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PARTON DISTRIBUTION FUNCTIONS AND THE QCD-IMPROVED PARTON MODEL -- AN OVERVIEW[†]

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Abstract

Some non-trivial features of the QCD-improved parton model relevant to applications on heavy particle production and semi-hard (small-x) processes of interest to collider physics are reviewed. The underlying ideas are illustrated by a simple example. Limitations of the naive parton formula as well as first order corrections and subtractions to it are discussed in a quantitative way. The behavior of parton distribution functions at small x and for heavy quarks are discussed. Recent work on possible impact of unconventional small-x behavior of the parton distributions on small-x physics at SSC and Tevatron are summarized. The Drell-Yan process is found to be particularly sensitive to the small x dependence of parton distributions. Measurements of this process at the Tevatron can provide powerful constraints on the expected rates of semi-hard processes at the SSC.

A. INTRODUCTION

Recent studies of SSC physics involve mostly the production of heavy new particles or conventional processes which extend the kinematic region of very small x (the Bjorken variable) far beyond the currently available experimental range.[2] In both cases the straightforward application of the naive parton model with common parameterizations of parton distribution functions becomes questionable. This talk reviews the relevant issues for studying these processes in the context of the QCD-improved parton model and summarizes recent progress made up to the conclusion of this workshop. The presentation is aimed at those interested in parton model applications who are not necessarily experts of QCD theory.

† Review Talk of the QCD Work Group at From Colliders to Super Colliders, Univ. of Wisconsin, Madison, May, 1987. This work was supported in part by the National Science Foundation, Grant No. PHY-85-07635.

[2] Proceedings of the 1984 Summer Study on the Design and Utilization of the SSC (1984), Ed. R. Donaldson and J. Morfin; and ibid (1986), Ed. R. Donaldson and J. Max.

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Most high-energy processes with reasonable cross-section at colliders in the multi-TeV range involve partons with small x. Parton distributions in hadrons are known to be strongly peaked towards small x at high energies; but the precise x-dependence is unknown. A specific issue that the QCD group deemed important to ask was: to what extent can current and planned experiments at the Tevatron, S$\bar{p}$pS, and HERA tell us about the small x behavior of parton distributions relevant to SSC physics? Although there was not enough time to complete a full survey, a modest study of cross-sections and various final state particle distributions for some prominent small-x processes (Drell-Yan processes, as well as b-quark and "mini-jet" productions) at Tevatron I and SSC based on different assumptions on the small-x behavior of parton distributions was started. Preliminary results are summarized here and given in detail in a separate contribution to these proceedings.[3]

B. THE QCD-IMPROVED PARTON MODEL

In order to apply the parton model correctly to high energy processes involving heavy particles and/or for very small values of x, it is necessary to understand its theoretical basis in Quantum Chromodynamics beyond the "naive parton picture." The validity of the parton model is founded on factorization theorems for high-energy cross-sections which have been established in perturbative QCD to varying degrees of rigor for different types of processes. We shall assume that factorization holds for the processes under consideration, and refer the audience to a recent review[4] on this subject for details and for further references.

1. "Hard Cross-Sections" and "Parton Distribution Functions" in the Factorization Formula

To illustrate the basic ideas involved, we consider a simple example: inelastic scattering of neutrino on a hadron target A to produce a heavy quark U with mass M. The factorization theorem in this case states that, in the Bjorken limit, the physical cross-section[5] σ can be written in the following factorized form, (cf. Fig.1),

$$\sigma_A^i(x,Q,M,m_A) = \hat{\sigma}_a^i(x,Q,M,\mu) \otimes f_A^a(x,m_{a,A},\mu) + O(\frac{m_{a,A}}{Q}) \qquad (1)$$

[3] F. Olness and Wu-Ki Tung, "Small x Physics at the Tevatron and SSC", these proceedings.

[4] J.C. Collins and D. Soper, "The Theorems of Perturbative QCD" (submitted to Ann. Rev. Nucl. Part. Sci.)

[5] For this example, it is more appropriate to refer to the "structure functions" for the effective W-boson - hadron scattering process. We use "cross-section" for its general applicability to other hard processes.

where the variables (x,Q) are standard, $\hat{\sigma}(x,Q,M,\mu)$ is the "hard part" of the neutrino-parton cross-section (or structure function[5]), and $f(x,m,\mu)$ is the "parton distribution function" which pertains to soft physics at the hadron mass scale. The symbol $\otimes$ represents a convolution integral in the variable x defined as

$$f(x) \otimes g(x) = \iint d\xi\, d\eta\, f(\xi)\, g(\eta)\, \delta(x-\xi\eta) \\ = \int d\xi\, f(\xi)\, g(x/\xi) = \int d\eta\, f(x/\eta)\, g(\eta) \quad . \tag{2}$$

Several labels and variables, often left implicit in the literature, are explicitly displayed in Eq.(1) in order to make clear the full content of the factorization,

i	:	label depending on the "current" probe
A	:	hadron (target) label
a	:	parton label (to be summed over)
$m_{a,A}$	:	masses of parton a and hadron A
μ	:	renormalization scale parameter .

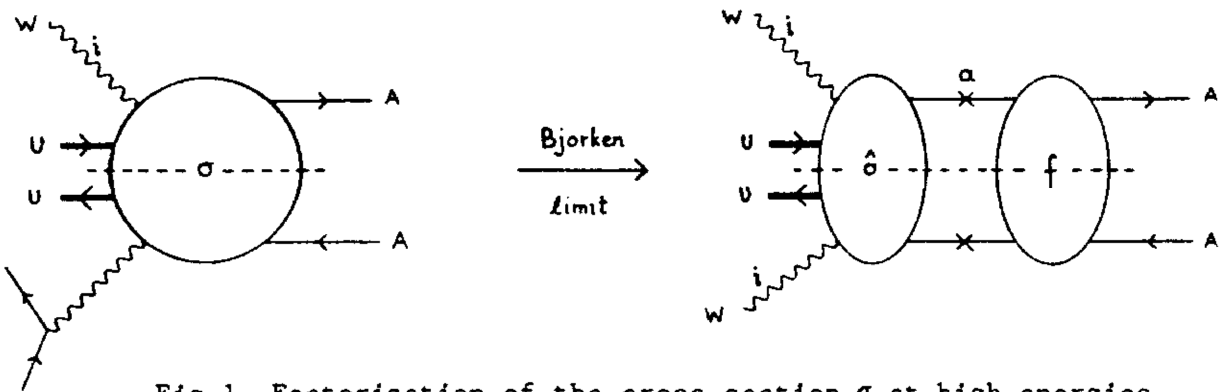

Fig.1 Factorization of the cross-section σ at high energies, Eq.(1). The parton lines marked with an x are on the mass-shell and collinear to the hadron parent. The dashed lines indicate a summation over all intermediate states.

The physical interpretation of Eq.(1) and Fig.1 is well-known. We shall only call attention to some details relevant for non-trivial applications. First, it is important to note that the two factors on the right-hand side of the equation are both theoretical constructs, hence are dependent on the renormalization scheme (used to make sense of perturbative field theory) and on a renormalization scale parameter (μ) which is arbitrary in principle. The physical cross-section σ on the left-hand side must, of course, be independent of these theoretical artifices. Hence, it is essential that the two factors on the right-hand side be calculated consistently using the same renormalization scheme and with the same renormalization scale so that the required matching takes place. This point is not always taken into account in common applications of the parton model. We shall come back to this subject in Sect. B.3.

The hard scattering cross-sections (sometimes called Wilson coefficients) $\hat{\sigma}^i_a$ have the following properties:

i) they depend on the particular process (label i) and the parton label (a) but are independent of the physical initial-state hadron (A);

ii) they are "hard" cross-sections because they are free of (soft) mass-singularities (cf. Sect. B.3); this makes them perturbatively calculable in asymptotically-free theories -- provided the renormalization scale μ is chosen to be of order Q (or M) which is large;

iii) they are not, however, just sums of lowest order Feynman diagrams for the relevant partonic cross-sections (Fig. 2): since the latter do have mass-singularities (except in the most trivial cases), these must be subtracted before the two can be properly related.

The last point also bears on the problem of "double counting" in applying the parton model formalism. We shall discuss these points in Sect. B.2.

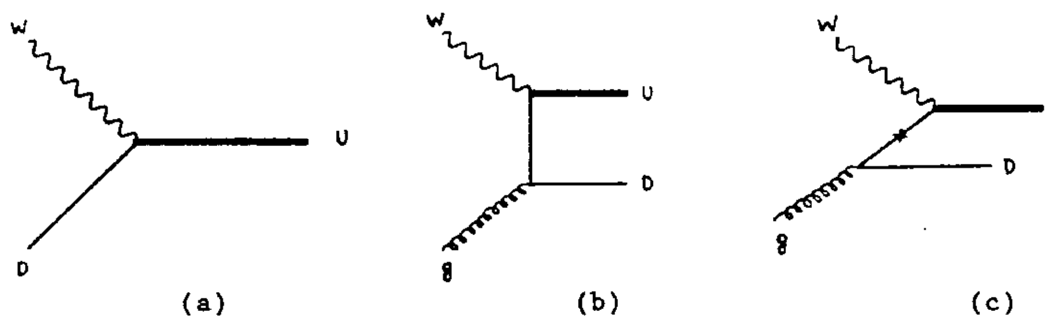

Fig.2 Feynman diagrams contributing to the production of heavy particle U at the parton level: (a) zeroth order; (b) first order; and (c) the subtraction term in Eq.(6). The D-line marked by an ⁎ is on mass-shell and collinear to the gluon.

The Parton Distribution Functions (abbreviated sometimes as PDF's in the following) f^a_A have the following properties:

i) they are process-independent (i.e. universal) but renormalization scheme-dependent;

ii) they are "soft" in that they depend on detailed hadron dynamics at the energy scale $m_{a,A}$ (hence cannot be calculated ab initio in conventional perturbation theory no matter what value is chosen for the renormalization scale parameter μ);

iii) they are, in principle, independent of the hard variables Q and M; however, the choice of μ = Q brings in Q-dependence indirectly (cf. point (ii) on hard cross-sections above);

iv) their dependence on the variable μ is governed by a renormalization group equation (the Altarelli-Parisi equation); the renormalization group coefficients (splitting functions or evolution kernel) can be calculated order by order in perturbation theory.

We note that parton distribution functions are not really "structure functions" as they are often referred to in common parlance. In contrast to points (i)-(ii) above for PDF's, the structure functions are physical deep inelastic scattering cross-sections which are process-dependent, renormalization scheme-independent, and do not a priori pertain to partons. **The two are simply related** (as linear combinations of each other) **only in lowest order perturbation theory, or in certain renormalization schemes.** (Cf. the DIS scheme, Sect. B.3.)

The above formulation of factorization can be generalized to hadron-hadron processes: it then involves a double convolution (one for each initial state hadron). The choice of the scale parameter μ is in general not unique, especially when there are more than one large energy variable of comparable magnitude. In principle, the difference resulting from two distinct choices is one order higher than that included in the calculation, hence is relevant only to the next order. In practice, the size of the difference may vary depending on the kinematic region involved. Theoretical issues associated with processes involving more than one large variable of distinct orders of magnitude are complex; we refer the audience to the literature for details.[4]

2. Hard Cross-Sections, Partonic Cross-Sections, and Subtractions

The lowest order contribution to the hard cross-section in our sample process comes from the partonic process W-boson - quark scattering as depicted in Fig.(2a). In this case the partonic cross-section is just a constant; it is clearly regular, and hence directly identifiable with the corresponding "hard cross-section";

$$\hat{\sigma}^i_D = {}^0\sigma^i_D \tag{3}$$

where D is the initial state quark flavor label and the left-superscript 0 on the right-hand side of the equation indicates that this partonic cross-section is of order zero in the effective strong coupling parameter α_s. For simplicity we shall assume that only one quark flavor, D, couples to U via the W-boson. The mass of the D-quark shall be denoted simply as m.

Substituting Eq.(3) into Eq.(1), of course, yields the familiar minimal parton model formula.

The first order contribution to the process under consideration arises from W-boson - gluon scattering as depicted in the Feynman diagram of Fig.(2b). It is easy to see that the partonic cross-section due to this diagram depends on the mass of the D-quark (m) as well as that of the heavy quark (M). In particular it diverges in the limit as the mass m becomes negligible compared to the dominant energy scale Q or M. The singularity arises from the vanishing of the denominator of the D-quark propagator in the collinear configuration when the 4-momentum associated with the propagator goes on-the-mass-shell. This "mass-singularity" must be subtracted from the partonic cross-section in order to obtain a sensible "hard cross-section" appropriate for use in the basic parton-model formula, Eq.(1).

The easiest way to derive the proper procedure of subtraction is by observing that the W - g partonic cross-section σ_g^i itself satisfies the same factorization theorem as the W - hadron cross-section, Eq.(1),

$$\sigma_g^i(x,\ Q,\ M,\ m) \ = \ \hat{\sigma}_D^i(x,\ Q,\ M,\ \mu) \ \otimes \ f_g^D\ (x,\ m,\ \mu)\ . \tag{4}$$

Factorization guarantees that the hard cross-section $\hat{\sigma}$ which appears here is exactly the same as in Eq.(1). Eq.(4) holds order-by-order in perturbation theory. It serves as the basis for evaluating $\hat{\sigma}$ as follows. For W - g scattering, the perturbation series begins at order α_s. To this order, the right-hand side of Eq.(4) is of the form

$$ {}^1\hat{\sigma} \ \otimes \ {}^0f \ + \ {}^0\hat{\sigma} \ \otimes \ {}^1f \tag{5}$$

where, as in Eq.(3), the left superscripts on a quantity refer to the order in α_s of that quantity and all other indices have been suppressed. Taking into account Eq.(3) for ${}^0\hat{\sigma}$ and the fact that ${}^0f_b^a(x) = \delta^a{}_b\ \delta(x-1)$, we obtain,

$$\hat{\sigma}_g^i(x,Q,M,\mu) = {}^1\sigma_g^i(x,Q,M,m,\mu) \ - \ {}^0\sigma_D^i \otimes {}^1f_g^D(x,m,\mu) \tag{6}$$

where all quantities on the right-hand side are calculable in perturbation theory. Note that both terms on the right-hand side depend on the quark mass m, and are singular in the limit as m → 0; but these two singularities cancel to yield a singularity-free hard cross-section.

The truncated first-order formula on the right-hand side of Eq.(6) still has some residual mass dependence after the cancellation of singularities. When m << M, this expression is almost mass-independent and is well approximated by its m → 0 limit. The latter is derivable by an alternative method using dimensional regularization and minimum subtraction on ${}^{1}\sigma^{i}_{g}$ in the zero-mass theory.[6] If the mass m is not very small compared to the large energy scale M there is no compelling reason for using the zero-mass formula, the full mass-dependent expression in Eq.(6) should give a more reliable answer.[7]

Combining Eqs.(1), (3), and (6) we arrive at the QCD-improved parton model master formula:

$$\sigma^{i}_{A} = {}^{0}\sigma^{i}_{D} \otimes f^{D}_{A} - {}^{0}\sigma^{i}_{D} \otimes {}^{1}f^{D}_{g} \otimes f^{g}_{A} + {}^{1}\sigma^{i}_{g} \otimes f^{g}_{A} \qquad (7)$$

The subtraction term on the right-hand side of Eq.(6) and Eq.(7) has a simple physical interpretation: it represents the contribution to the cross-section due to W-scattering off an D-quark which originates from the splitting of a gluon in the hadron in the collinear configuration (as illustrated in Fig.(2c)). In addition to being part of the first order cross-section, Eq.(5), this is a contribution also counted in the zeroth-order term through the evolution of the PDF of the D-quark. Thus, **the subtraction of the mass-singularity is intimately related to the removal of double counting** in the parton picture. Eq.(7) is written in a way especially designed to exhibit this inter-relationship: on the one hand, factoring out $f^{g}{}_{A}$ in the last two terms makes it obvious that we are subtracting off a part of the first order cross-section as given in Eq.(6); on the other hand, factoring out ${}^{0}\sigma^{i}{}_{D}$ in the first two terms reveals that the subtraction can equally well be regarded as a correction to the D-quark distribution function in the zeroth order term.[8] Figs.(2a-c) illustrate the same point in graphic form.

3. Renormalization Scheme Dependence

To calculate the first-order quantities on the right-hand side of Eq.(6) in perturbation theory, one must adopt a specific renormalization scheme in order to regulate the inevitable divergences. The resulting $\hat{\sigma}$ is clearly renormalization scheme dependent in general. For a consistent application of the QCD-improved parton model, the parton distribution functions which enter the master equation, Eq.(1), have to be defined and generated in the same renormalization scheme. In practice, perturbative QCD

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[6]This is, in fact, the most often used method to calculate hard cross-section formulas. Without the mass m, the calculation of $\hat{\sigma}$ simplifies. It also does not require an independent calculation of the ${}^{1}f$ term in Eq.(6).

[7]F. Olness and Wu-Ki Tung, to be published.

[8]The reader can easily verify from the definition of the convolution operation, Eq.(2), that the triple-convolution in the subtraction term is associative, hence the two ways of grouping terms are equivalent.

calculations of the hard cross-sections are most conveniently (and hence most frequently) carried out in the MS-bar scheme. On the other hand, the commonly used PDF's are defined by an ad hoc scheme in which they are related as simply as possible to the fully inclusive deep inelastic scattering structure functions. We shall call this the DIS scheme. In order to achieve consistency it is necessary either to convert the hard cross-section to the ad hoc DIS scheme,[9] or to employ parton distribution functions defined in the MS-bar scheme. In either case, the conversion entails a convolution of the relevant quantities with a set of hard cross-section coefficients for deep inelastic scattering in the MS-bar scheme.

It appears logical and natural that the preferred choice is to use PDF's defined in the MS-bar scheme, since it is convenient in practice to use directly the MS-bar hard cross-sections for all processes.[10] Deep inelastic scattering has no a priori significance over the other hard scattering processes. The gluon distribution, which makes the largest contribution to many high-energy processes, has no natural definition in the DIS scheme. In addition, experience with hard-scattering processes that have been investigated in detail seems to indicate that higher order corrections to hard cross-section coefficients may be smaller in the MS-bar scheme than in the DIS scheme.[11] Nonetheless, the commonly used PDF sets[12,13] are all effectively defined in the DIS scheme.

When heavy particles are produced and when the mass(es) of some parton(s) in the theory are themselves non-negligible, the MS-bar scheme is not a convenient renormalization scheme for practical calculations. For these high energy processes the choice of renormalization scheme becomes an important issue, both from the theoretical and from the practical point of view. Several theoretically equivalent choices are possible, they may differ considerably in the ease of application in practice.[14,15,16] We shall use our elementary sample process to explain the issues underlying this problem in the next section.

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[9] G. Altarelli, R.K. Ellis, G. Martinelli, Nucl. Phys. <u>B157</u>, 461(1979).

[10] W. Furmanski, and R. Petronzio, Z. Phys. <u>C11</u>, 293(1982).

[11] As an example, see Ref.9 for lepton-pair production (Drell-Yan process) coefficients in the two schemes.

[12] D. Duke and J. Owens, Phys. Rev. <u>D30</u>, 49(1984).

[13] E. Eichten et. al., Rev. Mod. Phys. <u>56</u>, 579(1984), and Erratum <u>58</u>, 1065(1986)

[14] J.F. Gunion, H.E. Haber, F.E. Paige, Wu-Ki Tung, and S. Willenbrock, UCD preprint (UCD 86-15) (to be published).

[15] H.E. Haber, D.E. Soper, and R.M. Barnett in the Proceedings of "Simulations at High Energies", Madison, World Scientific Pub.(1986).

[16] F. Olness and Wu-Ki Tung, to be published.

C. HEAVY QUARKS AND THE PARTON MODEL

A heavy quark will not contribute to physical processes at an energy scale very much smaller than its mass (m). This intuitively obvious fact is formally proved in field theory in the form of the Decoupling Theorem.[17] The MS-bar renormalization scheme is mass-independent, it does not distinguish heavy quarks from light ones. Hence, **the decoupling of heavy quarks is not manifest in the MS-bar scheme.** It is evident then that this scheme is not suitable for parton model calculations of processes involving heavy quarks.

1. Renormalization Scheme with Manifest Decoupling of Heavy Quarks

Although in principle the decoupling of heavy quanta can be established in various renormalization schemes, it is obvious that adopting a scheme with manifest decoupling of heavy particles is imperative in practical calculations. One convenient scheme of this kind involves defining all Feynman diagrams with only light particle lines by the standard MS-bar subtraction, and all diagrams containing heavy particle line(s) by the BPHZ subtraction.[18] Among the many advantages of this scheme is the fact that, when applied to physical processes at a given energy scale, **it yields an effective theory equivalent to the conventional QCD in the MS-bar scheme with only the light quarks included.** (Here "light" means light compared to the given energy scale.) Consequently, all the well-known MS-bar results for renormalization group coefficients and hard cross-sections in the literature can be readily used.

Calculation of parton distribution functions in this renormalization scheme using arbitrary input distributions at a fixed value of Q is fairly straightforward. One solves the Altarelli-Parisi evolution equation numerically in a given Q-range between quark mass thresholds using running coupling function $\alpha_s(\mu)$ and renormalization group coefficients appropriate for the effective theory of that energy range, and adopts matching conditions at the thresholds (boundaries of the ranges) to ensure continuity of $\alpha_s(\mu)$ and the distribution functions. This has been conveniently implemented.[19] Since the heavy quarks (say, b and t) are effectively decoupled from physics at the $Q^2 = 5 - 50\ \mathrm{GeV}^2$ range where most current data used for PDF

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[17] T. Appelquist and J. Carazzone, Phys. Rev. D11, 2856(1975). For a lucid treatment, see: J.C. Collins, Renormalization, Cambridge Univ. Press, 1984.

[18] J.C. Collins, F. Wilczek, and A. Zee, Phys. Rev. D18, 242(1978).

[19] J.C. Collins and Wu-Ki Tung, Nucl. Phys. B278, 934(1986); and Wu-Ki Tung, in the Proceedings of "Simulations at High Energies", Madison, World Scientific Pub.(1986).

fits lie, it is reasonable to use the common PDF's at low-Q without heavy quarks as input to the evolution and generate heavy quark distributions at high values of Q relevant for high-energy collider studies. Results of such calculations are available in current literature.[13,19]

2. Heavy Quark Distributions and Heavy Quark Initiated Processes

It is not hard to get some qualitative idea about the behavior of the distribution function $f^a{}_A(x, \mu)$ of a heavy quark in a hadron. Below the threshold (which is of the order of its mass m) the quark is decoupled, hence f = 0. Just above the threshold, the quark is predominantly generated by the splitting of the gluons inside the hadron. For a range of μ not too large compared to m, we can obtain a fairly good approximate solution to the Altarelli-Parisi equation for $f(x,\mu)$ by keeping only the gluon term in the equation and by ignoring the mild μ-dependence of the evolution kernel and the gluon distribution.[20] The solution is

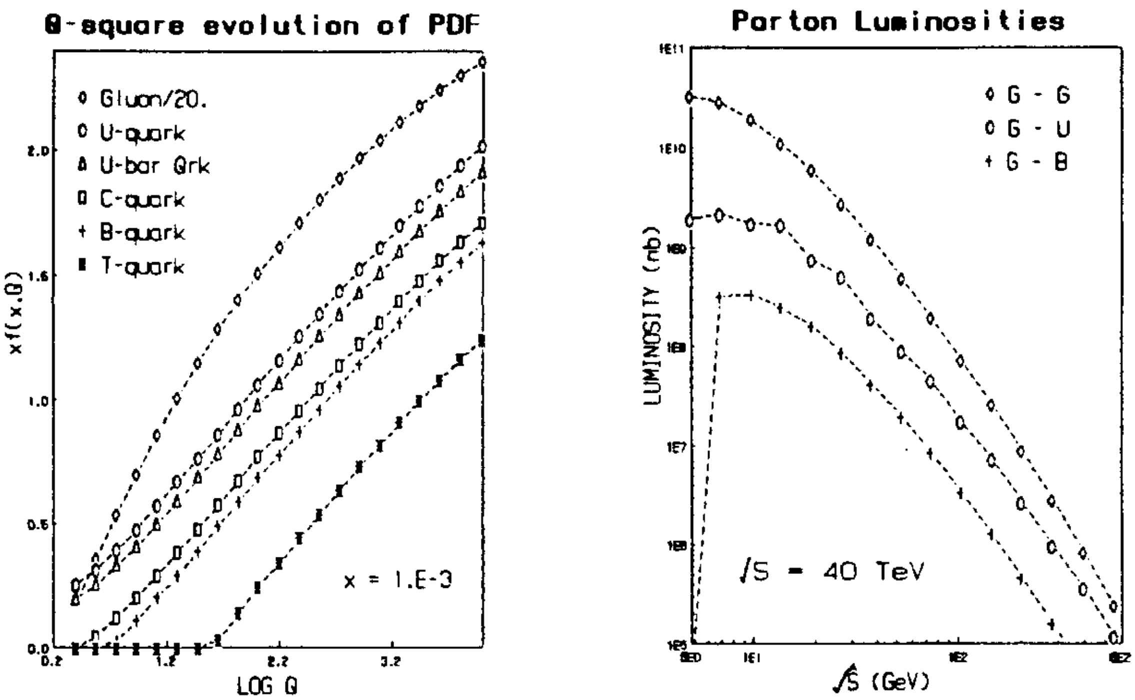
$$f^a_A(x, \mu) \simeq \ln \left(\frac{\mu}{m}\right)^2 \cdot \frac{\alpha_s}{2\pi} \cdot P^a{}_g \otimes f^g_A(x, \mu) \tag{8}$$

where m is the mass of the quark in question and $P^a{}_g$ is the relevant Altarelli-Parisi splitting kernel. We see that as long as $\log(\mu/m)$ is not large f^a is of order α_s compared to the gluon distribution f^g and that it becomes negligible as $\mu \to m$. However, When $\mu \gg m$ (i.e., when $\alpha_s \cdot \log(\mu/m)$ is not much smaller than 1) f^a will not remain of order α_s and the right-hand side of Eq.(8) ceases to be a good approximation to $f^a{}_A$; it becomes necessary to solve the complete evolution equation. $f^a{}_A$ grows eventually to the same order of magnitude as the light quark and gluon distribution functions.

For practical applications, it is useful to have a concrete knowledge of how the above expectations translate into real numbers. In Fig.(3a) we show the Q-evolution of six parton distribution functions at a fixed value of x (= 10^{-3}). The gluon curve has been scaled down by a factor of 20. In order to make some quantitative comparison of the integrated distributions, we show in Fig.(3b) three parton luminosity curves, G-G, G-U, and G-B, which are rather directly related to physical total cross-sections. The ratios between these curves give a good overall measure of the relative sizes of the gluon, the u-, and the b-quark distributions. From both graphs, we see that the distribution functions of the heavier quarks seem to "catch up" slowly as the energy scale increases. However, even at the very high end of the "SSC range" they still have a long way to go before

[20] It is easy to see that the neglected terms are of higher order in the coupling.

becoming comparable in magnitude to those of the light quarks, not to say the gluon. With these facts in mind, let us consider the sample process discussed in previous sections.



(a) (b)

Fig.(3)

i) Case of small m and the validity of the naive parton formula. If the mass of the quark-parton D (m) is very much smaller than the "large energy scale" represented by Q or M, then $f^D{}_A$ can in principle be of the same order as $f^g{}_A$. An inspection of the master equation, Eq.(7), tells us that the first term on the right-hand side is the dominant one -- the others are down by a factor of α_s. (Recall that a potential mass-singularity in each of these terms which might spoil this argument actually cancel between the two terms.) By ignoring the latter terms, we recover the naive parton model formula as expected.

How good is this approximation in practice? The answer depends largely on the size of the factor $\alpha_s f^g{}_A / f^D{}_A$ (assuming that the ratio of hard cross-sections, stripped of α_s, are of order one). A closer look at Fig.(3) will reveal that this number is fairly large for all practical cases even at SSC energies. Thus we should expect corrections to the naive parton model to be important for all heavy quark initiated processes in this energy range.

ii) Case of m ~ M and the dominance of ${}^1\sigma$. Now consider the case where the mass of the D-quark is not too small compared to M or, in general, when

$\alpha_s f^g{}_A / f^D{}_A$ is not much less than 1. Then all three terms in Eq.(7) are comparable in magnitude and must be kept, at least in principle (see next section). **The complete expression is expected to give a reliable estimate of the physical cross-section,** because it is not hard to argue (by analyses similar to those presented above) that higher order terms will be indeed smaller in magnitude then the ones shown.

It turns out that, in the MS-bar subtraction scheme, the product of the first three factors on the right-hand side of Eq.(8) is nothing but the first order perturbative quark distribution function in a gluon ${}^1f^D{}_g$ (cf. Eq.(5) or (6)). Thus, if we substitute the approximate expression for $f^D{}_A$ from Eq.(8) into Eq.(7), the first two terms (corresponding to Figs.(2a) and (2c), respectively) will almost cancel each other -- provided log(m/M) is not large of course. We reach the conclusion that **the last term in Eq.(7) (Fig.(2b)) is close to giving the true answer** in this case. This result is not hard to understand -- when the D-quark is almost as heavy as U, it too will be almost decoupled. The decoupling is manifested by the fact that most of the "D-quark distribution" in the hadron is contained already in the first-order diagram of Fig.(2b) (as explicitly represented by Fig.(2c)), hence it is effectively canceled by the subtraction term as discussed at the end of Sect. B.2. Practically speaking, there is no ${}^0\sigma$ for the process under consideration, the last term on the right-hand side of Eq.(7) (Fig.(2b)) will effectively represent the leading contribution to the production of U in the parton model.

3. A Simplified Scheme for Practical Calculations

The master equation, Eq.(7), and its generalization to other processes as implemented in the renormalization scheme described in Sect. C.1 is comprehensive enough to cover all energy ranges. In practical parton model calculations, however, it is desirable to avoid as much unnecessary computation as possible, yet still maintain consistency in the framework of QCD. The near cancellation of two of the three terms under certain conditions suggests that parts of the complete calculation (usually involving non-trivial loops) can be avoided by a judicious choice of renormalization scheme. In particular, for the case $m < M(Q)$ but $\ln(M/m)$ is not too large, the discussion at the end of the previous subsection hints that we can formally exclude the canceling terms from entering the factorization formula in the first place by the following choice of renormalization scheme: D-quark lines are classified as "heavy" (thus subtracted according to the BPHZ prescription) irrespective of the size of the renormalization scale parameter μ (rather than only below the D-threshold as in Sect. C.1).

This is a perfectly well-defined and legitimate renormalization scheme. In this scheme there is no D-parton; diagrams with D-quark lines all have to be included explicitly in the Feynman diagram calculations (rather than be factorized into soft distribution functions). The only limitation of this scheme is that its perturbation series cannot be truncated to yield reliable answers when $\ln(M/m)$ (or $\ln(Q/m)$) becomes large. Within this limitation, it naturally leads to a simplified expression for the U-production cross-section involving only the last term in Eq.(7). We should point out that using this scheme is not entirely equivalent to just ignoring the other terms in Eq.(7) in the complete scheme of Sect. C.1. The parton distribution functions of the gluon and the other light quarks which enter the cross-section formula are different in the two renormalization schemes; they should be calculated separately.

4. The Production of Heavy Flavors

Of equal interest is the production of heavy quarks, leptons, and other new particles associated with super-symmetry and strings from initial state light partons. Many issues relating to factorization and "high order corrections" (which may be large, and in fact negative) are similar to those discussed above. We refer the audience to recent reviews on this subject for details and further references.[21,22]

D. PARTON DISTRIBUTIONS AT SMALL X

1. The Validity of the Independent Parton Picture at Small x

Although the parton model has proved to be extremely useful in describing a wide range of high energy processes, it cannot be expected to hold indefinitely for all ranges of variables. In particular, the steep increase of parton densities at small x must break down at some point when the "mean-free-area" of partons becomes smaller than the typical interaction cross-section between partons. Beyond that point, parton densities must saturate; parton recombination and shadowing effects must set in to compensate the familiar parton splitting. The important question to ask is for what values of x and Q does this take place. Much of the theoretical investigation of this issue conducted in Snowmass 84 and 86[2] has been inspired by the work on semi-hard processes of GLR.[23] Detailed study of

[21] J.C. Collins, "Production of Heavy Flavors" in Proceedings of the XXIII International Conference on High Energy Physics, Ed. S.Loken, World Scientific Pub., 1987

[22] R.K. Ellis, Proceedings of the 21st Rencontre de Moriond, Editions Frontieres, 1986.

[23] L. Gribov, E. Levin, and M. Ryskin, Phys. Rep. 100, 1 (1983).

recombination and shadowing effects[24] reinforced previous estimates based on qualitative dimensional arguments[25] that these non-linear corrections to the parton picture do not become numerically significant in the kinematic regions of interest to SSC studies.

2. The Singular Behavior of $f(x,Q)$ Near $x \sim 0$

Given that the independent parton picture is valid in the small x range relevant to SSC, the next important question is: what is the x-dependence of the parton distributions at small x? It is very important to find out since most processes of physical interest (and practically all those with significant cross-section) at SSC involve partons with "small x" by current standards. In the absence of concrete experimental data from deep inelastic scattering for x smaller than $x \sim 0.01$, it is conventional to assume a $1/x$ behavior for the gluon and sea-quark distributions at a fixed Q,[26] and let the QCD evolution determine the rest.

The canonical $1/x$ behavior is inferred from lowest order perturbative formulas for bremsstrahlung and related results on the first order Altarelli-Parisi kernel. Because both the PDF and the evolution kernel become singular as $x \to 0$, it is natural to ask whether the accumulative effect of higher order terms will qualitatively modify the canonical behavior.[27] Besides, even the ordinary QCD evolution has the effect of making the small x behavior of the PDF's more singular than $1/x$ as Q increases. The theoretical analysis needed to clarify this problem is very difficult technically, and there is yet no definitive answer. However, there is good reason to believe that a

$$f(x, Q_0) \sim 1/x^{\gamma} \tag{9}$$

behavior with

$$\gamma = 1 + A\,\alpha_s \tag{10}$$

can result from such an analysis.[28] Here the constant coefficient A is in principle calculable and it is positive. Numerical estimates indicate that γ can be as large as 1.5 for moderate Q_0. Collins has pointed out[29] that such a x-dependence will also help to stabilize the evolution of the PDF's,

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[24] A. Mueller and J. Qiu, Nucl. Phys. B268, 427(1986).

[25] See contributions to Proceedings of Snowmass 1984, Ref.2, by J.C. Collins and L. Durand.

[26] R. Feynman, 'Photon-Hadron Interactions,' Benjamin/Cummings, (1972).

[27] J. Kwiecinski, Z. Phys. C29, 561(1985); Phys. Lett. 184B, 386(1987).

[28] T. Jaroszewicz, Phys. Lett. 116B, 291 (1982).

[29] J.C. Collins, Contribution to the Proceedings of "Simulations at High Energies", Madison, World Scientific Pub., 1986.

so that parton distribution functions generated by the usual method will not become totally meaningless as Q decreases. Assuming a value of γ greater than 1 for some Q_0 leads to similar behavior of f(x, Q) at other values of Q with effective γ's which are far less sensitive to the choice of Q_0 then in the conventional case.

3. Experimental Information on f(x,Q) at Small x

The primary source of experimental input on parton distribution functions comes from structure functions measured in deep inelastic scattering. We obtain direct information on quark- and anti-quark distributions by using a variety of lepton probes and hadron targets. But the gluon distribution is derived only through indirect means, since there is no basic gluon - vector-boson coupling. The range in small x covered by this source is limited by the energy available in the relevant fixed-target experiments. Good data exist only for $x \gtrsim 0.05$. This range should be extended further (although not by very much) when new data from Tevatron II and HERA become available.

A second source of information on PDF's is Drell-Yan processes: e^+e^- and $\mu^+\mu^-$ pair creation (via virtual γ and Z) as well as real W- and Z-production. Only a limited number of combinations of PDF's can be measured in these experiments. For the nucleon, this is used mainly as a check on PDF's measured in deep inelastic scattering. For mesons, however, it represents the only source of information on their parton distributions. The accuracy in this method of determining PDF's and the range of its kinematical coverage in Q are limited in comparison to deep inelastic scattering by the low rates and by the huge background of hadron interactions in which the final state is embedded. At future hadron colliders, however, this type of processes may be the only one available to gain useful information on the parton distributions without too much theoretical uncertainty. (See next section.)

Recently, some information on the small x behavior of PDF's, especially the gluon distribution, have been extracted from jet-production in $S\bar{p}pS$.[30] Because the crude approximations used in making the comparison, and the ambiguities associated with jet physics, these preliminary results are not very restrictive and can only be regarded as useful first indications of the behavior of PDF's at high energies. Even so, the data already showed that the gluon distribution behaves in a way consistent with expectations at a small x range not explored before.

[30] For a review, see the report by F. Ceradini in the Proceedings of the XXIII International Conference on High Energy Physics, Ed. S.Loken, World Scientific Pub., 1987.

New information on parton distributions accumulated in the past 3 - 4 years from all these sources have not yet been systematically incorporated into the commonly used PDF's. In order to make the best possible assessment on expected rates and distributions for various processes at SSC, particularly the ones involving small x, it is highly desirable to have accurate PDF's which can accommodate a range of small-x behavior allowed by known data. It is also important to gain an overall view of the potential of experiments at Tevatron (I & II) and HERA to extend current knowledge on the small x behavior of the gluon and the quark distributions and to assess the possible impact on SSC detector planning and design. Much of this task remains to be done.

E. SMALL-X PHYSICS AT SSC AND TEVATRON

Some preliminary work on the estimated range of parton luminosities due to various assumptions on the small x behavior of PDF's have been reported in Snowmass 86 proceedings.[31] They showed that total cross-sections for physically interesting "small-x processes" can be larger by as much as a factor of 10 compared to previous estimates based on the assumed 1/x canonical behavior for the PDF's. The work performed at this workshop focuses on two extensions of this investigation. First, we extended the calculation to specific processes and to final state distributions as well as total rates. Secondly, we also obtained results for Tevatron I so that we can determine how sensitive the Tevatron experiments are to the theoretical assumptions, and how they can contribute to reducing the uncertainties in the extrapolation to SSC energies.

The specific processes which are strongly influenced by the small x behavior of PDF's and which have been discussed extensively at this workshop are: mini-jets, b-quark jets, and Drell-Yan processes. The expected rate for mini-jets is huge, and that for Drell-Yan process is tiny. As is often the case, the simplicity of the theoretical basis of a process is inversely proportional to the magnitude of the cross-section: the connection between the Drell-Yan (differential) cross-sections and parton distributions is much cleaner than for the mini-jets. The situation with b-jets lies somewhere in between. Thus, if the Drell-Yan processes are measurable, then they should be the best source of information on the parton distributions. Whether the b-jet events can be used for the same purpose, especially to yield information on the gluon distribution, is unclear at this moment. It is the general consensus of the QCD study group that the abundant mini-jet events may be very interesting on their own

[31] See contributions by Wu-Ki Tung and by J. Ralston & D.W. McKay in the Proceedings of Snowmass 86, Ref.2.

right, but our current lack of a firm grasp of the wide range of theoretical and experimental issues relating to these events does not encourage us to expect quantitative conclusions from them. Of course, this assessment may change when progress is made in jet physics in years to come.

The study is carried out with parton distributions generated by the QCD evolution method from initial conditions which correspond to $\gamma = 1.0$, 1.3, and 1.5 (cf. Eq.(9)). The three sets of initial distributions are taken from EHLQ[13] set 1 with the power γ modified as indicated with compensating changes in the normalization of the PDF's such that all momentum and quark number sum rules remain unchanged. The cross-sections and the various distributions for the three processes were generated by the PYTHIA Monte Carlo program. The results obtained for mini-jets are similar to those for b-jets, except in overall magnitude. A brief summary of the results for the Drell-Yan process and for b-jets are given below.

In order to account (in a crude way) for minimal experimental requirements to identify the relevant processes, events are generated with a transverse momentum cut of 5 GeV. Integrated cross-sections with this cut are found to increase by a factor of ~8 (~2.5) as γ is stepped from 1.0 to 1.5 at SSC (Tevatron I) for the Drell-Yan process, and by ~7 (~1.6) for b-pair production.

The differential cross-sections can, of course, provide many more clues on the underlying physics. Thus the distributions in the rapidity y, the transverse momentum P_T, and the scattering angle θ of the final state partons (μ or b as the case may be), as well as the invariant mass distribution of the parton pair were examined. It was found that there are dramatic differences in the shape of most of these distributions (in addition to the overall factor as noted above) for the Drell-Yan process at SSC due to the different assumptions on the small-x parameter γ. These differences are also quite significant at the Tevatron. In contrast, the corresponding differences in the b-pair production process are rather small for both colliders. For details on these results, see the separate report in these proceedings.[3]

We reach the preliminary conclusion that the Drell-Yan process offers the most sensitive test on the small x behavior of parton distribution functions. Even though the gluon is not a dominant contributor to this process, information derived on the x-dependence of the anti-quarks should give a very good indication of the behavior of the gluon distribution function as they are closely related in the QCD framework. Thus it would be very worthwhile to make the effort to measure the Drell-Yan cross-section at the Tevatron. **Any useful result obtained here can provide powerful constraints on the expected rates and distributions of particles coming out of all semi-hard processes at the SSC.**

F. SUMMARY

We have given a pedestrian review of some non-trivial aspects of the QCD-improved parton model which are relevant to the study of heavy particle production and processes initiated by heavy partons. There is much physics in this problem. A straightforward procedure to determine the applicability of the naive parton picture and to formulate the next order corrections is outlined. The origin and the numerical significance of subtraction terms are elucidated. A complete renormalization scheme as well as a simplified one relevant for heavy parton processes are presented.

Current status concerning the small-x behavior of parton distributions is reviewed. Some new results on the sensitivity of semi-hard physics to uncertainties on the PDF's obtained at this workshop are summarized. These preliminary results strongly indicate that the Drell-Yan process (if it can be measured) provides an especially useful tool in studying the small-x behavior of the parton distributions. The need to conduct a comprehensive survey of current and anticipated experimental information on PDF's at small x and to have available reliable PDF's with a reasonable range of allowed small x behavior is emphasized.

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