Fermi National Accelerator Laboratory
FERMILAB-Conf-87/32-A
February 1987

COSMOLOGICAL STABILITY OF QUANTUM COMPACTIFICATION

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ABSTRACT

We discuss the cosmological stability of higher dimensional models that feature internal manifolds given by the product of two spheres. In particular, we consider the case when the total number of dimensions is even. After we obtain the vacuum energy coming from one-loop fluctuations of scalars and spin-$\frac{1}{2}$ fermions, we show how a realistic cosmological scenario can arise by balancing the quantum energy with monopole-like contributions.

1. Introduction

The quest for a unified theory of the fundamental interactions based on higher- dimensional spacetimes has been the focus of much attention in recent years[1]. From the upsurge of the traditional Kaluza-Klein theories in the mid- seventies to superstring theories nowadays, the inclusion of "some" extra dimensions in the description of nature seems to be imperative if we are to unify gravity with the gauge interactions.

Once we accept the possible physical relevance of extra-dimensions, two questions come immediately to our minds; why do the scales characterizing the physical four-dimensional spacetime and the internal compact space differ by roughly 60 orders of magnitude nowadays and what determines the stability of the internal space. The physical motivations for both questions should be obvious. We find no trace of extra dimensions on scales between $10^{-16} cm$ to $10^{28} cm$ and also have very strict limits on the time variation of the fundamental couplings that would be induced by a time variation of the internal space. To put it concisely, the internal space is extremely small and static (or very nearly so)[2].

On view of the above comments, it is important to study the stability of the various compactification mechanisms proposed to date in order to select the proper ground-state to the physical theory coming from extra-dimensions. Different compactification schemes will provide different ground-state geometries with different stability properties. Although we

† Talk presented at the XIII Texas Symposium in Relativistic Astrophysics, December 1986. The work reported was done in collaboration with P. Jetzer and M. A. Rubin.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

expect the correct ground-state to be univocally determined once a better understanding of string theory is achieved, for the moment a stability analysis of compact manifolds is certainly useful.

To our knowledge, there are presently three possible contributions to the energy-momentum tensor that can play a role in stabilizing the internal space. One can introduce a $4+D$-dimensional cosmological constant and use it to try to balance the curvature of the internal space, one can calculate 1-loop quantum corrections that give rise to an attractive force coming from imposing boundary conditions to the fields in the effective action, very much like the Casimir effect in QED[3)], or one can use the fact that most actions include anti-symmetric tensor fields that can assume non-trivial topological configurations on the compactified background[4)] .

All the above possibilities have been extensively analysed in the literature, including some combinations of these effects with each other and with finite temperature[5)]. Without going into details, it has been shown that models with a cosmological constant are semi-classically unstable against quantum tunneling; for large values of the internal radius the cosmological constant dominates and the system behaves like a $4+D$-dimensional de Sitter spacetime. (There is some apparent controversy as to whether the tunneling actually occurs. The root of the discussion is in the Weyl rescaling of the metric that, according to some authors, is fundamental to the proper interpretation of the potential. We sustain that physical results will depend on the rescaling since the theory is not conformally invariant and that the absence of tunneling that occurs with the rescaled metric has to be reinterpreted in the light of the uncertainty principle, since the rescaling changes the definition of time. In any case, the necessity of rescaling has not been clarified yet. Work in these lines is on progress.)[6)]

Thus, the more interesting possibilities seem to include the quantum and monopole effects. Accordingly, we have studied the possibility of obtaining a stable compactification with these effects taken into account[7)]. We used, in ref. 7, an approximate expression for the Casimir effect in even dimensions and showed that, for certain theories in ten dimensions, it is possible to obtain a stable background. The promising model comes from $N=1$ supergravity with fermionic condensation and with an internal space given by the product of two 3-spheres. It is yet far from being related to the phenomenologically more promising Calabi-Yau manifolds but it represents an example where the calculations can be presently performed and that will be of relevance once a realistic compactification is achieved.

In the next section, we will present the results of a more complete calculation of the vacuum energy for even dimensional spaces with two internal spheres.[8)] In particular, we will present the results for the $M^4 \times S^3 \times S^3$ geometry that is of relevance to our previous stability analysis of ref. 7. In section 3 we will use these results within a cosmological context and write Einstein's equations for the ten-dimensional case including monopole-like contributions. These latter terms come from the presence of the Kalb-Ramond 3-form in the action of $N=1$ supergravity and can be used to induce the $S^3 \times S^3$ compactification of the internal space. We will then integrate the equations numerically to study the possibility of finding solutions that exhibit both stability of the internal space and a physical rate of expansion for the four-dimensional spacetime.

2. Vacuum Energy of $M^4 \times S^M \times S^N$ in Even Dimensions

We are interested in calculating the 1-loop quantum fluctuations of scalars and spin-$\frac{1}{2}$ fields in the background $M^4 \times S^M \times S^N$ geometry in even dimensions. We refer the reader to ref. 8 for details. In principle higher-spin fields should be included but we will concentrate on the simplest case at this level. The 1-loop quantum potential is given in general by

$$V_Q = bV_Q^{(0)} - 4fV_Q^{(1/2)} , \tag{1}$$

where b and f are the number of bosons and fermions, respectively.

If we use the zeta-function regularization, we can write V_Q as

$$V_Q^{(i)} = i\left(ln\bar{\mu}\varsigma^{(i)}(0) + \frac{1}{2}\varsigma'^{(i)}(0)\right), \quad i = 0, 1/2 , \tag{2}$$

where $\bar{\mu}$ is a constant coming from the measure of the path integral. By taking into account the eigenvalues and degeneracies of the Laplacian operator of scalars and fermions in the above product manifold, we end up with an extremely complicated expression for the quantum potential. We will thus make two simplifications; we will only consider the case with two internal 3-spheres (which is the case of interest anyway) and will restrict ourselves to small deviations from equality in the ratio of the two internal radii. Accordingly, we define ε , the deviation from equality of the radii, as

$$\frac{C}{B} \equiv 1 + \varepsilon , \tag{3}$$

where B (C) is the radius of the M (N) -sphere (here, $M = N = 3$) and retain, in what follows, terms of no higher than linear degree in ε . With this approximation we obtain, after some algebra,

$$\begin{aligned} V_Q = \frac{V_4}{B^4}[b(3.639 \times 10^{-4} - \varepsilon 6.053 \times 10^{-4} + 3.315 \times 10^{-4}(1 - 2\varepsilon)ln(B\bar{\mu}))+ \\ + f(3.657 \times 10^{-6} - \varepsilon 5.414 \times 10^{-5})]. \end{aligned} \tag{4}$$

This is the quantum potential we were looking for. Note that the original symmetry under the interchange of the two radii was lost due to our asymmetric choice of expansion. The next step is to try to solve Einstein's equations in the presence of this potential. Following Candelas and Weinberg (see ref.3), we should be able to find a constant solution for the internal radii by balancing the quantum correction with the classical curvature term. We can write Einstein's equation for the $M^4 \times S^3 \times S^3$ geometry whithin our approximation, as,

$$\frac{6(1-\varepsilon)}{B^2} = \frac{8\pi G_4}{V_4} V_Q \tag{5.1}$$

$$\frac{6(2-3\varepsilon)}{B^2} = \frac{8\pi G_4}{V_4} B \frac{\partial V_Q}{\partial B} \tag{5.2}$$

$$\frac{6(2-3\varepsilon)}{B^2} = \frac{8\pi G_4}{V_4}\frac{\partial V_Q}{\partial \varepsilon} \quad , \tag{5.3}$$

where we used that $G_D = G_4 \Omega_M \Omega_N$, with Ω_M being the volume of the M-dimensional sphere.

Now we impose that $V_Q(\bar{\mu}, B_0, \varepsilon_0) = 0$ and that B_0 and ε_0 are critical points of the quantum potential. The first condition is equivalent to fine-tuning the 4-dimensional effective cosmological constant to zero and allows one to solve for $\bar{\mu}$ in terms of B_0 and ε_0 . We note that in odd-dimensional theories the 1-loop potential is independent of the renormalization parameter $\bar{\mu}$ and one is forced to introduce a D-dimensional cosmological constant in order to obtain a flat 4-dimensional spacetime.

By using the expression of the quantum potential in equation 5, we find

$$\varepsilon_0 = 1.447 \times 10^{-1} + 6.871 \times 10^{-2}\frac{f}{b} \tag{6}$$

$$\frac{B_0^2}{8\pi G_4} = b10^{-6}(8.14 - 5.26 \times 10^{-1}\frac{f}{b}) \ . \tag{7}$$

In particular, if we take $b \sim f \sim 10^4$, we find $B_0 \simeq 1.428 L_P$, where L_P is the Planck length. Thus, for a sufficiently large number of matter fields, the 1-loop calculation is valid. Note also that ε_0 is small enough to justify our approximate solution. We have checked if this approximate solution is stable by explicitly calculating the second derivative of the effective potential obtained by taking the quantum and classical terms together. As expected, the solution given by equations (6) and (7) is not stable, representing a saddle point of the effective potential. This result, together with many others in the literature, seems to suggest that quantum effects alone are not enough to stabilise the internal space. We note however that the effective potential that will come from a realistic solution of a higher dimensional theory will be certainly more complex than the prototypes analysed so far. Hopefully, it will also exhibit the necessary properties for stability.

3. Cosmological Evolution of Quantum Compactification

Now that we have the expression for the quantum potential we are in a position to study the cosmological evolution of our model. Here, we will neglect the corrections that come from performing the quantum calculation in a time-varying background. These corrections may be of relevance in determining the sign of the kinetic term for the internal radius and thus may affect conclusions about the stability of perturbations.[9] As usual, we consider a generalized Robertson-Walker metric for the product space $M^4 \times S^3 \times S^3$. We will also include a general monopole-like term in the energy-momentum tensor, as in ref.7. It is the balance between this term and the quantum contribution that can make stability possible.

By writing the scale factors for the physical spacetime and for the internal space as $A(t)$, $B(t)$ and $\varepsilon(t)$ and by writing the constants in the monopole contribution as f_0^B and f_0^ε , Einstein's equations become,

$$\frac{\ddot{A}}{A}+\frac{\dot{A}}{A}\left(2\frac{\dot{A}}{A}+3\dot{\varepsilon}+6\frac{\dot{B}}{B}\right)=-\frac{1}{32\pi^4B^{10}}\Big\{b\big[8\alpha(1-3\varepsilon)+\beta(1-10\varepsilon)-\gamma(1-5\varepsilon)+$$

$$2\gamma(5-22\varepsilon)ln(B\bar{\mu})\big]+f\big[8\delta(1-3\varepsilon)+\xi(1-10\varepsilon)\big]\Big\}+$$

$$-\frac{1}{2B^6}\left[(f_0^B)^2+(f_0^\varepsilon)^2(1-6\varepsilon)\right] \tag{8.1}$$

$$\frac{\ddot{B}}{B}+\frac{2}{B^2}+\frac{\dot{B}}{B}\left(5\frac{\dot{B}}{B}+3\dot{\varepsilon}+3\frac{\dot{A}}{A}\right)=\frac{1}{96\pi^4B^{10}}\Big\{b\big[32\alpha(1-3\varepsilon)-\beta(3+26\varepsilon)-5\gamma(1-5\varepsilon)+$$

$$2\gamma(13-74\varepsilon)ln(B\bar{\mu})\big]+f\big[32\delta(1-3\varepsilon)-\xi(3+26\varepsilon)\big]\Big\}+$$

$$+\frac{1}{2B^6}\left[3(f_0^B)^2-(f_0^\varepsilon)^2(1-6\varepsilon)\right] \tag{8.2}$$

$$\ddot{\varepsilon}+\frac{2(1-2\varepsilon)}{B^2}+\frac{\ddot{B}}{B}+\frac{\dot{B}}{B}\left(9\dot{\varepsilon}+5\frac{\dot{B}}{B}+3\frac{\dot{A}}{A}\right)+3\dot{\varepsilon}\frac{\dot{A}}{A}=\frac{1}{96\pi^4B^{10}}\Big\{b\big[3\gamma(1-5\varepsilon)+$$

$$5(1-2\varepsilon)(\beta+2\gamma ln(B\bar{\mu}))\big]+5f(1-2\varepsilon)\xi\Big\}+$$

$$+\frac{1}{2B^6}\left[-(f_0^B)^2+3(f_0^\varepsilon)^2(1-6\varepsilon)\right] \tag{8.3}$$

where α, β, γ, δ and ξ are, in this order, the numbers appearing in the result for the quantum potential, eq.(4). The 00 equation was not written and can be used as a constraint equation.

The usual procedure now is to find the constant solutions for the internal radii by fine tuning the effective cosmological constant (the r.h.s. of eq. (8.1)) to zero and by setting the r.h.s. of eqs.(8.1) and (8.2) to zero. By doing this we can also express the regularization parameter $\bar{\mu}$ in terms of the constant values for the internal radii, as we mentioned earlier. This was done for the static case without the monopole term in ref.8. Note that the usual interpretation of eq.(8.2) as an equation for a scalar field $\phi = ln(\frac{B}{B_0})$ is not possible here since the potential is a function of the two internal radii. A detailed analysis of the constant solution and its stability is in progress[10)]. For now, we show in fig.1 the result of a numerical integration of eq.(8). Note that the perturbation ε oscillates around zero while the internal radius B oscillates around a constant value. The physical radius expands with a rate that is sensitive to the initial conditions used. This example suffices to show that, within certain approximations, it is possible to obtain a cosmological scenario coming from higher-dimensional theories that can be compatible with the usual Friedmann cosmology.

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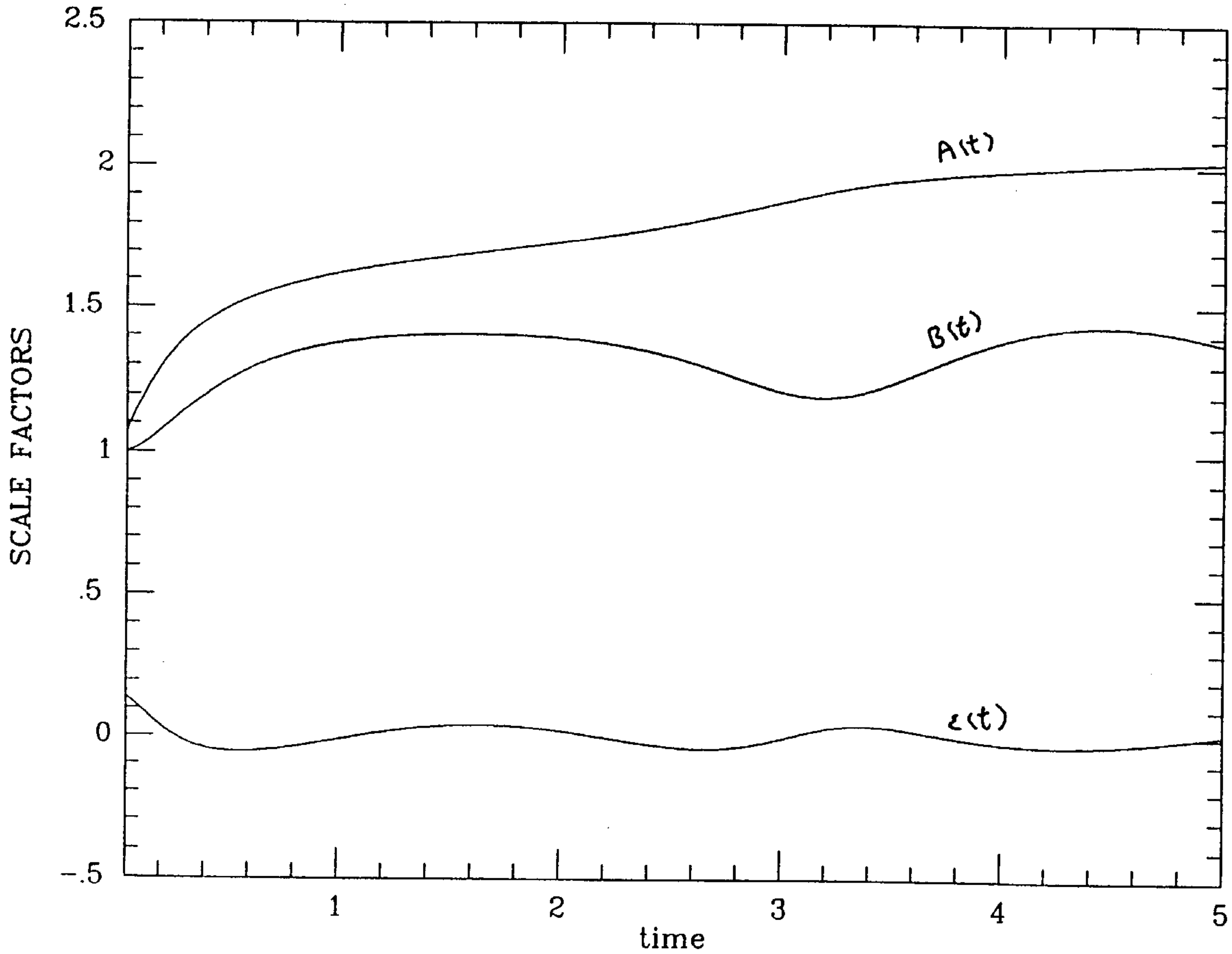

The time evolution of the physical scale factor (A) and of the internal radius (B) together with the perturbation (ϵ) are shown in Planck units.