) \, 10^4 \, \kappa \tag{5.1}$$

In this equation, λ is a number between 1.52 and 2.7 dependent upon g^2/λ. The constant κ is of order one. If there is significant damping (3.21), the rate of sphaleron decays per unit entropy is

$$\Gamma/S \sim \left(\frac{\alpha_w}{4\pi}\right)^2 \ \alpha_3^{-2}\, T\ N/S \tag{5.2}$$

The rate of baryon number changing processes is

$$\gamma_B \sim \frac{1}{2}\,\frac{\mu}{T}\,\Gamma/S \tag{5.3}$$

Notice that the factor of $(\alpha_w/4\pi)^2$ accounts for a suppression of about 10^{-6} of the rate relative to $T\ N/V$.

To get a more reliable estimate of the rate, a good computation of κ should be performed, since this is the largest uncertainty in our computation. This analysis is difficult, since the small fluctuations in the presence of the sphaleron do not seem to admit a simple angular momentum decomposition. Such a computation might be performed by Monte-Carlo methods, but we have no plans to do so.

There are also corrections arising from a non-zero value of the Weinberg angle. These contributions give only a small correction to the classical energy of a sphaleron, and we hope that these effects are small here. Again, the computation of such effects is complicated by the lack of spherical symmetry of the sphaleron for $\Theta_W \neq 0$.

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APPENDIX A

In this appendix, we shall discuss the dilute sphaleron-gas approximation and its validity. In general, an infinite universe will be filled with an infinite number of sphalerons. So, to evaluate ImF, we need to sum over multiple-sphaleron configurations. If the sphalerons are dilute enough that they do not overlap, then we may express multiple-sphaleron configurations as the superposition of single-sphaleron ones. We shall justify this assumption *a posteriori*.

Let us consider the path integral about two sphalerons:

$$\int [D\phi] e^{-\int \mathcal{L}_{(2\ sp)}}. \tag{A1}$$

Now divide space into two volumes V_1 and V_2, each containing one of the sphalerons. Then we may approximately split the path integral into

$$\frac{1}{2!}\int [D\phi]_{V_1} e^{-\int_{V_1} \mathcal{L}_{sp}} \int [D\phi]_{V_2} e^{-\int_{V_2} \mathcal{L}_{sp}} \tag{A2}$$

where the factor of 1/2! avoids double counting. This may be rewritten as

$$\frac{1}{2!}\frac{\int[D\phi]e^{-\int \mathcal{L}_{sp}}}{\int[D\phi]_{V_2}e^{-\int_{V_2}\mathcal{L}_{sp}}}\times\frac{\int[D\phi]e^{-\int \mathcal{L}_{sp}}}{\int[D\phi]_{V_1}e^{-\int_{V_1}\mathcal{L}_{sp}}}=Z_0\frac{1}{2!}\left(\frac{Z_{sp}}{Z_0}\right)^2. \tag{A3}$$

In general, summing all the N-sphaleron configurations, we get:

$$Z_0+Z_0\left(\frac{Z_{sp}}{Z_0}\right)+Z_0\frac{1}{2!}\left(\frac{Z_{sp}}{Z_0}\right)^2+Z_0\frac{1}{3!}\left(\frac{Z_{sp}}{Z_0}\right)^3+\cdots=\exp\left(\ln Z_0+\frac{Z_{sp}}{Z_0}\right). \tag{A4}$$

The imaginary part of the free energy is then

$$\mathrm{Im}F=T\ \mathrm{Im}\frac{Z_{sp}}{Z_0} \tag{A5}$$

as claimed.

To find the sphaleron density, we wish to find what number of sphalerons contributes most to the ImF. If we replace (A4) by

$$\begin{aligned}Z_0+e^{\gamma}Z_0\left(\frac{Z_{sp}}{Z_0}\right)+e^{2\gamma}\frac{1}{2!}Z_0\left(\frac{Z_{sp}}{Z_0}\right)^2+e^{3\gamma}\frac{1}{3!}Z_0\left(\frac{Z_{sp}}{Z_0}\right)^3+\cdots\\=\exp\left(\ln Z_0+e^{\gamma}\frac{Z_{sp}}{Z_0}\right),\end{aligned} \tag{A6}$$

then $(d(\beta\mathrm{Im}F)/d\gamma)_{\gamma=0}$ will give the average number. We find

$$N=\mathrm{Im}\frac{Z_{sp}}{Z_0}. \tag{A7}$$

Another, more physical, way to derive this answer is to say that the number should be the total rate of transitions multiplied by the time of a single transition:

$$N \sim \Gamma \frac{2\pi}{\omega_-} \sim \mathrm{Im} \frac{Z_{sp}}{Z_0}. \tag{A8}$$

From Eq. (2.14), we can pull out the $\mathrm{Im}(Z_{sp}/Z_0)$, which gives

$$\frac{N}{V} = \mathcal{N}_{\mathrm{tr}} (\mathcal{N}\mathcal{V})_{\mathrm{rot}} \left(\frac{\alpha_w T}{4\pi} \right)^3 \alpha_3^{-6} e^{-\beta E_{sp}} \times \mathrm{O}(1). \tag{A9}$$

The average spacing between sphalerons must be compared to the radius of the sphaleron. For the dilute gas approximation to be valid, we need $(N/V)^{1/3} R \ll 1$. Using (A9), we find that the dilute gas approximation is also breaking down at the peak of Figure 3 where $(N/V)^{1/3} R$ is about 1. At $\alpha_3 = 0.1$, however, it's about 0.04.

APPENDIX B

In this appendix, we discuss the normalization and volume factors $\mathcal{N}$ and $\mathcal{V}$ obtained from integrating the spatial zero-modes of the sphaleron using the method of collective coordinates. For each symmetry group under which the sphaleron is not invariant, we obtain a set of zero-modes $\delta\Phi$ corresponding to infinitesimal transformations of the fields. The zero-mode integration gives

$$\mathcal{N}\mathcal{V} = \mathcal{V} \prod \left(\frac{1}{2\pi\hbar} \int (\delta\Phi)^2 \right)^{\frac{1}{2}} \tag{B1}$$

where $\mathcal{V}$ is the volume of the group of symmetries (appropriately normalized with respect to the $\delta\Phi$).

First, let us discuss global gauge rotations of the sphaleron and of the vacuum. At spatial infinity, the sphaleron and vacuum fields approach

$$\begin{aligned} \vec{A}_{sp} &\to 0 \qquad & \phi_{sp} &\to \frac{v}{\sqrt{2}}\hat{r}\cdot\vec{\sigma} \\ \vec{A}_{vac} &\equiv 0 \;, \qquad & \phi_{vac} &\equiv \frac{v}{\sqrt{2}} \end{aligned} \quad . \tag{B2}$$

Global gauge rotations change ϕ at infinity in both cases. We can therefore ignore the rotations of these fields if we fix the boundary condition of our path integrals so that the fields must have the asymptotic behavior (B2). Fixing the boundary condition is not necessarily enough. Conceivably, there could be configurations which look like a (global) gauge-rotated sphaleron out to some large distance R and then, in the region $R < r < \infty$ return to the boundary conditions (B2) at an infinitesimal cost in energy by closely approximating a pure gauge there.[23] This cannot occur in our gauge fixing $A_r = 0$ because the only pure gauges are independent of r. We shall henceforth ignore global gauge rotations except to note that any other zero-mode we consider must preserve the boundary conditions (B2).

For translations, working with the dimensionless fields (2.2),

$$\begin{aligned} \delta\vec{A} &= (\vec{\epsilon}\cdot\vec{\nabla})\vec{A}_{sp} + \vec{D}\Lambda \\ \delta\phi &= (\vec{\epsilon}\cdot\vec{\nabla})\phi_{sp} + i\Lambda\phi \end{aligned} \tag{B3}$$

where the gauge transformation given by

$$\Lambda = \frac{k(\xi)}{\xi}\hat{r}\cdot\vec{\epsilon}\times\vec{\sigma}, \qquad k(\xi) \equiv \xi\int_{\xi}^{\infty} d\xi' \frac{f(\xi')}{\xi'^2} \tag{B4}$$

puts $\delta\phi \to 0$ at infinity so that it preserves the boundary condition. Inserting the sphaleron solution (1.9) above then yields:

$$(\mathcal{N}\mathcal{V})_{\rm tr} = \mathcal{V}_\xi\left\{\frac{2}{3}\int_0^\infty d\xi\,\left(\frac{8}{\xi^2}\left[(f+k-2fk)^2+(f-k-\xi f')^2\right]\right.\right.$$
$$\left.\left.+\left[h^2(1-k)^2+\frac{1}{2}(\xi h')^2\right]\right)\right\}^{\frac{3}{2}} \tag{B5}$$

For the case $\lambda = g^2$, this gives $\mathcal{N}_{\rm trans} = 26$.

For rotations $\delta\vec{r} = \vec{\epsilon}\times\vec{r}$, we must again also make a gauge rotation to preserve boundary conditions. We find that

$$\begin{aligned}\delta\vec{A} &= \frac{1-f}{\xi}\left[\vec{\epsilon}(\hat{r}\cdot\vec{\sigma}) - 2\hat{r}(\hat{r}\cdot\vec{\epsilon})(\hat{r}\cdot\vec{\sigma}) + \vec{\sigma}(\hat{r}\cdot\vec{\epsilon})\right]\\ \delta\phi &= 0.\end{aligned} \tag{B6}$$

The volume of SO(3) in this normalization is $8\pi^2$. We then find

$$(\mathcal{N}\mathcal{V})_{\rm rot} = 8\pi^2\left\{\frac{32}{3}\int_0^\infty d\xi(1-f)^2\right\}^{\frac{3}{2}}. \tag{B7}$$

For the case $\lambda = g^2$, this gives $(\mathcal{N}\mathcal{V})_{\rm rot} = 5.3\times10^3$.

As mentioned in the main body of the paper, global $SU(2)_R$ does not give a new zero mode; its action on the sphaleron is equivalent to rotations.

APPENDIX C

In this appendix, we derive the quasi-particle representation of Z_{sp}/Z_0 from the imaginary-time path integral. The imaginary-time path integral for a free theory gives:

$$Z \propto \left[\det(p^2 + m^2)\right]^{-\frac{1}{2}}.$$

For interacting field theories, one can to good approximation generalize this result to

$$Z \propto \left[\det(p^2 + \Sigma^\star)\right]^{-\frac{1}{2}}$$

where $\Sigma^\star$ is the proper self-energy.[20] This relation holds so long as the excitations of the system are well approximated as non-interacting particles together with non-interacting collective excitations. In the analysis of the decay of the sphaleron, we used the dispersion relation for such excitations extracted from the weak boson propagator. The important excitation was a damped plasma oscillation, which should be properly resolved within a pseudo-particle approximation. So, in the case of the sphaleron,

$$\frac{Z_{sp}}{Z_0} = e^{-\beta E_{sp}} \left[\det\left(\frac{p^2 + \Sigma_0^\star}{p^2 + \Sigma_{sp}^\star}\right)\right]^{\frac{1}{2}} \tag{C1}$$

where $\Sigma^\star$ is calculated in the appropriate background. (We will consider just the case of boson fields here.) We may rewrite this as

$$\begin{aligned}\frac{Z_{sp}}{Z_0} &= e^{-\beta E_{sp}} \exp\left\{\frac{1}{2}\sum_{\nu_n} \mathrm{tr}_{\vec{p}} \ln\left(\frac{p^2 + \Sigma_0^\star}{p^2 + \Sigma_{sp}^\star}\right)\right\} \\ &= e^{-\beta E_{sp}} \exp\left\{\frac{\beta}{2}\,\frac{1}{2\pi i}\int_C \frac{d\omega}{e^{\beta\omega} - 1} \mathrm{tr}_{\vec{p}} \ln\left(\frac{\omega^2 - \vec{p}^2 - \Sigma_0^\star}{\omega^2 - \vec{p}^2 - \Sigma_{sp}^\star}\right)\right\}\end{aligned} \tag{C2}$$

where have used the standard trick of turning the summation into a contour integration. The contour C is shown in Figure 9. Integrating by parts in ω,

$$\frac{Z_{sp}}{Z_0} = e^{-\beta E_{sp}} \exp\left\{\frac{1}{2}\frac{1}{2\pi i}\int_C d\omega \;\ln(1-e^{-\beta\omega}) \right.$$
$$\left. \mathrm{tr}_{\vec{p}}\left[\frac{2\omega}{\omega^2-\vec{p}^2-\Sigma_0^\star} - \frac{2\omega}{\omega^2-\vec{p}^2-\Sigma_{sp}^\star}\right]\right\}. \tag{C3}$$

Now, deforming the contour to pick up the poles of the propagator (C' in Figure 9), we find

$$\begin{aligned}\frac{Z_{sp}}{Z_0} &= e^{-\beta E_{sp}} \exp\left\{\frac{1}{2}\mathrm{tr}_{\vec{p}}\left[\ln(1-e^{-\beta\omega_0(p)}) + \ln(1-e^{+\beta\omega_0(p)}) \right.\right.\\ &\qquad \left.\left. - \ln(1-e^{-\beta\omega_{sp}(p)}) - \ln(1+e^{+\beta\omega_{sp}(p)})\right]\right\} \\ &= e^{-\beta E_{sp}} \prod \frac{\sinh(\beta\omega_0/2)}{\sinh(\beta\omega_{sp}/2)},\end{aligned} \tag{C4}$$

Here, the frequencies ω are the *real-time* response frequencies of the system. We can treat the system as harmonic oscillators having these real-time frequencies by rewriting this in the familiar form for the partition function:

$$\frac{Z_{sp}}{Z_0} = e^{-\beta E_{sp}} \prod \frac{\sum_n \exp(-\beta(n+\frac{1}{2})\omega_{sp})}{\sum_n \exp(-\beta(n+\frac{1}{2})\omega_0)}. \tag{C5}$$

APPENDIX D

In this appendix, we shall examine in more detail the argument of section 4 that, at very high T, the suppression of the rate is algebraic rather than exponential. Our argument here is not rigorous, and is meant only to be suggestive and to clarify the more general discussion of section 4 by making the algebra more explicit.

For a given size R, let us loosely consider the lowest-energy configuration of that size which 'sits on the edge' between the two vacua. That is, one fluctuation will cause it to fall into one vacuum, the opposite fluctuation into the other. (For example, consider all the points along the ridge of a saddle.) Except for the sphaleron, these will not be static solutions to the equations of motion.[24]

We shall focus on $R \ll M_w^{-1}$. As we saw in section 4, the Higgs fields are then irrelevant and we may concentrate on the gauge fields. By scaling R out of the coordinates, we then find the action at high temperature becomes

$$S_3 = \frac{1}{g^2(RT)} \int d^3\xi \bar{\mathcal{L}}_3[\bar{A}(\xi)]. \tag{D1}$$

This is the analog of (2.4) where now

$$\alpha_3 \equiv \frac{g_3^2}{4\pi} \equiv \alpha_w(RT). \tag{D2}$$

So we expect

$$E(R) \equiv \bar{E}\alpha_3^{-1} \sim \alpha_3^{-1}. \tag{D3}$$

Note that $\alpha_3 \ll 1$ when $R \ll (\alpha_w T)^{-1}$.

We shall now attempt to evaluate the rate at which the barrier is crossed by methods similar to those of section 2. But we will *explicitly* integrate over R while we treat everything else as Gaussians or zero modes. The calculation proceeds the same as before, but now we have one more power of g_3^{-1} in (2.13) because we have not performed a Gaussian integration for the R direction. Treating R as a collective coordinate and normalizing appropriately gives $\sim \int dR/R$. So

$$\Gamma \sim \int_{\sim M_w^{-1}}^{\infty} \frac{dR}{R} \frac{\omega_-}{2\pi} (\mathcal{N}\mathcal{V}) g_3^{-7} e^{-\bar{E}/\alpha_3}. \tag{D4}$$

This time, $V_\xi = R^{-3}V$. For simplicity, let us consider the undamped case $\omega_- \sim R^{-1}$. Then, using $x \equiv 1/\alpha_3$,

$$\frac{\Gamma}{V} \sim T^4 (\mathcal{N}\mathcal{V})' \left(\frac{\alpha_w}{4\pi}\right)^4 \frac{1}{\sqrt{\pi}} \int dx \bar{x}^{\frac{13}{2}} e^{-\bar{E}x}. \tag{D5}$$

The upper cut-off of the x-integration is $\sim M_w/\alpha_w T$. In the limit $T \gg M_w/\alpha_w$, we get

$$\frac{\Gamma}{V} \sim T^4 (4 \times 10^{-8}) (\mathcal{N}\mathcal{V})' \bar{E}^{-15/2} \tag{D6}$$

If we were to use sphaleron values for $\bar{E}$ and $\mathcal{N}_{\mathrm{tr}}(\mathcal{N}\mathcal{V})_{\mathrm{rot}}$, we would get $(2 \times 10^{-5})T^4$. The exact numerical value is not to be taken seriously; the point is to see that it need not be vanishingly small.

One should note that the most important part of the x-integration (D5) is where our approximations break down because $\alpha_3 \sim 1$. So, at best, this approach could be used to set a lower-bound on the rate by restricting the integration to $\alpha_3 \ll 1$.

It would be interesting to put more flesh on this argument by identifying the configuration (or set of configurations) $A(\vec{x})$ which do the job. We have not done so.

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FIGURE CAPTIONS

1. Potential for a one-particle analogy in quantum mechanics.
2. $f(\xi)$ and $h(\xi)$ for the sphaleron when $\lambda = g^2$.
3. $dN_B/N_B dT$ as a function of T, for $\lambda = g^2$ and three families, ignoring the damping effects discussed in section 3.
4. Our estimate for $dN_B/N_B dT$ (solid line) plotted against the simple estimate of eq. (1.2) (dashed line).
5. Leading one-loop contributions to the equations of motion.
6. Possible contributions of order g^2 to the equations of motion.
7. Unstable quadratic potential in x,y for (a) $a < b$ and (b) $b < a$.
8. $dN_B/N_B dT$ where Landau damping has been included.
9. The integration contours for deriving the pseudo-particle formula.

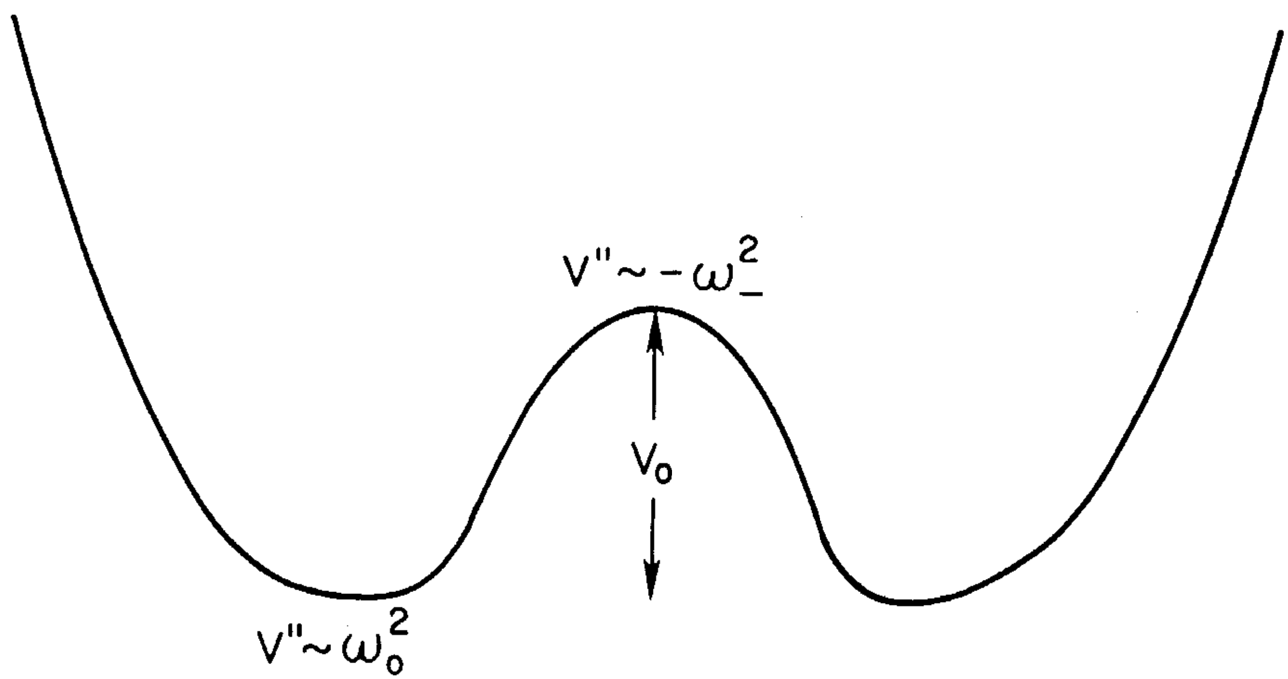


Figure 1.

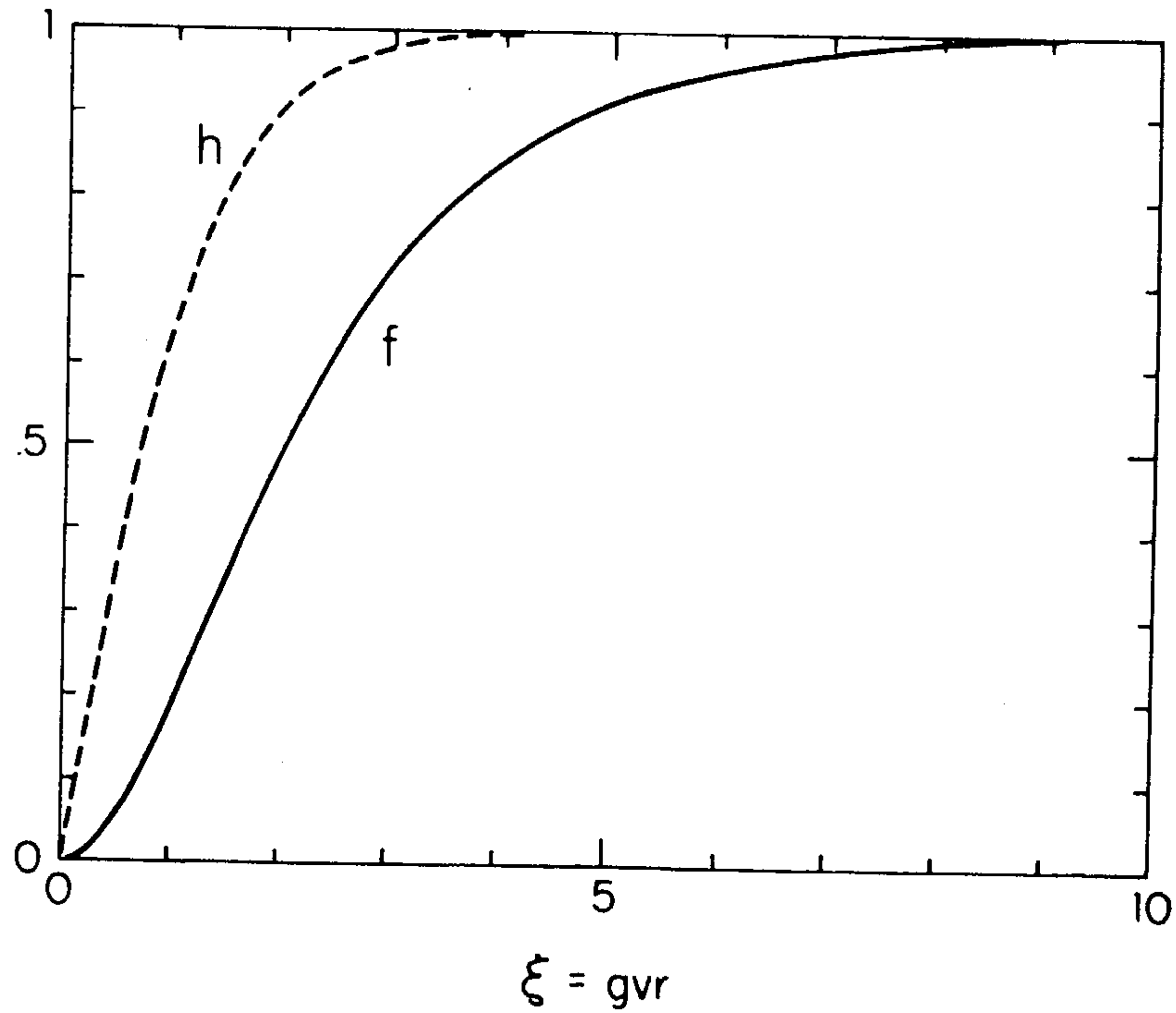
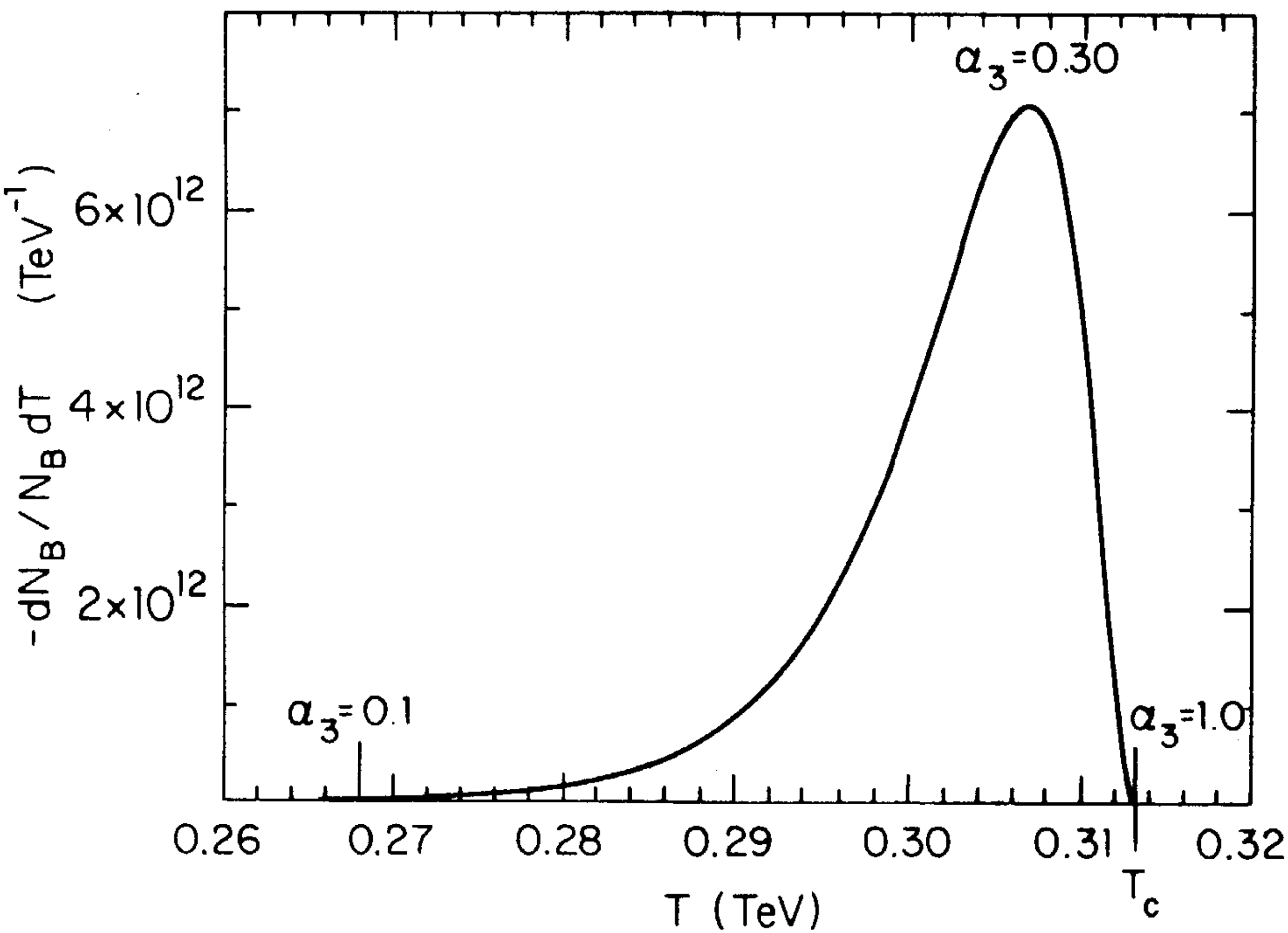


Figure 2



$-dN_B/N_B\,dT$ (TeV^{-1})
6×10^{12}
4×10^{12}
2×10^{12}
$\alpha_3=0.30$
$\alpha_3=0.1$
$\alpha_3=1.0$
0.26
0.27
0.28
0.29
0.30
0.31
0.32
T_c
T (TeV)

Figure 3

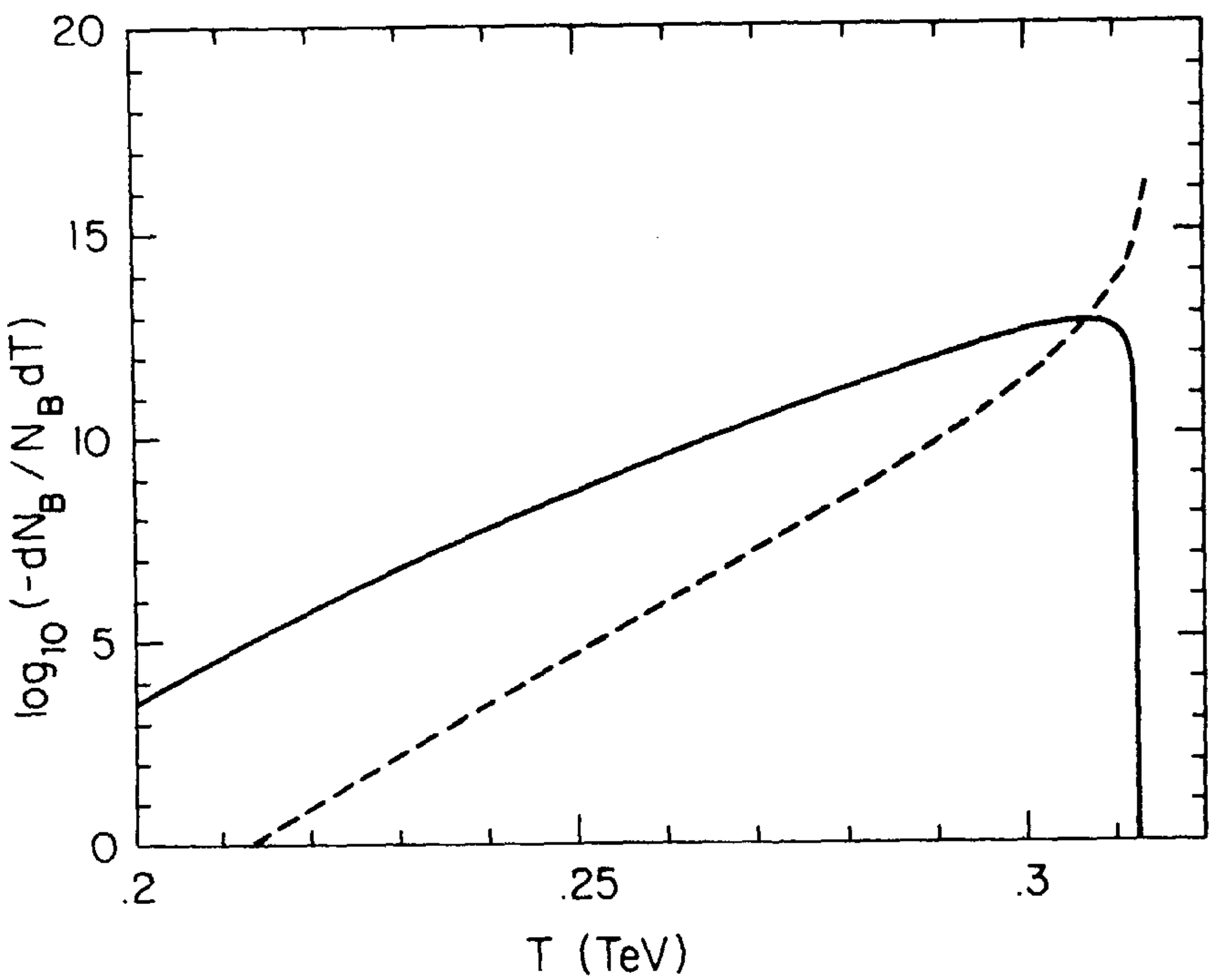


Figure 4.

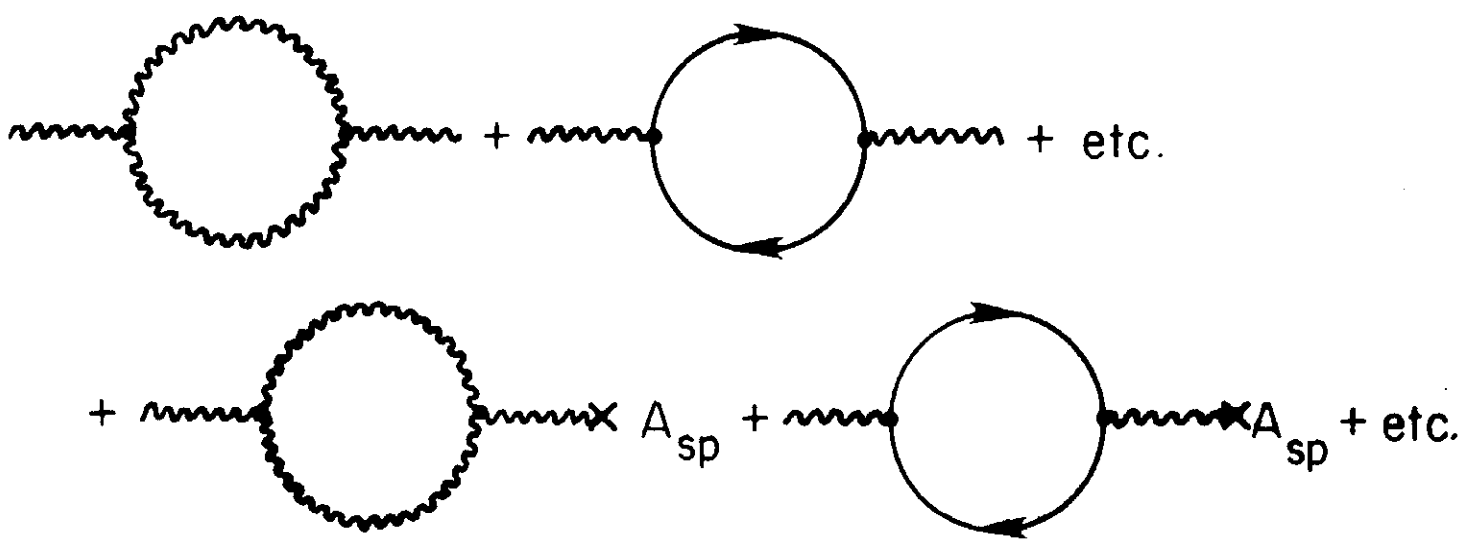


Figure 5.

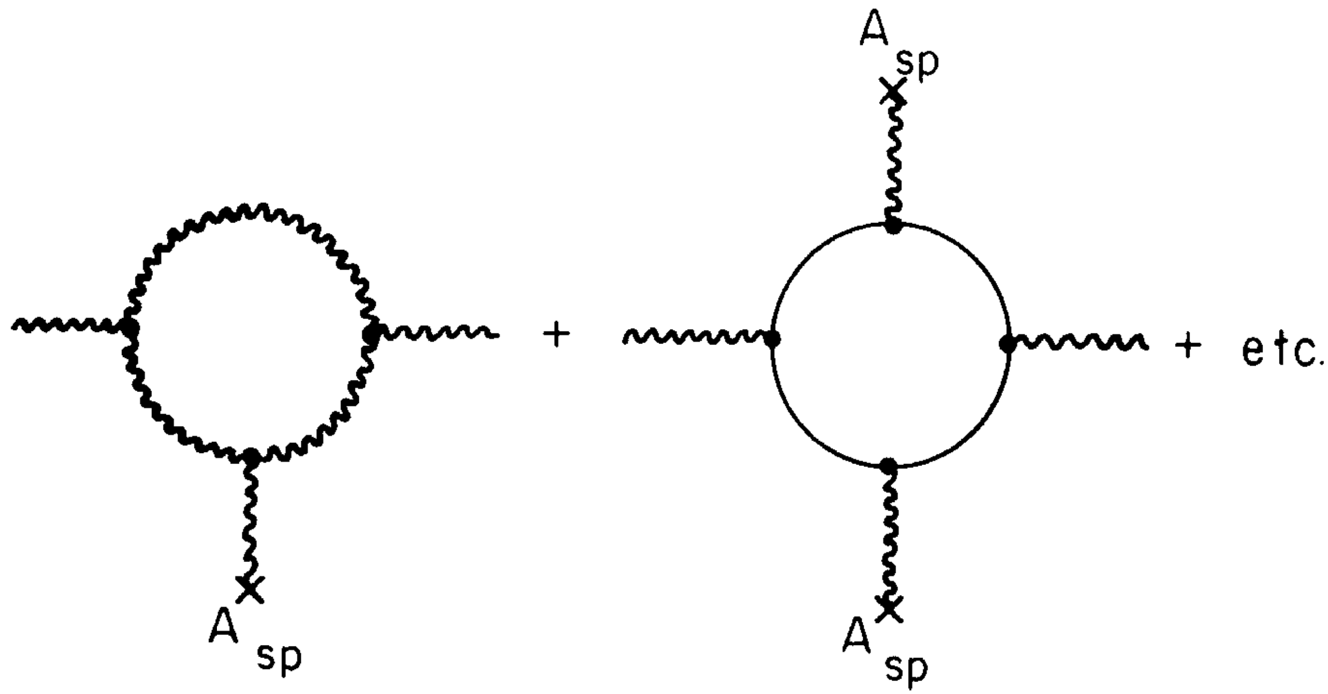
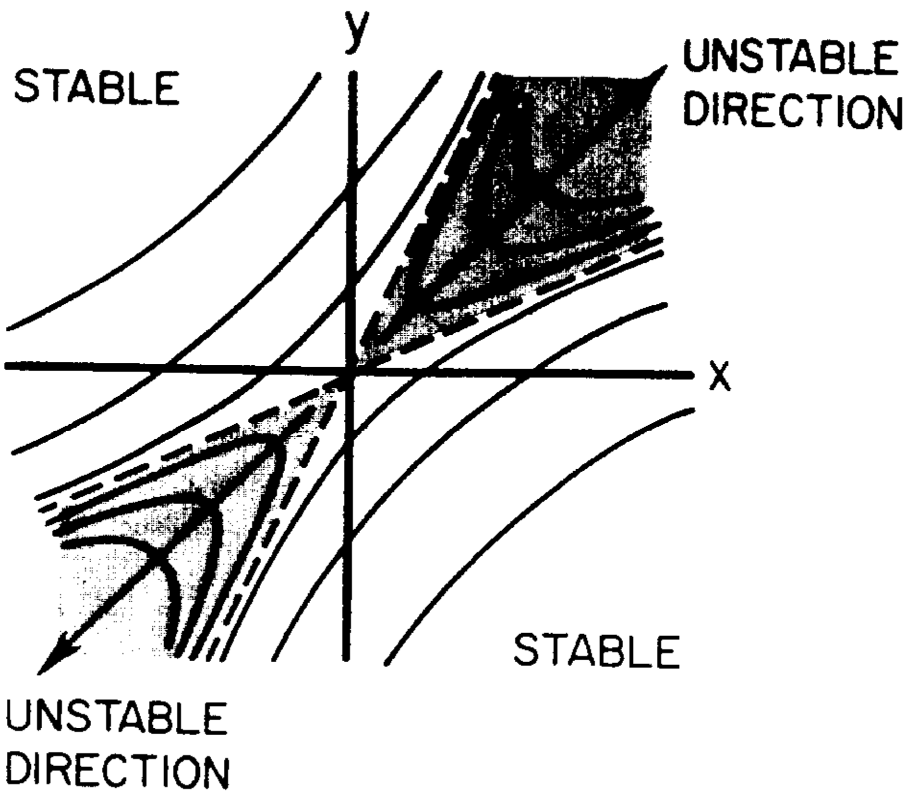
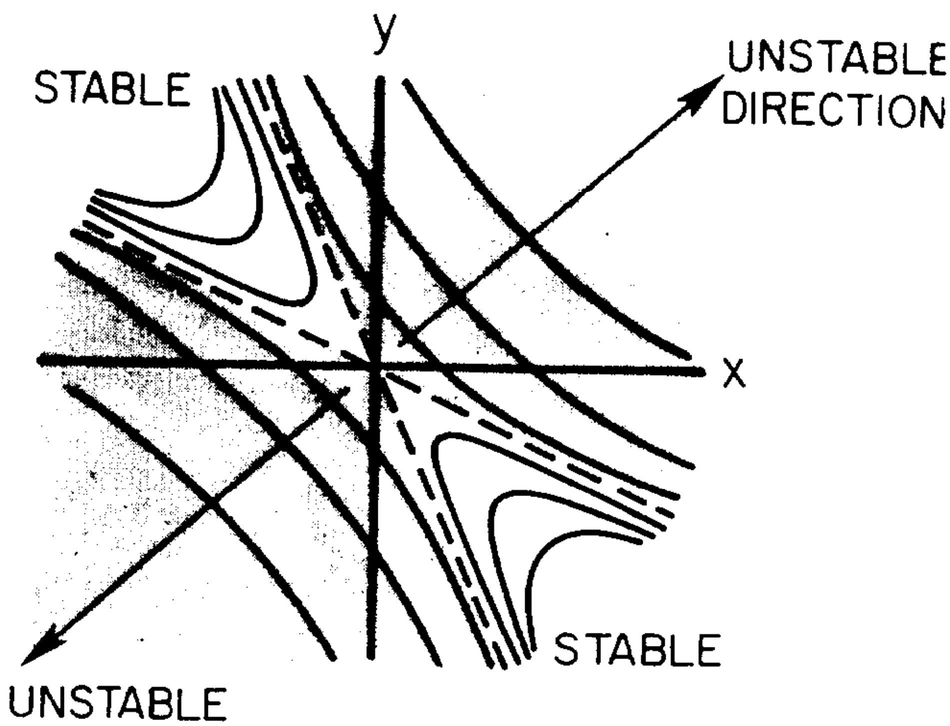


FIGURE 6


y
STABLE
UNSTABLE
DIRECTION
x
STABLE
UNSTABLE
DIRECTION

(a)


y
STABLE
UNSTABLE
DIRECTION
x
STABLE
UNSTABLE
DIRECTION

(b)

Figure 7.

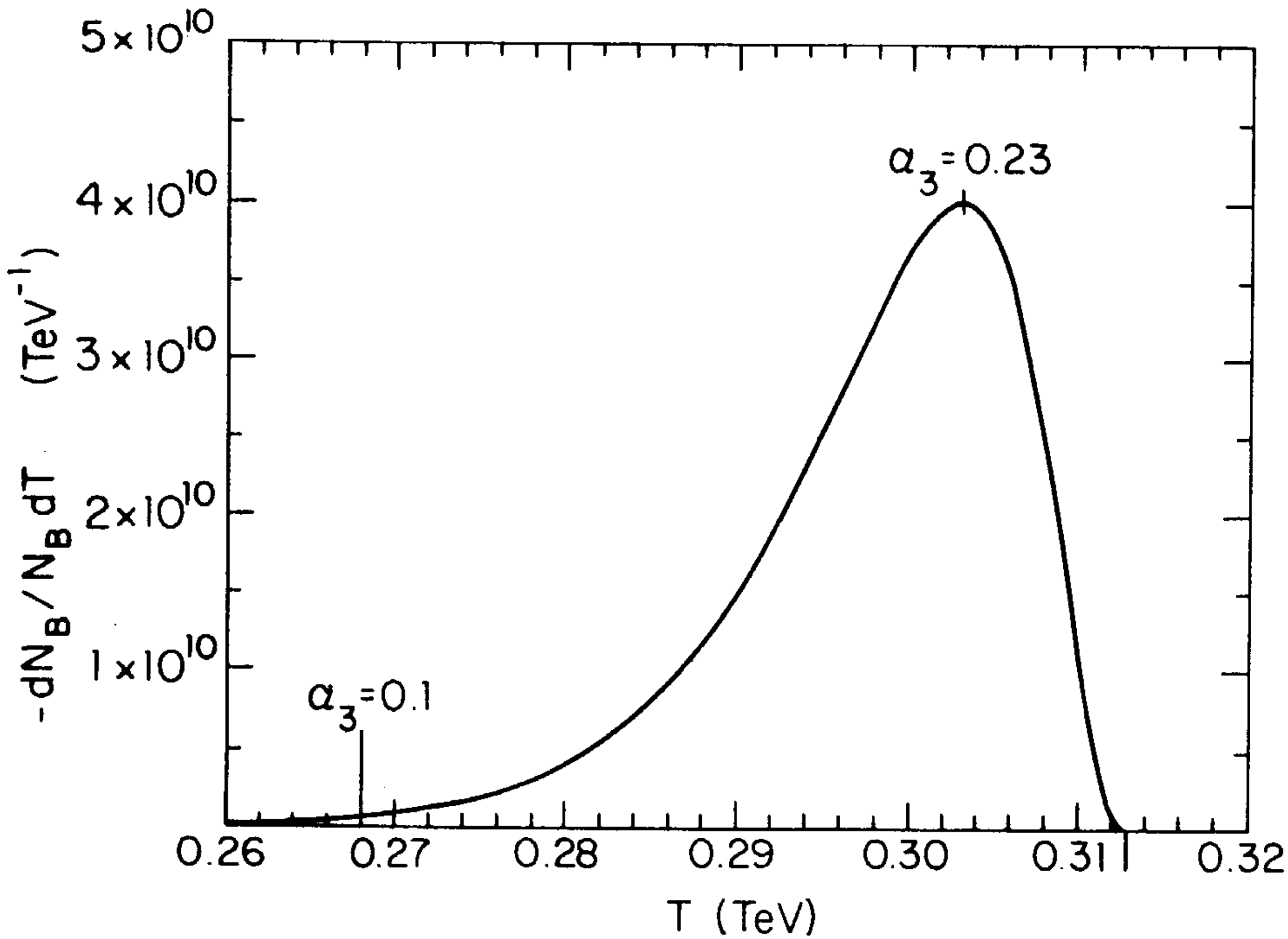

5×10^10
4×10^10
3×10^10
2×10^10
1×10^10
-dN_B/N_B dT (TeV^-1)
α_3=0.23
α_3=0.1
0.26
0.27
0.28
0.29
0.30
0.31
0.32
T (TeV)

Figure 8.

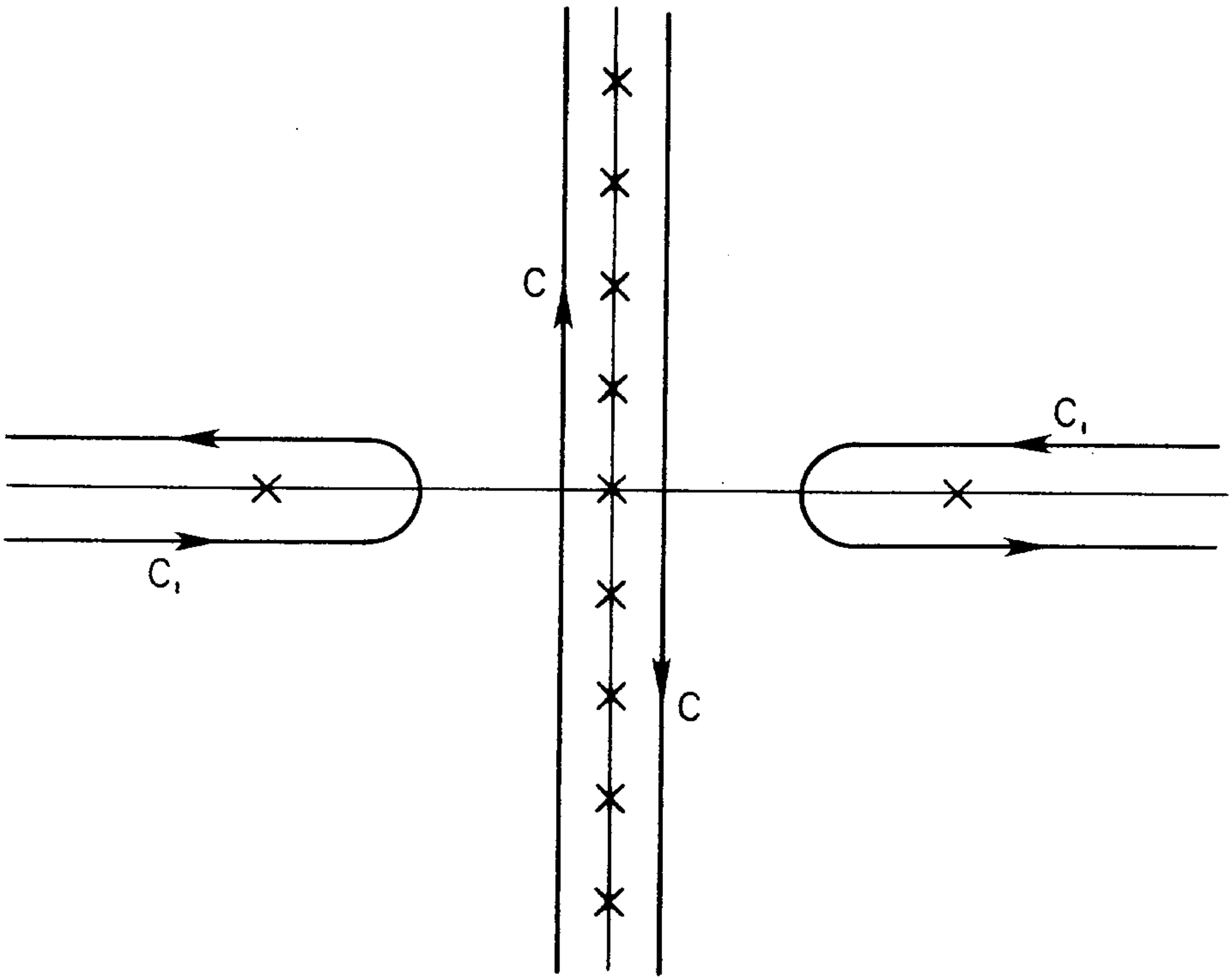


Figure 9.