Fermi National Accelerator Laboratory

FERMILAB-Conf-87/53-T
March 25, 1987

# Theoretical Expectations for Mass Scales of the Fourth Generation and Higgs Bosons[1]

**Christopher T. Hill**
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Ill. 60510

## Abstract

We discuss two classes of theories that have something to say about fourth generation quark and lepton masses (as well as Higgs). One is based upon universality of low energy parameters as determined by the renormalization group: infra–red fixed points. The other is a toy model of geometric heirarchy which may arise in a novel scheme of dimensional reduction without compactification.

[1] *Invited Talk, Fourth Generation Workshop, Santa Monica, Calif., Feb. 1987*

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

# I. Introduction

A major problem of modern particle physics is to understand the origin and to compute the values of the quark and lepton masses and mixing angles. It is not clear that this will be possible in short order since, if considerations from grand unified theories are relevant, the details at inaccessibly high energy scales may be involved. Furthermore, the mechanism that breaks the electroweak symmetries is fundamentally distinct form the mechanism that produces the masses (certainly this is true in the standard model since the Higgs–Yukawa couplings are parameters independent of the gauge couplings and Higgs potential; in many models this distinction is maintained).

Nature may oblige us with helpful new information at the TEV scale which sheds light on *The Theory.* However, there is another interesting possibility which, with a fourth generation, becomes somewhat likely: heavy quark and lepton masses are *universal.* That is, perhaps the details of the origin of the Higgs-Yukawa couplings are buried at the GUT scale, but are *irrelevant* in determining the masses measured at low energies due to the evolution of the parameters by the renormalization group. The low energy values may essentially be fixed points and can be computed by a knowledge of the (i) renormalization group equations up to the GUT scale (ii) existence of a desert (this is really a simplifying assumption) (iii) existence of sufficiently heavy fermions which are governed by these fixed points. This idea was first proposed by Pendleton and Ross [1] and subsequently refined and applied to the fourth generation in ref.[2]. We shall review the application of fixed points to fermion masses, mixing angles, and the Higgs system for which the ideas apply and suggest natural scales [3]. We emphasize that the natural scale for Higgs–Yukawa couplings in superstring inspired models is already large ($\approx g_{gauge}$) and this possibility is quite plausible there.

There are many other theories of mass and mixing angles, including horizontal unification, extended technicolor, etc. We will not discuss these at present but rather propose a novel idea based upon the essential teachings of superstrings and Kaluza–Klein theories on the problem of fermion masses: fermions emerge as zero-modes on the compact manifold associated with dimensional reduction and the number of families is associated with a topological index. Little has been done to understand the resulting systematics of masses and mixings. We will show how a geometric heirarchy might emerge from a novel realization of these ideas.

## II. Renormalization Group Fixed Points

Consider the evolution of the Higgs–Yukawa coupling constant of a standard model $+2/3$ charged quark. The renormalization group equation is:

$$D \ln g_{+2/3} = \frac{3}{2} g_{+2/3}^2 - \frac{3}{2} g_{-1/3}^2 + \sum_{all\ flavors} g_f^2 \ - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \qquad (2.1)$$

where $16\pi^2 D = d/d \ln E$. We may translate this into an effective "running mass" by multiplying the coupling constant by the Higgs VEV of 175 Gev. The resulting evolution given by eq.(1) from a large scale, presumeably the GUT scale, for various choices of initial value is seen in Fig.(1). Here we consider two candidate GUT scales, $M_X \approx 10^{15}$ Gev and $M_X \approx 10^{19}$ Gev. We notice that for a wide range of of initial values the resulting low energy top quark mass is universal, i.e., it is insensitive to the choice of initial value and it is insensitive to the choice of GUT mass. We would predict at one loop $m_{quark} \approx 240$ GeV. Fischler and Oliensis [4] have given a comprehensive two–loop analysis and obtain $224 < m_{quark} < 232$ GeV with $\Lambda_{\overline{MS}} = .16$ Gev. Also, to one loop we may compare supersymmetric $SU(5)$ which yields $m_{quark} \approx 205$ Gev [5]. Such a prediction for the top quark mass cannot yet be ruled out by experiment, though it may be in conflict with the best estimates of the weak isospin splitting permitted by the $\rho$ parameter [6]. Moreover, some other considerations in standard GUTology may be relevant [7]

This may be stated another way; given no a priori knowledge of the value of the quark mass at $M_X$, i.e. a flat distribution of initial values at $M_X$, we determine the probability distribution at low energies as in Fig.(2). We see it is peaked at the fixed point, which corresponds to an upper limit as first discussed in ref.[8].

We turn to the fourth generation and consider the evolution equations (assuming a massless neutrino) [2]:

$$D \ln(g_T) = \frac{9}{2} g_T^2 + \frac{3}{2} g_B^2 + g_E^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \qquad (2.2)$$

$$D\ln(g_B) = \frac{9}{2}g_B^2 + \frac{3}{2}g_T^2 + g_E^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{12}g_1^2 \tag{2.3}$$

$$D\ln(g_E) = \frac{5}{2}g_E^2 + 3g_B^2 + 3g_T^2 - \frac{9}{4}g_2^2 - \frac{15}{4}g_1^2 \tag{2.4}$$

We have here set all Cabibbo angles to zero. The results of a numerical integration are best presented as scatter plots for various initial points as shown in Fig.(3). It should be noted that no particular point on the fixed 2-surface in the 3-space of the 3 coupling constants is particularly favored, though the point: *up* $\sim$ 220 (204); *down* $\sim$ 215 (198); *lepton* $\sim$ 60 (68) Gev, appears to be an attractor (we give some two-loop results from [4] in parentheses). There is insufficient running time to significantly attract the three couplings to this point. However, the surface is definitely non-spherical; yet one can simply quote a sum rule [9]. Knowledge, however, of two masses determines the third.

# III. Application to the Higgs Sytem

It is quite interesting to apply these ideas to the Higgs system [3]. First we consider a single doublet and then generalize to the case of the two-doublet model.

First we assume the standard potential:

$$V(\phi) = -\frac{m_H^2}{2}|\phi^\dagger\phi| + \frac{\lambda}{4}|\phi^\dagger\phi|^2 \tag{3.1}$$

whence $v^2 = m_H^2/\lambda = (175\ Gev)^2$ and the Higgs boson has mass $m^2 = v^2\lambda$ Hence, a knowledge of $\lambda$ determines the Higgs boson mass and $\lambda$ is governed by a logarithmic renormalization group. We find for $\lambda$ the RG equation:

$$\begin{aligned} D\lambda &= 12\lambda^2 - 3\lambda(g_1^2 + 3g_2^2) + 3g_2^4/2 + 3(g_1^2 + g_2^2)^4/4 \\ &\quad +4\lambda(3g_t^2 + 3g_b^2 + g_l^2) - 4(3g_t^4 + 3g_b^4 + g_l^4) \end{aligned} \tag{3.2}$$

where we have included the contributions from a single heavy generation.

Curiously, we find that if we set the fermion contributions to zero (all fermions assumed light) then $D\lambda > 0$ and since we are descending in energy from a presumed $M_X$ small initial $\lambda$ values can be driven negative. Mass scales at which

$\lambda$ becomes negative are essentially Coleman–Weinberg symmetry breaking scales. This is shown in Fig.(4). Moreover, there is no real fixed point as described above, but there is a practical upper limit to the low energy value of $\lambda$ and thus an upper limit on the Higgs mass. This is no longer universal, Fig.(5).

Allowing for a single heavy quark we may study the joint evolution of the Higgs–Yukawa coupling, eq.(1) (there is no backreaction of the Higgs quartic coupling onto the Higgs-Yukawa coupling). This is indicated in Fig.(6). Thus, at the fixed line we may construct a relationship between the Higgs mass and the quark mass valid for $m_{quark} > 100$ Gev as shown in Fig.(7).

We turn now to the two–doublet model which has the general potential:

$$\begin{aligned} V(\phi_1, \phi_2) &= \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \frac{\lambda_1}{2} |\phi_1^\dagger \phi_1|^2 + \frac{\lambda_2}{2} |\phi_2^\dagger \phi_2|^2 + \\ & \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 |\phi_1^\dagger \phi_2|^2 + \frac{\lambda_5}{2} e^{i\epsilon} (\phi_1^\dagger \phi_2)^2 + h.c. \end{aligned} \tag{3.3}$$

There are obvious quartic stability constraints:

$$\lambda_1, \ \ \lambda_2 > 0 \tag{3.4}$$

and several others involving $\sqrt{\lambda_1 \lambda_2}$ for various ranges of the other parameters [3]. Finally, there are ten ways of sewing the two doublets together with a generation of quarks and leptons (modulo $\phi_1 \leftrightarrow \phi_2$), such as (this is a fairly obvious notation; schemes **I.** through **X.** are listed in ref.[3]; no coupling may be viewed as coupling to light fermions and we always assume a light neutrino):

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \begin{pmatrix} u \\ d \\ l \end{pmatrix} \ \mathbf{I.} \qquad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \begin{pmatrix} u \\ d \\ l \end{pmatrix} \ \mathbf{II.} \qquad \ldots \tag{3.5}$$

Since we have nothing to say about the evolution of the masses, we use as input parameters the known combination, $v_1^2 + v_2^2 = (175 \ Gev)^2$ and an unknown vacuum angle, $\tan\theta = v_1/v_2$.

Now we must face several issues with the two–doublet model:

1. Why is the breaking ferromagnetic, $\uparrow\uparrow$ as opposed to $\uparrow\downarrow$, so as to preserve the $U(1)_{EM}$? This requires $\lambda_4 < 0$, so why should such a value be naturally selected?

2. Are the quartic stability constraints naturally respected by evolution across the desert (if not we get Coleman–Weinberg at the appropriate scale)?

3. The model admits an axion in the ferromagnetic breaking when $\lambda_5 = 0$. Is there anything special, or natural, about that value?

4. Does the model admit infra-red RG fixed points?

In fact we find that the RG behavior of the theory is interesting only in the presence of a heavy quark or lepton (or both) and appears to develop quartic instabilities otherwise. For example, the evolution of the quartic coupling constants is plotted in Figs.(8a–b) in scheme (I). Remarkably, $\lambda_4 < 0$ occurs naturally as a fixed point result. Also, though there is insufficient evolution time to produce a bonafide low mass axion, the coupling $\lambda_5$ is driven small and we get a neutral pseudoscalar as the lightest Higgs boson. Hence, $\lambda_5 = 0$ is a stable IR fixed point. The heavy neutrals have masses at the fixed point determined up to $v_1/v_2$, Fig.(9). In Table I we give the other masses at this fixed point, again for scheme I. Here, in addition to a heavy fourth generation, we include the effects of the top quark.

For comparison, if there are no heavy $> 100$ Gev fermions, the evolution of the parameters is unstable, [3]. In terms of a reasonable fundamental theory in the presence of the desert the two Higgs doublet scheme (and presumeably multi–Higgs) seems to make sense only in presence of a fourth generation.

# IV. Topological Theory of Masses and Mixing Angles

One of the major lessons of compactified higher dimensional unified theories, such as Kalusza–Klein and Superstring models is that the low energy laboratory fermions may be (i) zero–modes on a compactified manifold and generally (ii) the number of generations is determined by an index theorem on the compactified manifold. This latter point is important since the parent higher dimensional theory may have only one fermionic field which "spawns" the larger observed number of generations. There has been, however, little understanding to date as to how the fermion mass matrix arises, how it may lead to an approximate geometrical mass heirarchy, and how the texture of the CKM matrix emerges. Here we give some preliminary sketches as to how these questions may be answered. Furthermore, we choose to study a modified problem which may either be viewed as a laboratory for the study of compactified higher dimensional theories or as a fundamentally new approach to dimensional reduction *without employing compactification.*

Spontaneous symmetry breaking of gauge theories can give rise to objects such as solitons, vortices, and monopoles. In $D$ space dimensions the generalization of the flux tube is to a $D-2$ dimensional object and the solutions go over directly. If fermions obtain a mass by coupling to the complex scalar field of an abelian vortex, massless states localized on the vortex, known as Jackiw–Rossi zero-modes, can exist. Here a single fermion field in higher dimensions spawns $n$ zero–modes if the vorticity of the flux tube is $n$. Presumeably higher spin local bosonic condensates can give vector and spin–2 zero–modes, candidate effective Yang–Mills and gravitational fields, in a manner analogous to Witten's bosonic superconducting strings [10].

An intriguing possibility then arises. If the Universe is actually an infinite $1+D$ spacetime, there might occur such objects in such a Universe. The scale of symmetry breaking forming the soliton, $M_s$ may be quite large, $M_s >> M_W$ and probably $M_s \sim M_{Planck}$. The trapped zero–mode fermions would then be described by an effective $1+D-d$ theory and we choose $D-d=3$ to correspond to the observed physical Universe. Thus, *we have dimensionally reduced the world without compactifying the extra dimensions.*

The vortex is a non-trivial axially symmetric configuration of a complex scalar field $\phi$ and an optional gauge field $A$ which we set to zero. The scalar field will be taken as a fixed c-number of the form: $\phi(r) = e^{in\theta} f(r)$, where, $(r, \theta, z)$ are cylindrical coordinates normal to the string. For definiteness, we will take $n > 0$. The asymptotic forms of the fields are $f(r) \to 0$ for $r \to 0$ and $f(r) \to f_o$ for $r \to \infty$.

The simplest way to exhibit the heirarchy phenomenon is to write the Lagrangian for the fermions:

$$\begin{aligned} L &= i\psi^\dagger \partial\!\!\!/ \psi + i\chi^\dagger \partial\!\!\!/ \chi + i\lambda^\dagger \partial\!\!\!/ \lambda + i\delta^\dagger \partial\!\!\!/ \delta \\ &\quad + ig\phi(r,\theta)\psi^T \epsilon \chi - ig\phi^*(r,\theta)\lambda^T \epsilon\delta \\ &\quad + h\sigma(\psi^T\epsilon\delta + \chi^T\epsilon\lambda) + h.c + L(\sigma) \end{aligned} \tag{4.1}$$

$\psi$, $\chi$, $\lambda$ and $\delta$ are 2-component (left-handed Weyl) spinors. Here $\sigma$ is a scalar field which develops a VEV localized on the string, $\langle\sigma\rangle = \sigma_0 \exp(-\kappa r)$. This is analogous to Witten's bosonic superconducting string [10] and can easily be engineered by dynamics. It is presented here to simplify the arguments to come and is not essential for the present effect (we can set $g = g'$ with the $\sigma$ condensate; alternatively we can generate the geometric hierarchy with $\sigma = constant$ and $g' >> g$).

Far from the flux tube the Higgs field may be regarded as having a constant VEV. Then we see that the pairs of Weyl spinors, $(\psi,\ \chi)$ and $(\lambda,\ \delta)$ form two four-component massive Dirac fields. On the vortex there occur Jackiw–Rossi zero-modes and $(\psi,\ \chi)$ produce $n$ chiral left-movers while $(\lambda,\ \delta)$ produce $n$ chiral right-movers for a vorticity $n$.

For any vorticity $n$ we have to a good approximation the sequence of normalized zero-modes from $p = 0$ to $p = \pm(n-1)$: $\psi_p = \beta_p(r)\alpha(z,t)e^{ip\theta}$ and one finds that the $\beta_p(r)$ have the behaviors:

$$\beta_p(r) \to c_p r^{|p|} \qquad r \to 0; \qquad \beta_p(r) \to e^{-|gf_0|r} \qquad r \to \infty \tag{4.2}$$

We shall assume $\kappa >> |gf_0|$, i.e., the radial profile of the $\sigma$ condensate varies rapidly compared to the fermionic zero modes. This simply requires a choice of small $g$.

Let us now consider the effects of the mass terms. We may now compute the mass matrix for the fermionic zero modes by integrating the transverse wave–function radial profile as given in eq.(4.2):

$$M_{pq} = \int r\ dr\ d\theta \beta_p \beta_q^* h \sigma_0 e^{-\kappa r} e^{i(p-q)\theta} \tag{4.3}$$

Clearly, owing to the angular dependence of the modes, the mass-matrix is diagonal; that is, generation number is topologically conserved at this level and no flavor mixing occurs. We see, however, that a geometrical heirarchy emerges:

$$M_{pq} \approx 2\pi c_p^2 \delta_{pq} \int r\ dr\ r^{|2p|}\sigma_0 e^{-\kappa r} \approx \delta_{pq} K_p \epsilon^{|2p+1|} \tag{4.4}$$

where $\epsilon \sim |gf_0|/\kappa << 1$ and $K_p \sim 1$. (We have numerically solved the transverse Dirac equation and verified that this heirarchy does indeed occur; the $K_p$ are interesting for making model predictions and must be determined numerically [11]).

Mixing can occur by introducing higher dimension operators into the Lagrangian as fermionic mass terms, but also involving the vortex Higgs. For example, we might add terms involving $\psi^\dagger\Phi^2\psi$ and $\psi^\dagger\Phi^3\psi$, etc. to pairs of Weyl spinors with the appropriate charges (this requires introducing different charged fermions). Such terms will naturally be suppressed by $1/M_{heavy}$, $1/M^2_{heavy}$, etc. These terms will lead to mixing between families and a rudimentary KM matrix emerges.

Of course, the principal difficulty is to give a convincing explanation of gravity and Yang–Mills fields trapped on the flux tube. These issues will be addressed in a forthcoming paper [11].

**References**

1. Pendleton, B. and G. G. Ross, Phys. Lett. 98B, 291 (1981)

2. Hill, C. T., Phys. Rev., D24, 691 (1981)

3. Hill, C. T., C. N. Leung, S. Rao, Nucl. Phys., B262, 517 (1985)

4. Fischler, M. and J. Oliensis, Phys. Rev. D28, 194 (1983); Phys. Lett., 119B, 385 (1982)

5. Bagger, J., S. Dimopoulos, E. Masso, Phys. Rev. Lett., 55, 920 (1985); H. Goldberg, Phys. Lett. 165B, 292 (1985)

6. Marciano, W., talk presented at this meeting.

7. Fischler, M., C. T. Hill, Nucl. Phys., B193, 53 (1981); J. Bagger, S. Dimopoulos, E. Masso, Phys. Lett., 145B, 211 (1985)

8. Cabibbo, N., *et. al.* Nucl. Phys., B158, 295 (1979)

9. Bagger, J., S. Dimopoulos, E. Masso, Phys. Rev. Lett., 55, 1450 (1985); Nucl. Phys. B253, 397 (1985)

10. Witten, E., Nucl. Phys. B249 (1985) 557.

11. Hill, C. T., in preparation.

Table I. Higgs Masses (Gev) at the Fixed Point (Scheme I):

| $m_{quark}$ | $m_{\pm}$ | $m_1^0$ | $m_2^0$ | $m_{axion}$ |
|---|---|---|---|---|
| 0 | 188 | 190 - 230 | 0 - 105 | 60 |
| 50 | 186 | 188 - 225 | 0 - 105 | 60 |
| 172 | 136 | 164 - 216 | 0 - 131 | 25 |

## Figure Captions

1. Evolution from (A) $10^{15}$ Gev and (B) $10^{19}$ Gev of effective mass of single heavy $+2/3$ charge quark.
2. Probability distribution of masses of heavy quark with equal liklihood of any mass as defined at $M_X$.
3. Scatter plots of $g_t$, $g_b$ $g_\tau$ from inital array at $M_X$ (integer initial values).
4. Higgs-quartic coupling integrated from $10^{19}$ Gev. Coleman–Weinberg breaking effectively occurs at zero crossing.
5. Evolution of Higgs effective mass from $10^{15}$ Gev. No apparent fixed point occurs in absence of heavy fermions.
6. Joint evolution of large Higgs-Yukawa coupling and quartic Higgs coupling.
7. Fixed line relationship between a heavy "t–quark" and Higgs boson masses.
8. (A) Evolution of $\lambda_1$ and $\lambda_2$ in two doublet model for sample initial values (B) Evolution of $\lambda_1$ and $\lambda_3$ (C) Evolution of $\lambda_1$ and $\lambda_4$; note ferromagnetic ( $\lambda_4 < 0$) fixed point occurs.
9. Scheme I masses of neutral Higgs bosons in two doublet model.

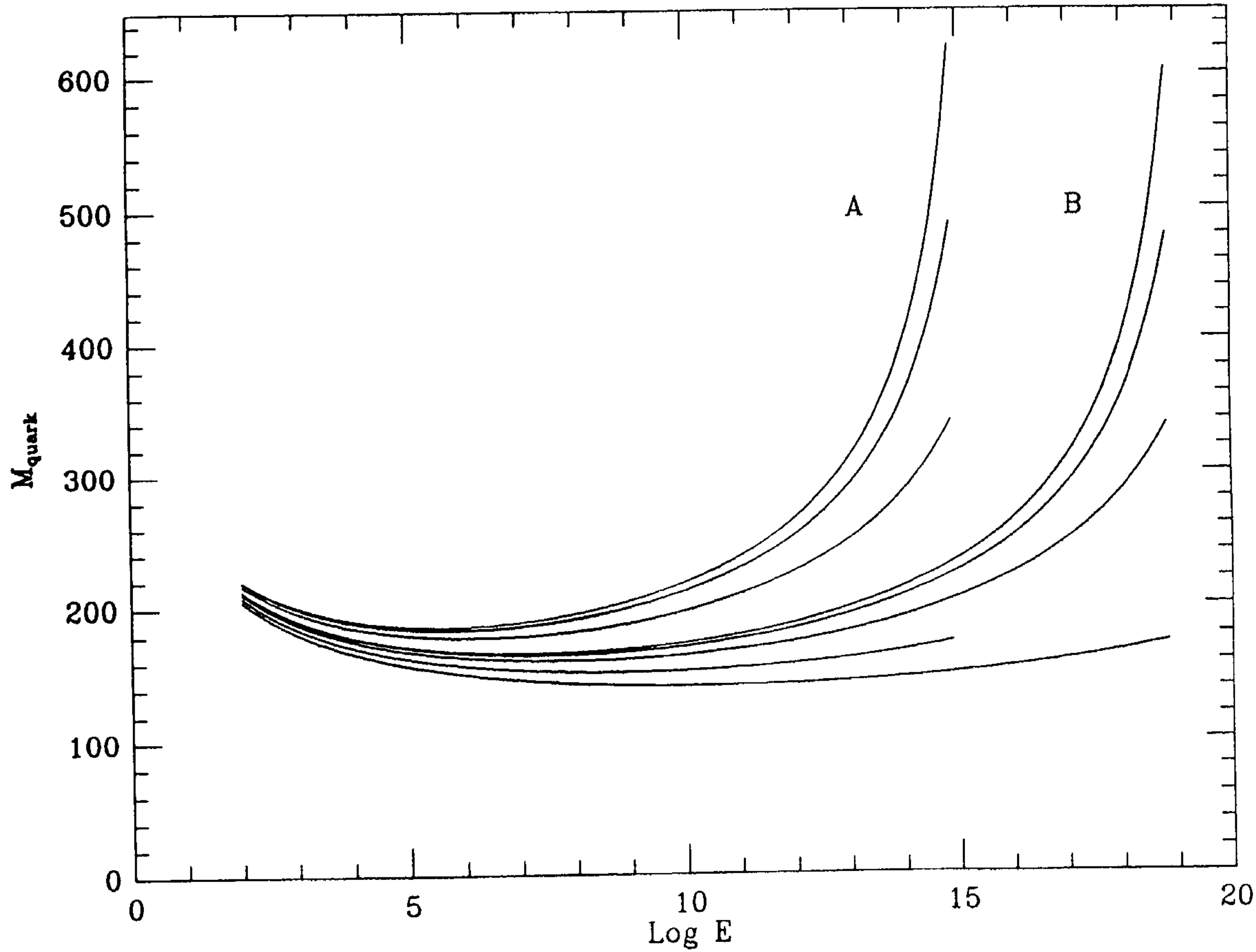


Figure 1

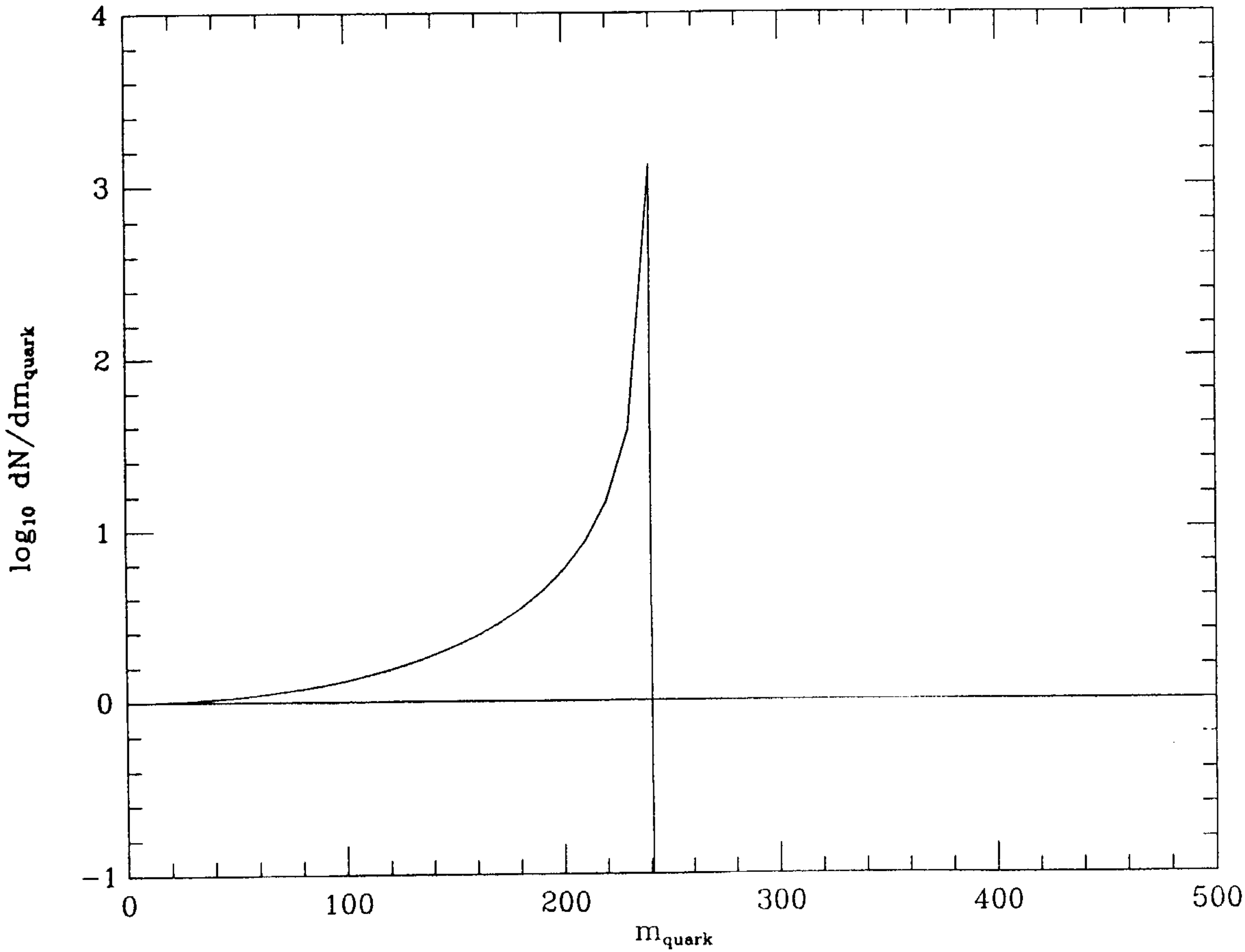


Figure 2

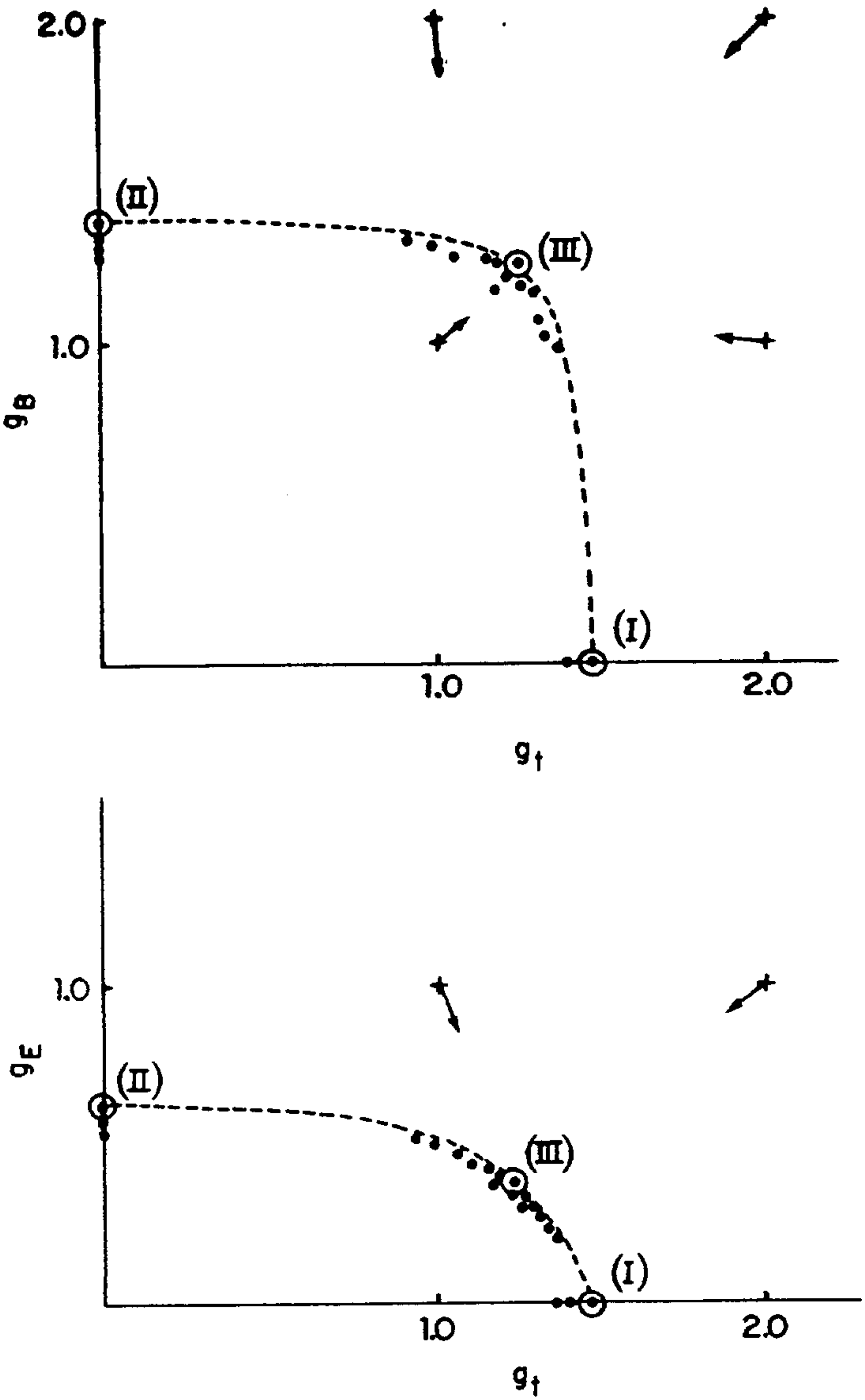


Figure 3

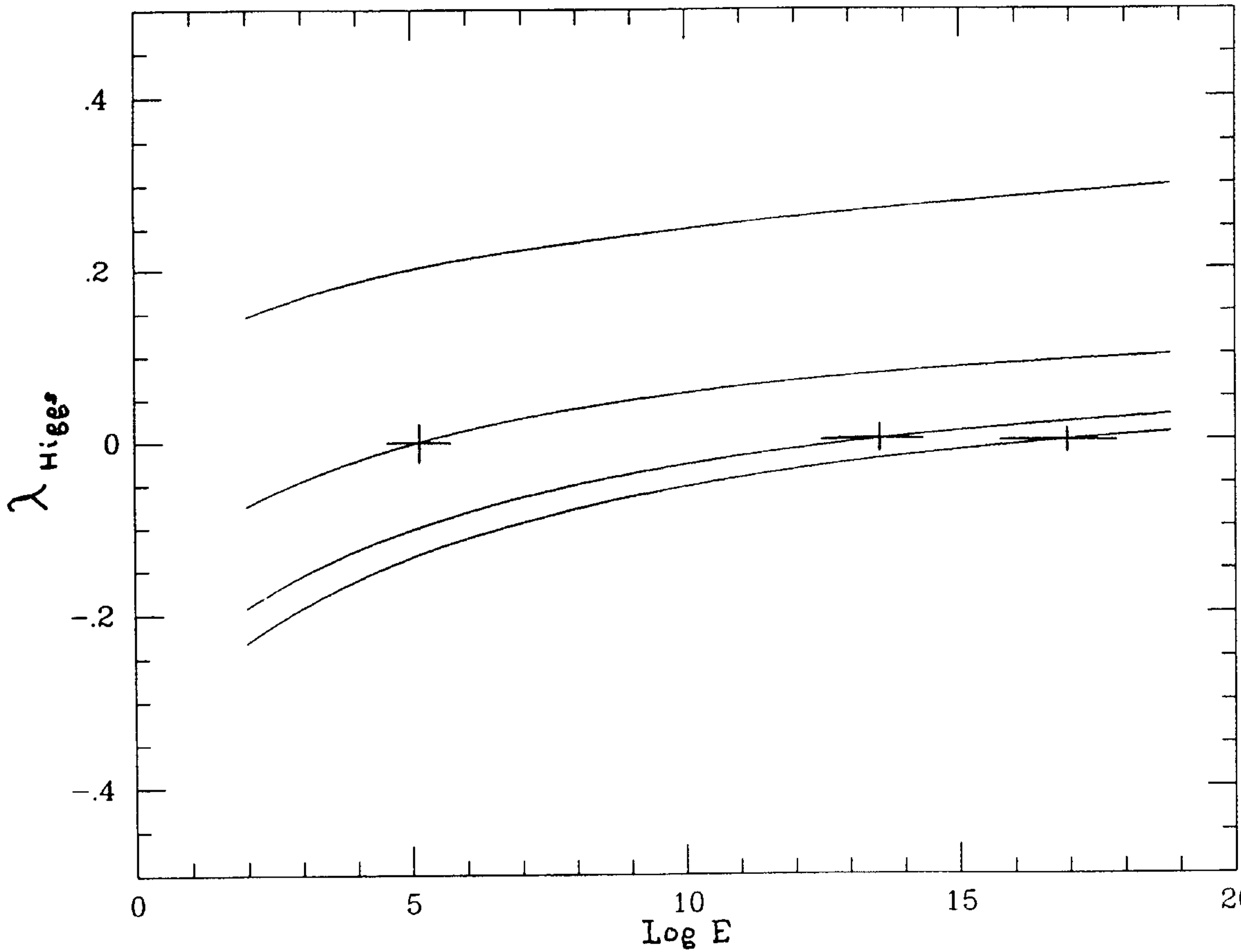


Figure 4

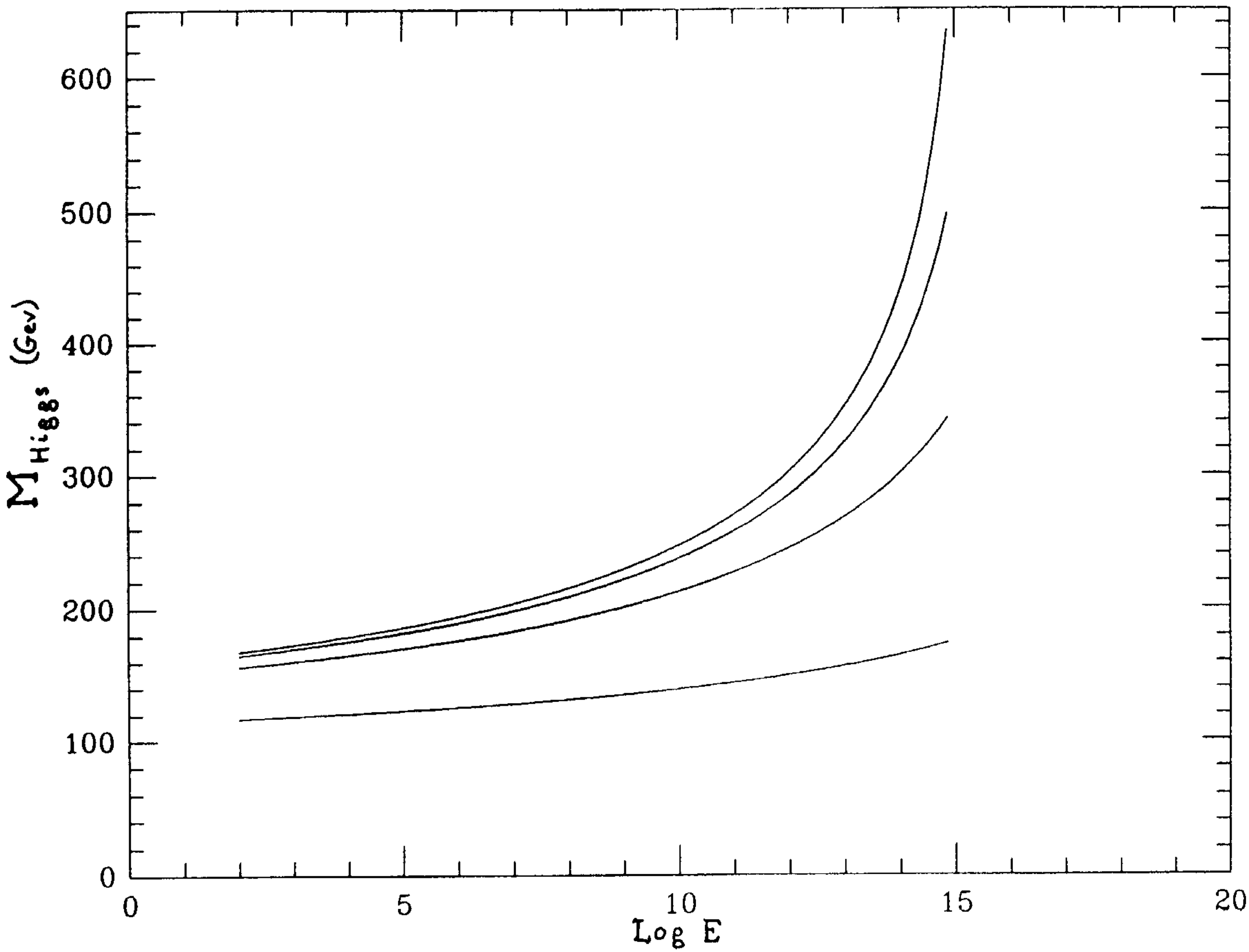


Figure 5

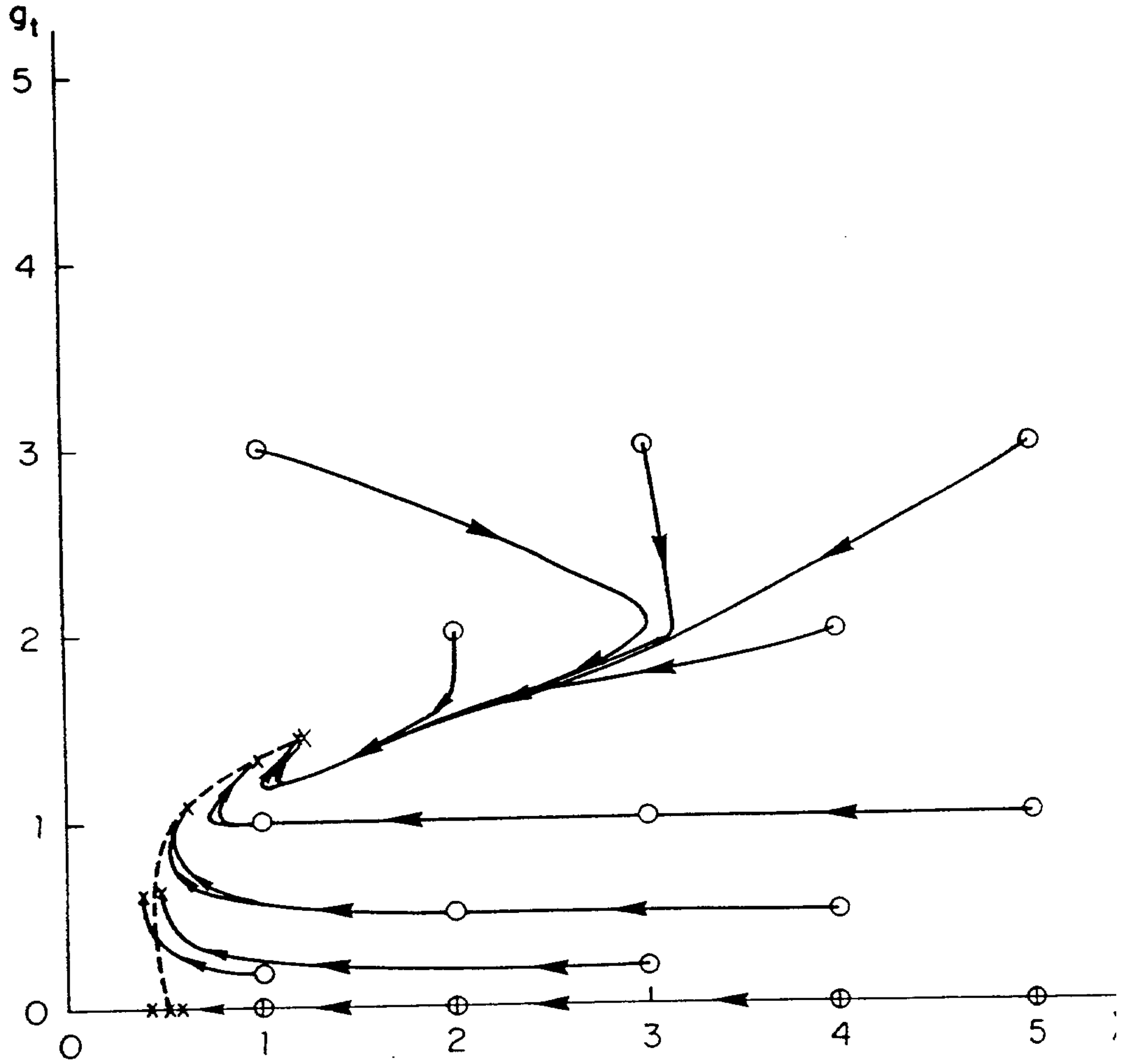


Figure 6

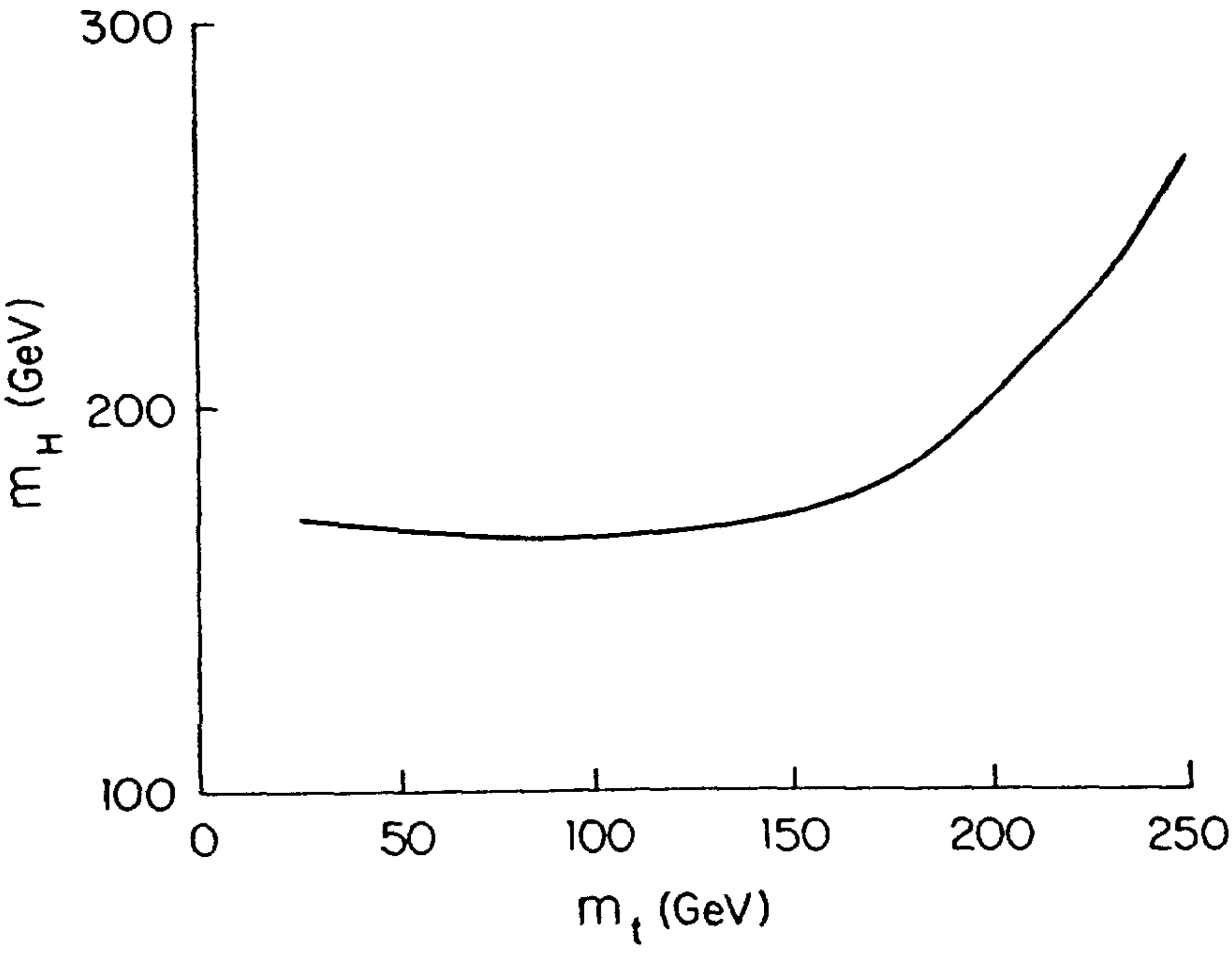


Figure 7

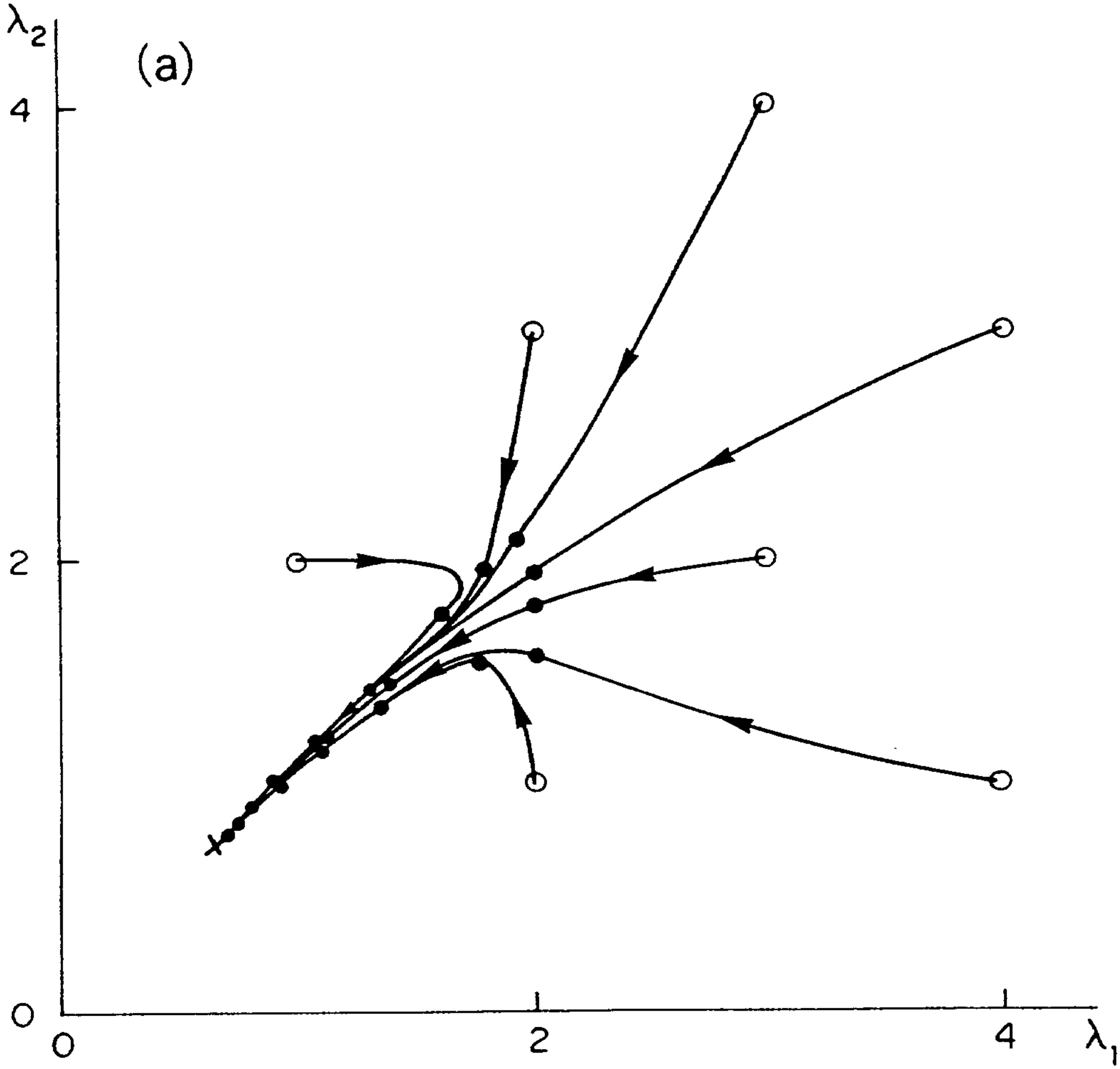


Figure 8a

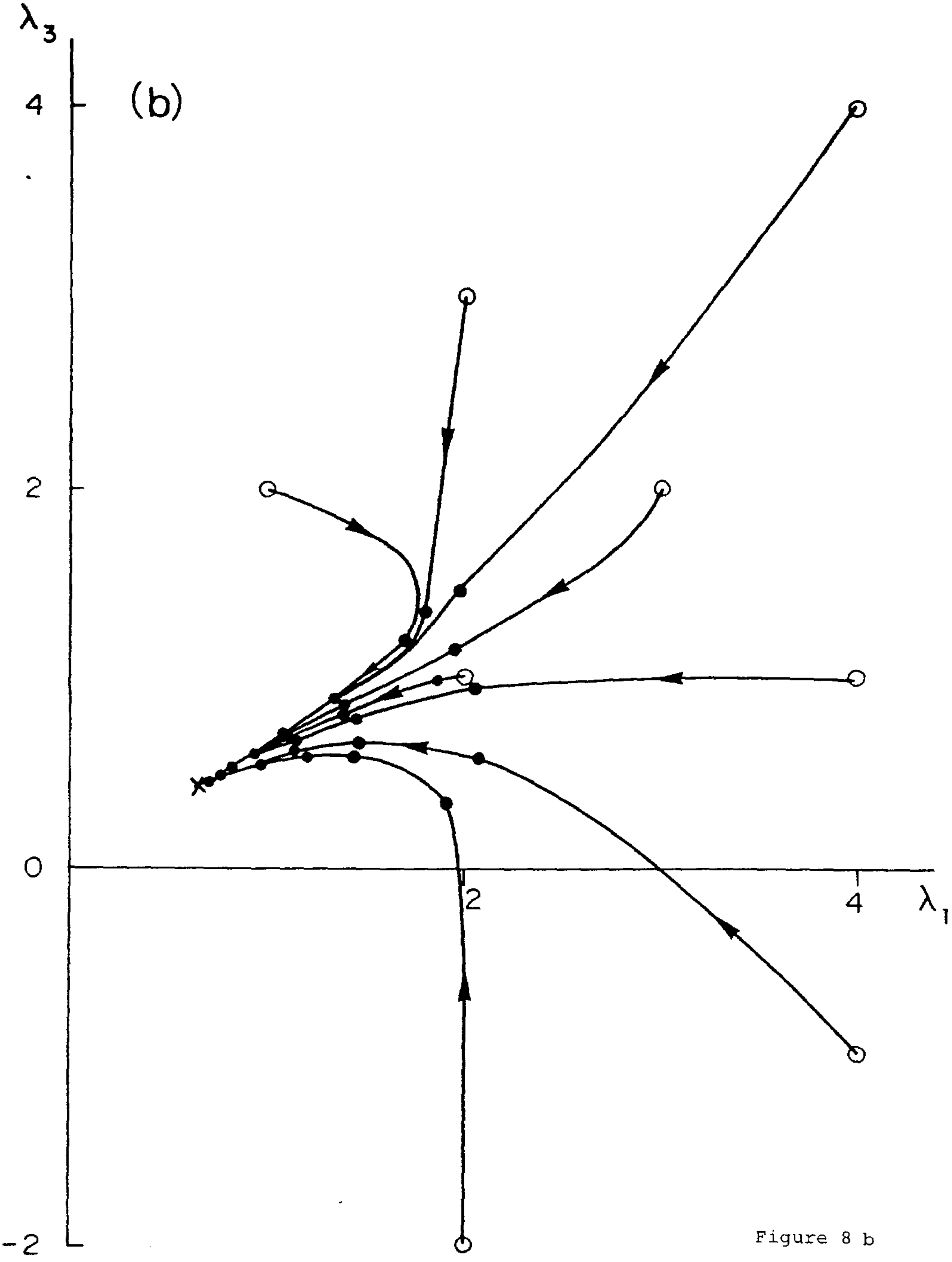


Figure 8 b

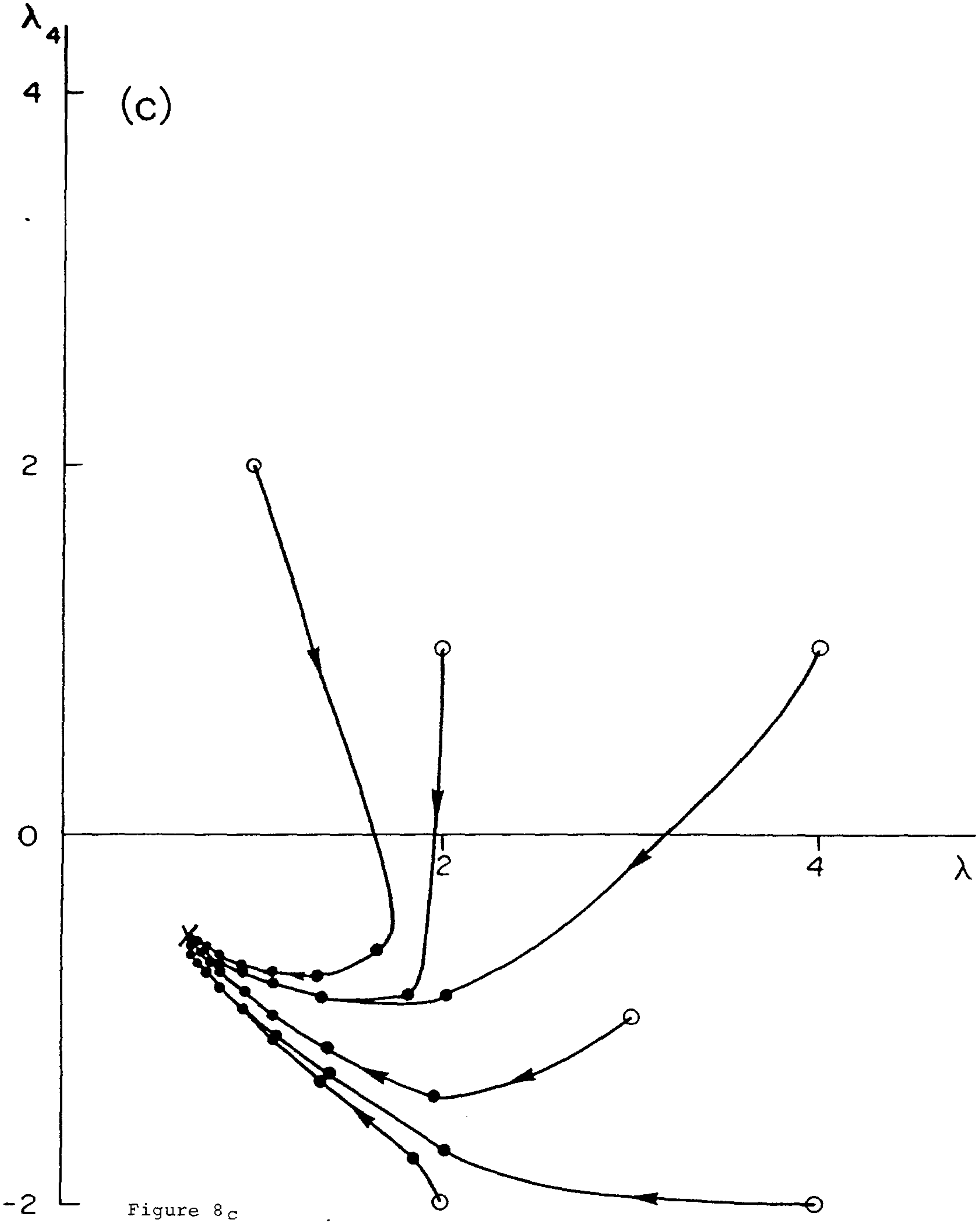

Figure $8_c$

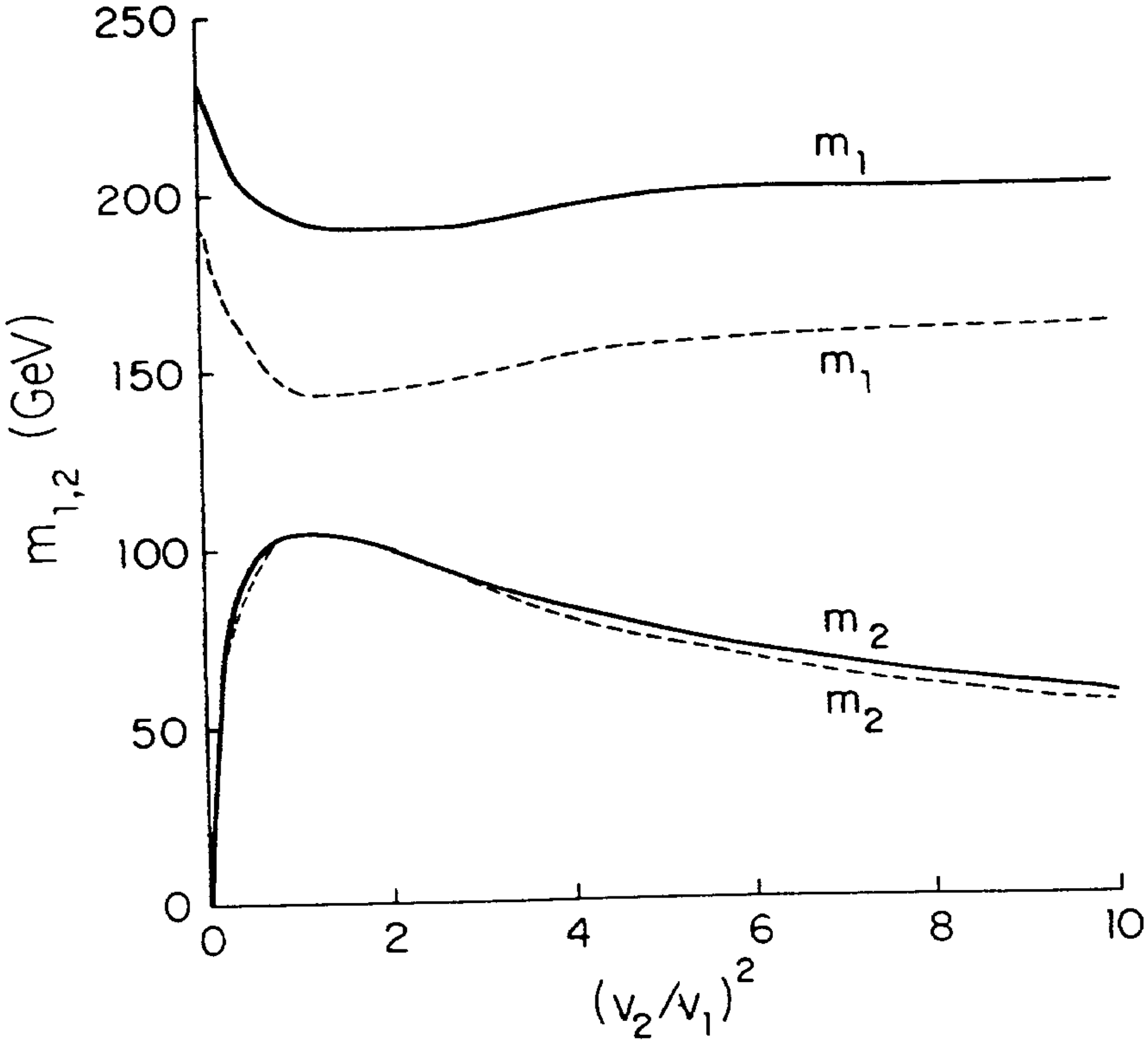


Figure 9