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COSMIC STRINGS: A PROBLEM OR A SOLUTION?

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ABSTRACT

We address, by means of Numerical Simulations, the most fundamental issue in the theory of cosmic strings: the existence of a scaling solution. The resolution of this question will determine whether cosmic strings can form the basis of an attractive theory of galaxy formation or prove to be a cosmological disaster like magnetic monopoles or domain walls. After a brief discussion of our numerical technique, we present our results which, though still preliminary, offer the best support to date of this scaling hypothesis.

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INTRODUCTION

Although Cosmic strings are often discussed as possible sources of the primordial density fluctuations[1] the fundamental question of how a string network evolves in the early universe has not been resolved. Strings form when the universe is at a GUT scale temperature, but they are expected to be relevant at much later times because the majority of the string length at the time of formation is in the form of infinitely long strings. These infinitely long strings cannot radiate away into gravitational radiation, so they are expected to survive indefinitely. This is the reason that cosmic strings might be relevant for galaxy formation, but it also poses a potential problem for the cosmic string scenario.

If the interactions between strings are neglected, it can be shown that the energy density of the infinite strings scales roughly like non-relativistic matter. So if interactions were not important this would imply that imply that cosmic strings (or their gravitational radiation) would dominate the universe soon after the strings are formed. This "disaster" is supposed to be avoided by the process by which the infinitely long strings cross themselves and break off small loops which can decay into gravitational radiation. This scenario has been studied analytically by Kibble[2] and Bennett[3] Their work has shown that there are two possibilities: either the loop production is not sufficient to avoid a string-dominated universe, or the strings will settle down to a scaling solution in which the number of strings crossing a given horizon volume is fixed.

A great deal of work has already been done on the cosmic string theory of galaxy formation, *assuming* that a scaling solution does indeed exist, and there has been a great deal of speculation as to the *characteristics* of the assumed scaling solution. So far, however, all of this work is on uncertain ground because the basic details of string evolution are not understood. Albrecht and Turok[4] have published preliminary results from their simulation two years ago. However, their program was fairly crude, and these results were criticized as inconsistent on the basis of analytical work[3]

NUMERICAL TECHNIQUE

To Generate the initial conditions, we follow the general procedure introduced by Vachaspati and Vilenkin[5] To minimize the problems caused by the degeneracies of these initial conditions, we have replaced the generated sharp corners by arcs of circles, and added small transverse velocities on the curved segments.

To evolve the generated configuration, we solve the partial differential equations[6] which describe cosmic string motion in an expanding universe using a modified leapfrog scheme. A major difficulty is that the strings have physical discontinuities in $\dot{\mathbf{x}}$ and $\mathbf{x}'$, which result from the reconnections that occur when strings cross. These "kinks" have a long lifetime, and may have important implications for the loop fragmentation pattern. To avoid the development of short wavelength instabilities near the kinks, we have introduced limited amount of numerical diffusion. This is accomplished by averaging the velocities over neighboring points *only* when instabilities start to develop. The onset of this instability is detected by comparing the values for the energy per unit proper length $\epsilon = \sqrt{\mathbf{x}'^2/(1-\dot{\mathbf{x}}^2)}$ which is evolved with its own equation even though it is not independent of $\dot{\mathbf{x}}$ and $\mathbf{x}'$. When the evolved value for ϵ differs from the value calculated from $\dot{\mathbf{x}}$ and $\mathbf{x}'$ by a few per cent, the velocity averaging is invoked. This procedure prevents the development of instabilities, and seems to preserve the kinks fairly well.

The majority of the computer time is devoted to the detection of string crossings. To determine if two string segments crossed during the time step, we check the volume of the tetrahedron spanned by the four points on the two segments. If it changed sign during the step, the configuration is checked at the time the volume is zero, to see if a crossing did really occur (the positions of the points are extrapolated linearly between time steps). This procedure is *exact.*

Finally, when two segments have been determined to cross, we interchange partners, and average the positions and the velocities in the crossing region to minimize the amount of diffusion required in the subsequent evolution. At this stage, we also update a "genealogical tree", which records entire loop fragmentation and reconnection history. This enables us to get a posteriori a detailed picture of the string system evolution.

RESULTS

We have performed several runs in boxes of size $28\xi_0$, and one run on a $36\xi_0$ box (ξ_0 is the correlation length of the strings in the initial configuration). The simulation on the $36\xi_0$ box had $\sim$ 350,000 points and 1000 strings initially. After 870 steps (21 cpu hours), the universe had expanded by a factor of 2.9 and 14000 new loops were produced. In an effort to "bracket" the scaling solution, we evolved several configurations with different initial horizon sizes, and thus different initial energy densities in long strings $\rho_L(t_0)$, to see if ρ_L scales as radiation as required for a scaling solution (long strings are defined to be of proper length $> ct$). Fig. 2 shows the behavior of $\rho_L t^2/\mu$ as a function of time for several different runs. It is apparent that the different runs seem to be converging toward similar (constant) values with $\rho_L t^2/\mu \simeq 25$ or so. Thus, our preliminary conclusion is that we are seeing evidence for a scaling solution. It should be stressed, however, that our value for $\rho_L t^2/\mu$ is an *order of magnitude* larger than the value quoted by Albrecht and Turok, so our results cannot be considered to be consistent with theirs.

In fact, our results differ substancially from many of the generally accepted ideas about string evolution. The standard scenario for loop production holds that horizon sized "parent" loops break off the infinite string network and fragment into roughly 10 "child" loops of roughly equal sizes, but we find that in addition to the horizon sized parent loops, the infinite strings lose significant amounts of energy directly into small loops. The reason for this is presumably that the strings have a lot of short wavelength structure in the form of the kinks that are formed whenever strings cross. In addition, the large parent loops fragment much more efficiently than was previously thought, so that most (but not all) of the loops that are created are close enough to our lower cutoff on loop size (usually 10 points per loop) so that their chances of fragmenting further are significantly (or completely) suppressed.

The fact that these very small loops play such an important role presents some difficult numerical problems. The most obvious problem is we cannot be sure of the correct loop distribution function because we don't know which of these small loops would fragment further if we had better resolution. Another difficulty is that our result for $\rho_L t^2/\mu$ has some dependence on our small loop cutoff. This is mainly because loop reconnection to the infinite strings is much less efficient for very small loops than for larger ones. Thus, reducing the lower cutoff increases the efficiency of loop production and decreases the scaling solution value of ρ_L/t^2. Thus, our determination of ρ_L/t^2 suffers from a systematic error due to our small loop cutoff. Only by fitting the free parameters of the analytic model of Refs. 2 and 3 to these numerical results will we be able to resolve these questions.

CONCLUSION

Our results, though not definitive, lend support to the hypothesis of *existence* of a scaling solution although the string density at the scaling solution is an order of magnitude larger than that reported by Albrecht and Turok. Nevertheless more work is necessary in order to disentangle the precise characteristics of the scaling regime from numerical effects such as the cutoff on small loops.

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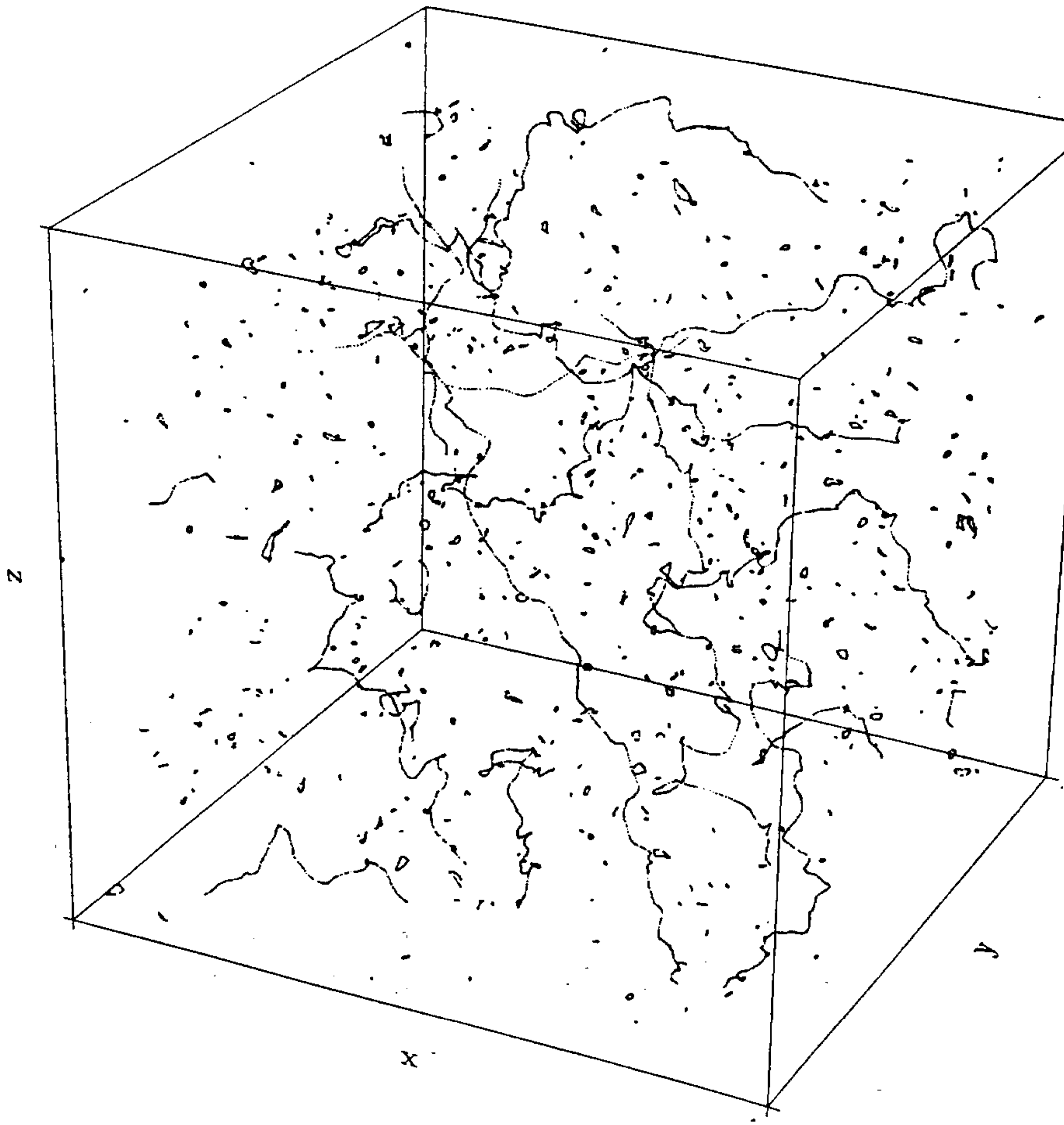

Figure 1
1/10th of a $28\xi_0$ configuration after the universe has expanded by a factor of 2.25. The cube sides are equal to $ct/2$.

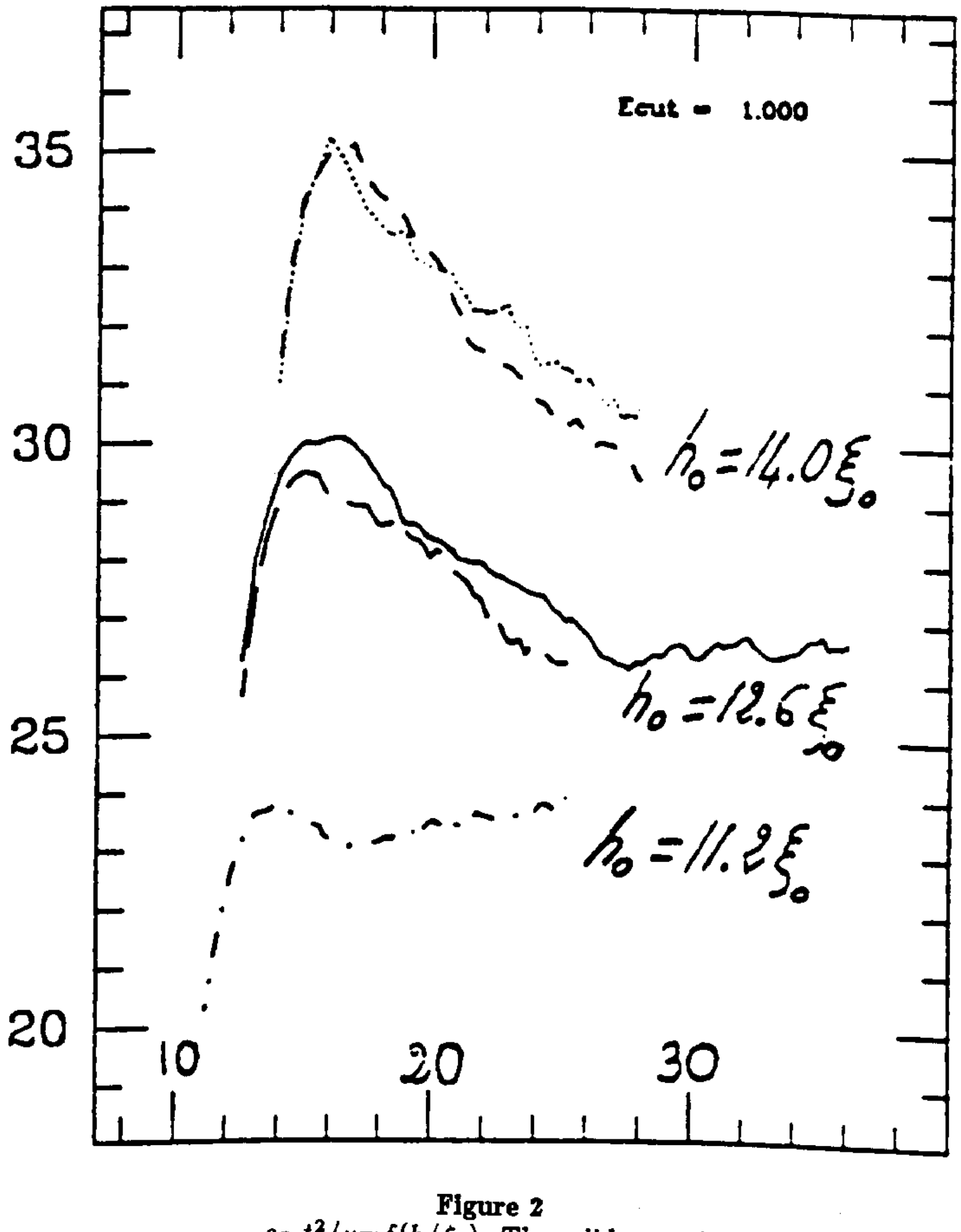

Figure 2
$\rho_L . t^2/\mu = f(h/\xi_0)$. The solid curve is for the run in a box of side $L = 36\xi_0$, the others correspond to $L = 28\xi_0$.