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SMALL COSMOLOGICAL CONSTANT IN STRING MODELS

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The problem of cosmological constant has been known as one of the biggest discrepancies between the world we observe and what quantum field theories of particles predict. It is, therefore, reasonable to ask if string theory can provide a hint to a resolution of this problem. In the calculations based on local field theories, even getting a finite answer requires some effort and one often invokes supersymmetry. The string provides a *bona-fide* cutoff to ultraviolet divergences and a manifestly finite answer at least to one-loop calculations. In this talk, we would like to review a simple model studies in ref [1] with a small cosmological constant [2]. The model itself is by now pretty much standard or elementary due to the quick progress in this field. (A partial list includes [3]~[14]. For a more extensive reference, see a recent review by J. H. Schwarz [15]). We would like to present it as an illustrative example that tachyon-free nonsupersymmetric string models can have an exponentially small cosmological constant when the number of massless fermions and bosons are equal.

The first point to observe is the following. In the simplest model with internal compactified coordinates, namely 26 dimensional bosonic string compactified on a torus, the one-loop cosmological constant is dual with respect to the radius r of the compactified coordinate[16].

$$V(r) = V(\alpha'/r) \tag{1}$$

Here, I ignored the divergence due to the presence of a tachyon. Eq.(1) immediately tells that the Frenkel-Kac point $r = \sqrt{\alpha'}$, where one sees symmetries enhanced, is also energetically favored, leading to the cosmological constant which is $0((2\pi\alpha')^{-25/2})$. A way to construct models with a small cosmological constant is to first eliminate this duality through "twisting".

A simplest model one would imagine having such property is obtained from the $0(16) \times 0(16)$ model [6,7] with extra twist in the compactified coordinate. Let's first study the nine dimensional case. The requirement of modular invariance forces the compactified momenta to lie on an even self dual Lorentzian lattice σ^1 in $\mathcal{R}^{1,1}$[4]. At the Frenkel-Kac point,

$$p = (p_R, p_L) = \left(\frac{m-l}{\sqrt{2}}, \frac{m+l}{\sqrt{2}}\right) \quad , \quad m, l \epsilon Z \quad , \alpha' = 1/2 \tag{2}$$

The momenta at an arbitrary radius is obtained by boosting the lattice with rapidity y = log a with $a = \frac{\sqrt{\alpha'}}{r}$.

Remember that the $0(16) \times 0(16)$ model itself is obtained by the Z_2 twist [6]

$$R = (-1)^{F_{s.t.}} exp(2\pi i j_{12}) exp(2\pi i j'_{12}) \tag{3}$$

Here, $F_{s.t.}$ is the spacetime fermion number and j_{12} and j'_{12} are generators of $0(16) \times 0(16')$. In our example, the twist is simply

$$R' = Re^{2\pi i p.\delta} \tag{4}$$

Here, δ is a shift vector in the compactified coordinate and half the lattice vector in σ^1. It turns out that there are only two inequivalent shift vectors:

$$\delta^I = (\delta^I_R, \delta^I_L) = (\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}) \ , \delta^{II} = (\delta^{II}_R \ , \delta^{II}_L) = (-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}) \ . \tag{5}$$

We call these twist I and twist II. The physical effect these shift vectors bring is rather obvious: twist I (II) shifts the spacetime fermion number by winding number l(momentum m),

$$(-1)^{F_{s.t.}} \to (-)^{F_{s.t.}+l}, (-1)^{F_{s.t.}+m} \ . \tag{6}$$

So far, we are at $r = \sqrt{\alpha'}$, and twist I and twist II are identical due to the symmetry between momenta and winding numbers unique to this point. This is no longer true as soon as we go away from the point. The shift vectors δ^I and δ^{II} are rescaled by a and a^{-1} respectively. For this reason, there is no relation like eq. (1) in twist I or twist II. Rather, they are dual to each other; in the small radius limit, twist I behaves exactly like the large radius limit of twist II and vice versa . From now on, we concentrate on twist I and analyse the model further. The partition function $\mathcal{P}$ turns out to be expressed as

$$\mathcal{P} = (\epsilon_0 - o_0)\mathcal{P}_{un} + \epsilon_{1/2}\mathcal{T} + o_{1/2}\mathcal{S}. \tag{7}$$

Here, $\mathcal{P}_{un(\mathcal{T})}$ means the untwisted (twisted) part of the partition function of the $0(16)\times 0(16)$ model. The S is obtained by interchanging the right moving spacetime bosons and fermions in $\mathcal{T}$. The $\epsilon_{o,1/2}, o_{o,1/2}$ are standard factors seen, for instance in ref.[17]; $\epsilon(o)$ refers to even (odd) winding numbers and the subscript $0(1/2)$ to integral (half-integral) momenta. The numerical plot is seen in the figure.

In the large radius limit, only the $l = 0$ states survive and the twist introduced becomes irrelevant. The model approaches the original $0(16) \times 0(16)$ model: $a\sqrt{\tau_2}\mathcal{P} \to \mathcal{P}_{un} + \mathcal{T}$. In the small radius limit, the opposite thing happens: only those states carrying $m = 0$ survive, but all states carrying non-zero l become nearly degenerate with the $l = 0$ massless states. It is a phenomenon unique to the string theory. The low lying spectrum is schematically

$$\begin{array}{cccc}
\vdots & B & & F \\
l = +1 & (8_s,(128,1)),(8'_s,(1,128)) & \nearrow\!\!\!\!\!\!\searrow & (8_V,(1,120)+(120,1)),(8_V,8_V) \\
l = 0 & (8_V,(1,120)+(120,1)),(8_V 8_V) & \swarrow\!\!\!\!\!\!\nwarrow & (8_s,(128,1)),(8'_s,(1,128)) \\
\vdots & & &
\end{array} \tag{8}$$

We interpret that the states having winding number l and $l+1$ form broken $E_8 \times E_8$ heterotic super-multiplet with mass splitting $M_s^2 = 1/\alpha' a^2$. We now see

$$\frac{2\sqrt{\tau_2}}{a}\mathcal{P} \overset{a\to\infty}{\to} 4\sum_{n=0}^{\infty} e^{-(2n-1)^2\pi a^2/4\tau_2}\mathcal{P}_{un} \tag{9}$$

Luckily, the asymptotic behavior of the one-loop cosmological constant can be evaluated analytically with exponential accuracy;

$$\Lambda_{10} \overset{a\to\infty}{\to} (n_f - n_B)\frac{24}{(2\pi^2)^5}\frac{\varsigma(10,1/2)}{(2\pi\alpha')^5}\frac{1}{a^8} + 0(e^{-a}), n_F - n_B = 64 \tag{10}$$

The salient feature of the formula which is not completely obvious is that the leading power suppression factor is entirely due to the massless degrees of freedom of the untwisted sector in 10 dimensions. (We, of course, sum over all the winding number excitations in 9 dimensions). Therefore, once the number of bosonic and fermionic massless degrees of freedom is matched, we are left with a model with an exponentially small cosmological constant. In fact, if is not difficult to construct such a model in four dimensions by tuning radii of compact dimensions. Let r_9, r_8, r_7, and r_6 be equal to $\sqrt{\alpha'}$ and r_5 be away from $\sqrt{\alpha'}$. We then get eight additional massless 8_V bosons corresponding to the nonzero roots of $(SU(2))^4$, cancelling the first term in eq. (9). The resultant cosmological constant is

$$\Lambda^4_{cosm} = 0(exp(-1/M_s\sqrt{\alpha'})/\alpha'^2) \tag{11}$$

This appears to be the first example of an exponentially small one-loop cosmological constant. The scale of supersymmetry breaking need not be too small compared with the string tension: $M_s = 10^{15}$ GeV/c^2 makes Λ to be well below the observed bound. It remains to be seen to which extent this mechanism holds in chiral four dimensional string models.

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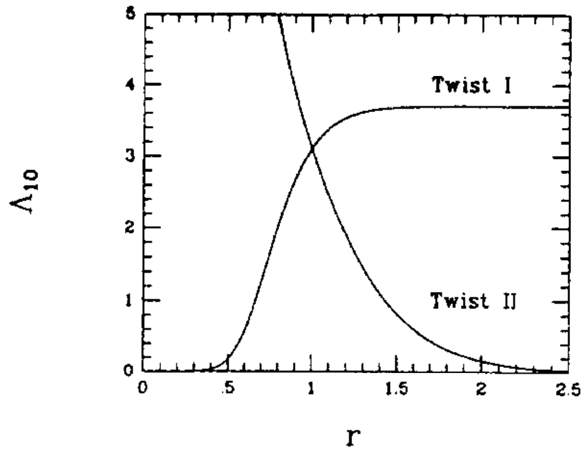

Fig. 1. Cosmological constant Λ_{10} [in units of $0.01(2\pi\alpha')^{-5}$] for the twist I and twist II models, plotted as a function of the radius r of the compact dimension [in units of $\sqrt{\alpha'}$].