Fermi National Accelerator Laboratory

FERMILAB-Pub-87/17-A
January 1987

ELECTROWEAK ANOMALY AND LEPTON ASYMMETRY

Edward W. Kolb
Michael S. Turner

Theoretical Astrophysics Group
Fermi National Accelerator Laboratory
Batavia, IL 60510 USA
and
Department of Astronomy and Astrophysics
Enrico Fermi Institute
The University of Chicago
Chicago, IL 60637 USA

Abstract

It has recently been pointed out that shortly after the electroweak phase transition ($t \simeq 10^{-11}$ sec, $T \simeq 100 - 200$ GeV) there was a period of very effective baryon (and lepton) number violation driven by non-perturbative effects arising from the electroweak anomaly. Here we argue that as a result of these electroweak interactions the *total* asymmetry in leptons must be equal and opposite to the baryon asymmetry: $L \equiv \sum_i L_i = L_e + L_\mu + L_\tau = -B$. Since the individual lepton numbers can be positive or negative, this does not preclude any large individual contribution, *i.e.*, $|L_i| > B$. This fact has important consequences for primordial nucleosynthesis if $|L_i| \gtrsim 1$.

Submitted for publication to *Modern Physics Letters A*

The baryon number of the Universe, B, defined as the ratio of the number density of baryons, n_b, minus the number density of antibaryons $n_{\bar{b}}$, to the entropy density, s (s is related to the photon number density by $s \simeq 7.04 n_\gamma$ for three light neutrino species), is one of the fundamental numbers in cosmology. It is a crucial input parameter to the calculation of primordial nucleosynthesis, in the determination of when the Universe became matter-dominated, and in the scenarios of galaxy formation. The baryon number density is only known to tolerable accuracy. The dynamics of galaxies, clusters of galaxies, etc., and the age of the Universe provide a limit to the total mass density in the form of non-relativistic matter, hence a limit to the number density of baryons. The photon number density is known to good accuracy from measurements of the temperature of the microwave background radiation ($T = 2.75K \pm 0.05K$), and the inferred entropy density is therefore known up to small uncertainties in the number of light neutrinos. B can be expressed in terms of Ω_B, the ratio of the mass density in baryons to the critical density, $\rho_c \equiv 3H_0^2/8\pi G = 1.88 \times 10^{-29} h^2 \text{ g cm}^{-3}$,

$$B \equiv \frac{n_b - n_{\bar{b}}}{s} = 4.0 \times 10^{-9} \Omega_B h^2 / T_{2.7}^3, \tag{1}$$

where the present photon temperature is $T = 2.7 T_{2.7}$K and the present Hubble parameter is $H_0 = 100\ h$ km s^{-1} Mpc^{-1}.

Requiring that the age of the Universe be larger than 10 Gyr and that $h \geq 0.4$ implies that $\Omega_{TOT} h^2 \leq 1.1$. The amount of luminous material in the Universe (presumably baryonic!) amounts to $\Omega_{lum} \simeq 0.01$. Together these observations imply

$$0.01 \lesssim \Omega_B h^2 \lesssim 1.1 \tag{2a}$$

$$4.0 \times 10^{-11} \lesssim B \lesssim 4.5 \times 10^{-9} \tag{2b}$$

where for simplicity $T_{2.7}$ is taken to be 1. Clearly our direct knowledge of Ω_B implies that $B \ll 1$. For a detailed discussion of the amount of baryonic material in the Universe see ref. 1.

The electric charge of the Universe is known to be very small, $n_Q/s \lesssim 10^{-27}$ (ref. 2). The charge neutrality of the Universe implies that the observed excess in the number of protons in the Universe compared to the number of antiprotons must be compensated for by a corresponding excess in the number of electrons compared to the number of positrons. If $L_{e^\pm} \equiv (n_e - n_{\bar{e}})/s$ is the lepton asymmetry in the electrons, and Y_e is the ratio of protons to baryons in the Universe, then $L_{e^\pm} = Y_e B \simeq O(10^{-10})$. The lepton number of the Universe is

$$L = L_e + L_\mu + L_\tau \tag{3a}$$

$$L_e = L_{e^\pm} + (n_{\nu_e} - n_{\bar{\nu}_e})/s \tag{3b}$$

$$L_j = (n_{\nu_j} - n_{\bar{\nu}_j})/s \qquad (j = \ \mu, \ \tau) \tag{3c}$$

Since $L_{e^\pm} \simeq 10^{-10}$, if the Universe has a large lepton number it must be hidden in the undetected cosmic neutrino seas.

The most reliable limit to the neutrino number of the Universe comes from the limit to the total energy density of the Universe. If neutrinos are cold (chemical potential, μ_ν, much greater than temperature, T_ν) and massless, the energy density in neutrinos is (per species)

$$\rho_\nu = \frac{\mu_\nu^4}{8\pi^2}\left[1 + 12\varsigma(2)\left(\frac{T_\nu}{\mu_\nu}\right)^2 + \cdots\right], \tag{4}$$

which results in a contribution to Ω_{TOT} of: $\Omega_\nu h^2 = 1.6 \times 10^8(\mu_\nu/\mathrm{eV})^4$. Since $\Omega_\nu h^2$ must be less than 1.1, this implies a limit to the neutrino chemical potential of $|\mu_\nu| \leq 9.1 \times 10^{-3}$eV. The neutrino temperature is comparable to the photon temperature (but depends upon the details of neutrino decoupling), and the limit from the energy density of the Universe gives[3]

$$|\mu_\nu/T_\nu| \leq 140. \tag{5}$$

The lepton number in neutrino species i is

$$L_i \simeq 9.8 \times 10^{-3}[\pi^2(\mu_i/T) + (\mu_i/T)^3] \tag{6}$$

Since $|\mu_i/T|$ is only constrained to be less than 140, the Universe can be endowed with a very large lepton number, $|L_i|$ as large as 3×10^4. For a more detailed discussion of the cosmological implications of and limits to neutrino degeneracy see ref. 3.

One of the most important effects of a large neutrino asymmetry would be upon primordial nucleosynthesis. Primordial nucleosynthesis with a large neutrino chemical potential has been considered in detail by Wagoner, Fowler and Hoyle,[4] by Yahil and Beaudet,[5] by David and Reeves,[6] by Fry and Hogan,[7] by Steigman,[8] and by Terasawa and Sato.[9] There are two primary effects of neutrino degeneracy on primordial nucleosynthesis. If $|\mu_\nu|/T_\nu > 1$ the neutrino energy density as a function of temperature is increased, by a factor of order $(\mu_\nu/T_\nu)^4$, and the equilibrium value of the n/p ratio is shifted (the latter effect is sensitive only to the chemical potential of the electron neutrino): $(n/p)_{eq} \simeq \exp(-\Delta m/T - \mu_{\nu_e}/T)$, where Δm is the neutron-proton mass difference. In addition a large neutrino chemical potential can effect neutrino decoupling and the neutrino-photon temperature ratio.[3]

The elements produced in significant amounts during primordial nucleosynthesis are ^{2}H, ^{3}He, ^{4}He, and ^{7}Li. The predictions of the standard ($|L_i| \ll 1$) big-bang cosmology are in agreement with the inferred primordial abundances of these elements for $6 \times 10^{-11} \lesssim B \lesssim 10^{-10}$ (ref. 10). In cosmological models with large lepton number ($|L_i| \gtrsim 1$) it is difficult, perhaps impossible, to obtain concordance between the predictions for the primordial abundances of D, ^{3}He, ^{4}He and ^{7}Li and their inferred abundances simultaneously[8]. In this regard ^{7}Li is crucial; obtaining concordance for D, ^{3}He, and ^{4}He is possible[3–9]. Six years ago it was believed that it was probably impossible to infer the primordial ^{7}Li abundance, due to the large contamination of contemporary astrophysical processes. The measurement made by Spite and Spite[11] in 1982 of the ^{7}Li abudance in the atmospheres of old halo and disk stars (extreme pop II) changed that. While all the recent data[12] suggests that the primordial ^{7}Li abundance has been deduced, the situation is far from being definitive (for further discussion see ref. 8). Even the primordial abundances (which must be inferred from present measurements) of D and ^{3}He are not beyond reproach. In addition, non-standard scenarios have been suggested for producing the primordial abundances of D, ^{3}He, and ^{7}Li (see ref. 13). We will adopt the point of view that it is not yet totally heretical to entertain the notion of a large lepton number in one or more of the neutrino species.

There are other cosmological consequences of large lepton number.[3] A Universe with a large neutrino asymmetry will be radiation dominated until a lower temperature than the standard cosmology (possibly well past the time of recombination), hence postponing the onset of structure formation and perhaps resulting in insufficient time for the formation of structure. This would argue against the possibility of a large neutrino asymmetry if structure formation proceeds solely as the result of gravitational instabilities with a primordial Harrison-Zel'dovich fluctuation spectrum[3]. However, this constraint might be circumvented in non-standard theories of structure formation, such as models where cosmic strings act as the seeds for structure formation.

In the standard out-of-equilibrium scenario for the generation of the baryon asymmetry the baryon and lepton asymmetries are both small ($B,\ L \ll 1$)[14,15]. However it is possible to arrange the intial conditions to produce a large lepton number *and* a small baryon number[16].

In this Letter we point out that if the electroweak anomaly effects discussed by Kuzmin, Rubakov, and Shaposhnikov (KRS)[17] are important, baryon- and lepton-number violating (but $B-L$ conserving) reactions will result in $B=-L$, hence the total lepton number *must* be small, although the *individual* lepton numbers could still be large.

The rate of baryon number violation due to the anomaly in an $SU(2)$ gauge theory with gauge coupling g_W at finite temperature was estimated by KRS. They pointed out that fermion quantum number violation occurs due to transitions between the different θ-vacua, and the rate for these transitions can be computed by considering vacuum decay at finite temperature[18]. The transition rate is proportional to $\exp(-S_3/T)$, where $S_3(T)$ is action for the appropriate solution to the 3-dimensional Euclidean field equations and T is the temperature. The dominant contribution to the action comes from the static "sphaleron" solutions for the gauge fields A_i^a (the $A_0^a=0$ gauge is used) and Higgs field ϕ[19–21]

$$A_i^a(\vec{x}) = \varepsilon_{iak}\frac{x^k}{|\vec{x}|}g(\xi)$$

$$\phi(\vec{x}) = i\frac{v}{\sqrt{2}}\frac{\vec{\tau}\cdot\vec{x}}{|\vec{x}|}\begin{pmatrix}0\\1\end{pmatrix}h(\xi), \tag{7}$$

where $\vec{\tau}$ are the Pauli spin matrices, and $g(\xi)$ and $h(\xi)$ are functions of $\xi = g_W v|\vec{x}|$ and set the scale for the size of the classical solutions ($g(0)=h(0)=1,\ g(\infty)=$

$h(\infty) = 0$). For the sphaleron configuration

$$S_3 = (4\pi v(T)/g_W)B(\lambda/g_W), \tag{8}$$

where λ is a Yukawa coupling and $B(x) \simeq 2$ for $\infty \geq x \geq 0$. For applications to the Weinberg-Salam model, at zero temperature $v = 253\text{GeV}$ and $g_W = 0.64$. The rate of baryon number violation (assuming for the moment that $B - L = 0$) found by KRS is given by

$$\dot{B} \simeq -BT \exp(-S_3/T) \simeq -B\Gamma_S(T). \tag{9}$$

The pre-exponential factor T was chosen by KRS on dimensional grounds.

A more detailed calculation of the pre-exponential factor was done by the authors of refs. 22,23. The more detailed treatment of the pre-exponential factor does not quantitatively change the conclusion, although these authors point out that if the Higgs mass is small enough, 9 GeV $\lesssim m_H \lesssim$ 45 GeV, the KRS effect may not be significant.

The rate for fermion quantum number violation (Γ_S) is much, much greater than the expansion rate of the Universe ($H = \dot{R}/R \simeq T^2/m_{Pl}$) for $T \simeq 200 - 300$ GeV: $\Gamma_S/H \gtrsim 10^8$ (refs. 22,23). Due to the exponential factor in Eqn(9), the freeze out of quantum number violation is abrupt (see refs. 17, 22, 23).

Although B and L are damped by these non-perturbative effects, $B - L$ is conserved; $(\dot{B} - \dot{L}) = 0$ is a consequence of the fact $B - L$ is anomaly free. It is clear that if $(B - L) = 0$ before the time of the electroweak transition, then the lepton number and the baryon number will be driven to zero. This undesirable result can be avoided if there is a non-zero value of $B - L$ present prior to the epoch of the electroweak transition. The non-perturbative effects will damp any initial baryon (and lepton) number that has a zero projection on $B - L$, and will result in $B = -L$. In $SU(5)$ $B - L$ is conserved, and any baryon asymmetry which evolves must necessarily have $B - L = 0$. For many larger gauge groups, such as $SO(10)$ and E_6, $B - L$ is not conserved, and $B - L \simeq O(B, L)$ also evolves[14].

In order to explicitly see how this works, it is necessary to write the Boltzmann equations governing the evolution of B and L_i. The effect of a sphaleron-induced B and L violating transition is to create a charge=isospin=0 color singlet $lqqq$ (or

$\bar{l}\bar{q}\bar{q}\bar{q}$) state from *each* generation, thereby violating B (L) by $\pm 3(\mp 3)$ units. The sphaleron transition rate, Γ_S, in a plasma with non-zero lepton and baryon number is proportional to

$$\Gamma_S \propto \exp(-S_3/T - \mu_e/T - \mu_\mu/T - \mu_\tau/T - \mu_1/T - \mu_2/T - \mu_3/T)$$

where the μ_i are the lepton chemical potentials ($i = e,\ \mu,\ \tau$) and the baryon chemical potentials ($i = 1,\ 2,\ 3$ label the generations). In the absence of C, CP violation in the interactions it is simple to show (see, e.g., ref. 14) by linearizing in the chemical potentials or by using Maxwell–Boltzmann statistics that

$$\dot{B}_i = \dot{L}_i = -\Gamma_S(T)(L+B) \qquad i = e,\ \mu,\ \tau,\ 1,\ 2,\ 3$$

where $L = L_e + L_\mu + L_\tau$, $B = B_1 + B_2 + B_3$, overdot indicates a derivative with respect to time, and $\Gamma_S(T) \propto \exp[-S_3(T)/T]$ is the sphaleron decay rate. Since strong interactions (through Cabbibo mixing) rapidly interconvert quarks of the different generations, separate bookkeeping is unnecessary (in any case, for $T \ll 1\mathrm{GeV}$ the entire baryon number resides in neutrons and protons). In the *standard model* there are no interactions which rapidly interconvert lepton asymmetries. Modifications to the standard model such as neutrino masses or non-diagonal Higgs interactions *could* mix the lepton asymmetries. For the moment we will assume that such interactions do not exist, or are always ineffective (rate $\Gamma \ll H$), and will return to this point later.

The four equations for the evolution of L_i and B are

$$\dot{L}_e = -\Gamma_S(L+B) \tag{10a}$$

$$\dot{L}_\mu = -\Gamma_S(L+B) \tag{10b}$$

$$\dot{L}_\tau = -\Gamma_S(L+B) \tag{10c}$$

$$\dot{B} = -3\Gamma_S(L+B). \tag{10d}$$

The four eigenmodes are easy to identify:

$$(\dot{L}_i - \dot{B}/3) = 0 \qquad (i = e,\ \mu,\ \tau) \tag{11a}$$

$$(\dot{L} + \dot{B}) = -6\Gamma_S(L+B) \tag{11b}$$

The first three correspond to conservation of '$B - L$' for each generation since $B_i = B/3$.

If the sphaleron transition rate Γ_S is very rapid ($\Gamma_S/H \gtrsim 10^8$ in the temperature range of 200–300 GeV), any pre-electroweak $B + L$ is exponentially damped:

$$(B+L)_f \simeq \exp\left(-\int \Gamma_S dt\right)(B+L)_0 \ll (B+L)_0,$$

while the $B - L$ for each generation is conserved. Using these facts the final lepton number asymmetries can be written as

$$\begin{aligned}
(B)_f &= (B-L)_0/2, \\
(L_e)_f &= (B-L)_0/6 + (L_e - B/3)_0, \\
(L_\mu)_f &= (B-L)_0/6 + (L_\mu - B/3)_0, \\
(L_\tau)_f &= (B-L)_0/6 + (L_\tau - B/3)_0.
\end{aligned}$$

where subscript 'f' refers to final ($T \ll 100$ GeV) and subscript '0' to initial ($T \gg 100$ GeV).

What conclusions can we draw? First, for any initial baryon asymmetry to survive there must be a net initial $B - L$. Second, since $(B+L)_f = 0$, the *total* lepton asymmetry which survives $(L)_f = -(B)_f = -(B-L)_0/2$. However, the individual lepton asymmetries can be large, provided that their sum is small. In order for any of the final lepton asymmetries to be large, there must be large initial $(L_i - B/3)$ asymmetries in at least two generations (otherwise $(B-L)_0$ would be large, implying a large B_f). If there are interactions beyond those in the standard model which rapidly interconvert lepton asymmetries (so that $L_i = L/3$), then $(L_i)_f = -(B)_f/3 \simeq -\frac{1}{3} \times 10^{-10}$.

In sum, if B, L violation via the electroweak anomaly is important, then the baryon asymmetry today must: (1) be due to a pre-electroweak $B - L$ asymmetry of twice the observed baryon asymmetry; or (2) evolve at a low temperature ($T \ll$ 100GeV) after the electroweak effects become impotent; or (3) evolve due to C, CP violating processes associated with the electroweak anomaly (which seems very unlikely[22,23]). In case (1) we can conclude that the total lepton number is equal to and opposite in sign to the total baryon number. Unless interactions beyond the standard model can insure that $L_i = L/3$ it is still possible that $|L_i| \gtrsim 1$, and necessarily for at least two flavors. If $|L_i| \gtrsim 1$, $L_e + L_\mu + L_\tau$ must still be small and equal to $-B \simeq -10^{-10}$; thus the allowed class of possibilities for large lepton asymmetries is restricted, and it may be that even this possibility can be definitely ruled out by primordial nucleosynthesis. We are currently investigating this possibility.[24]

Acknowledgment

This work was supported in part by the U.S. Department of Energy (at Chicago and Fermilab) and by the National Aeronautics and Space Administration (at Fermilab). MST is also supported by the A.P. Sloan Foundation.

References

1. V. Trimble, *Ann.Rev.Astron.Astrophys.* **25** (1987) in press; S. Faber and J. Gallagher, *Ann.Rev.Astron.Astrophys.* **17**, 135(1979); *Dark Matter in the Universe* (IAU 117), eds. J. Kormendy and G. Knapp (Reidel, Dordrecht, 1987).
2. R.A. Lyttleton and H. Bondi, *Proc.R.Soc.London* **A252**, 313(1959).
3. K. Freese, E.W. Kolb, and M.S. Turner, *Phys.Rev.* **D27**, 1689(1983).
4. R.V. Wagoner, W.A. Fowler, and F. Hoyle, *Ap. J.* **148**, 3 (1967).
5. A. Yahil and G. Beaudet, *Ap. J.* **206**, 26 (1976).
6. Y. David and H. Reeves, in *Physical Cosmology,* ed. R. Balian, J. Audouze, and D.N. Schramm (North-Holland, Amsterdam, 1980).
7. J.N. Fry and C.J. Hogan, *Phys. Rev. Lett.* **49**, 1783 (1982).
8. G. Steigman, in *Nucleosynthesis and its Implications on Nuclear and Particle Physics*, eds. J. Audouze and N. Mathieu, NATO ASI Series **163**, 45 (1985); A. Boesgaard and G. Steigman, *Ann.Rev.Astron.Astrophys.* **23**, 319(1985).
9. N. Terasawa and K. Sato, *Ap. J.* **294**, 9(1985).
10. J. Yang, M.S. Turner, G. Steigman, D.N. Schramm, and K.A. Olive, *Ap. J.* **281**, 493(1984).
11. F. Spite and M. Spite, *Astr. Ap.* **115**, 357 (1982).

12. M. Spite, J. P. Maillard, and F. Spite, *Astr. Ap.* **56**, (1984); F. Spite and M. Spite, *Astr. Ap.* **163**, 140 (1986); L. M. Hobbs and D. K. Dunkin, *Ap. J.* (in press) (1987); J. Beckman, R. Rebolo, and P. Molaro, preprint (1986).
13. J. Audouze and J. Silk, in *ESO Workshop on Primordial Helium*, eds. P. Shaver, D. Kunth, and K. Kjär (ESO, Garching, 1983) p. 7; S. Ramadurai, and M.J. Rees, *Mon. Not. Royal. Astro. Soc.* **215**, 53(1985).
14. For a review, see E.W. Kolb and M.S. Turner, *Ann. Rev. Nucl. Part. Sci.* **33**, 645(1983).
15. M.S. Turner, *Phys.Lett.* **98B**, 145(1981).
16. J.A. Harvey and E.W. Kolb, *Phys. Rev D* **24**, 2090(1981); I. Affleck and M. Dine, *Nucl. Phys.* **B249**, 361(1985).
17. V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposnikov, *Phys. Lett.* **155B**, 36(1985).
18. A.D. Linde, *Phys. Lett.* **70B**, 306(1977); **100B**, 37(1981).
19. R. Dashen, B. Hasslacher, and A. Neveu, *Phys. Rev. D* **10**, 4138(1974).
20. P. Forgács and Z. Horváth, *Phys. Lett.* **138B**, 397(1984).
21. F. R. Klinkhamer and N.S. Manton, *Phys. Rev. D* **30**, 2212(1984).
22. F.S. Accetta, P.Z. Arnold, E.W. Kolb, L. McLerran, and M.S. Turner, Fermilab Report (unpublished).
23. P. Arnold and L. McLerran, Fermilab preprint.
24. L. Kawano and E.W. Kolb, in preparation(1987).