



Fermi National Accelerator Laboratory

FERMILAB-Pub-86/77-A
May 1986

The Effect of Interacting Particles on Primordial Nucleosynthesis

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Abstract. We modify the standard model for big-bang nucleosynthesis to allow for the presence of a generic particle species, i.e., one which maintains good thermal contact with either the photons or the light neutrino species throughout the epoch of primordial nucleosynthesis. The production of D, ³He, ⁴He, and ⁷Li is calculated as a function of the mass, degrees of freedom, and spin statistics of the generic particle. We show that in general, the effect of an additional generic species cannot simply be parameterized as the equivalent number of additional light neutrino species. The presence of generic particles also affects the predicted value for the neutrino-to-photon temperature ratio.



I. Introduction

Big-bang nucleosynthesis has proved an invaluable tool for placing constraints on various aspects of particle physics models.¹ In particular, the effect of additional light (<MeV) neutrino species,² and of an additional massive neutrino species,³ have been addressed. It has been shown that the inclusion of more than one additional light neutrino species would ruin the concordance between the predictions and observations of light element abundances. The effect of other additional light particle species on primordial nucleosynthesis has also been considered.⁴ It is usually assumed that the effect of a new light particle species can be parameterized in terms of an equivalent number of additional light neutrino species:

$$\Delta N_{\nu} = \frac{4}{7} g_{\text{eff } x} (T_x/T_{\gamma})^4 \quad (1)$$

where $g_{\text{eff } x} = g_x$ for a boson and $7/8 g_x$ for a fermion (e.g., $g_{\text{eff } x} = 7/4$ for an additional neutrino and $T_{\nu} = T_{\gamma}$, so that $\Delta N_{\nu} = 1$). While this simple algorithm is correct for light particles which are decoupled from both the neutrinos and the photons before primordial nucleosynthesis ($T \geq$ few MeV), we find that for particles which remain tightly coupled to either the neutrinos (e.g., the Majoron⁵) or the photons (e.g., the axion⁶), it is not possible to parameterize their effect by such a simple formula. Such particles also change the prediction for the neutrino-to-photon temperature ratio, making it higher (for particles coupled to neutrinos) or lower (for particles coupled to photons) than the prediction of $T_{\nu}/T_{\gamma} = (4/11)^{1/3}$. In this paper we treat the effect

on primordial nucleosynthesis of a 'generic' particle species, by which we mean a particle which remains in thermal equilibrium with either photons or light neutrinos during the epoch of primordial nucleosynthesis ($T \sim 10 - 0.1$ MeV, $t \sim 0.01$ sec - 300 sec).

In section II we review the pertinent features of the standard model of big-bang nucleosynthesis and continue in section III to discuss the alterations of the standard model which are necessary to include 'generic' particles. Section IV contains the results of primordial nucleosynthesis for several types of generic particles and a discussion of the application of our results to specific particle physics models. Section V contains a summary of our paper and conclusions. Throughout we use units such that $\hbar = c = k = 1$.

II. Review of the Standard Model

The standard model of big-bang nucleosynthesis⁷ can be divided into two phases. During the first phase, characterized by temperatures between 1 and 10 MeV, the weak interaction physics describing the ensemble of electrons, positrons, light neutrinos and antineutrinos, photons, and a small number of nucleons, is most important. For $T >$ few MeV, the rate of weak interactions, $\Gamma_{wk} \sim G_F^2 T^5$, is larger than the expansion rate of the Universe, H , and light neutrinos maintain thermal equilibrium with photons via the weak interactions, e.g., $e^+ e^- \leftrightarrow \nu_i \bar{\nu}_i$. The nucleons also maintain chemical equilibrium via the weak interactions $p \nu_e \leftrightarrow e^+ n$, $n \leftrightarrow p e^- \bar{\nu}_e$, $p e^- \leftrightarrow n \nu_e$, and the neutron-to-proton ratio, (n/p) , follows the equilibrium value

$$\left(\frac{n}{p}\right)_{\text{eq}} = \exp(-\Delta m/T) \quad (2)$$

where $\Delta m = m_n - m_p = 1.29 \text{ MeV}$.

At a temperature of about 3 MeV the weak interactions can no longer keep up with the expansion rate of the Universe, and the neutrinos decouple from the electromagnetic plasma (e^\pm, γ). Thereafter, they freely expand and their phase space distribution is that of a Fermi-Dirac gas with a temperature which redshifts with the expansion: $T \propto R^{-1}$ (provided $m_\nu \lesssim T$).

Neutrons and protons remain in chemical equilibrium until the weak rates for the interactions which interconvert neutrons and protons become comparable to the expansion rate, at which point (n/p) 'freezes-out' at about the equilibrium value for the 'freeze-out temperature', $T_F \sim 0.7 \text{ MeV}$ ($n/p \sim 1/6$). This is not a 'freeze-out' in the strictest sense because (n/p) slowly decreases due to weak interactions (most notably neutron decay) which continue to occur (albeit with a rate less than H) (from $1/6$ to $1/7$) until the onset of the second phase of big-bang nucleosynthesis: nuclear interactions.

The second phase of nucleosynthesis is characterized by $T \lesssim 0.1 \text{ MeV}$ and involves various nuclear interactions between n , p , D , and other light nuclei, ultimately resulting in the production of D , ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$. These reactions occur until their rates drop below the expansion rate. For $T \gtrsim 0.1 \text{ MeV}$, the fractional deuteron abundance is given by its nuclear statistical equilibrium value

$$\frac{n_d}{n_N} \approx \eta (T/m_N)^{3/2} \exp(2.2\text{MeV}/T) \quad (3)$$

where η is the nucleon-to-photon ratio. For $T \gtrsim 0.1$ MeV the large photon-to-baryon ratio results in small equilibrium abundances of D and other light nuclei. When $T \sim 0.1$ MeV, there is sufficient deuterium (and also ${}^3\text{He}$ and t) so that n , p , and d reactions begin to produce ${}^4\text{He}$. The lack of stable mass 5 and 8 nuclei, Coulomb barriers, and the relatively large binding energy of ${}^4\text{He}$ result in most of the available neutrons being transformed into ${}^4\text{He}$.

The primordial yield of ${}^4\text{He}$ depends upon three parameters: the nucleon-to-photon ratio η , the neutron half-life $\tau_{1/2}$, and the expansion rate of the Universe when $T \sim 1$ MeV (often parameterized by the effective number of light neutrino species, N_ν). Since virtually all of the neutrons present at $T \sim 0.1$ MeV are synthesized into ${}^4\text{He}$, the amount of ${}^4\text{He}$ produced in the big-bang is approximately related to (n/p) at freeze out by

$$Y_p \approx 2 \left(\frac{n}{p}\right)_F / [1 + \left(\frac{n}{p}\right)_F] \quad (4)$$

where Y_p is the primordial mass fraction of ${}^4\text{He}$ produced. The earlier weak interactions freeze out, the larger $(n/p)_F$ and hence the larger Y_p . Since the expansion rate increases with more relativistic degrees of freedom ($H \propto (\text{const.} + N_\nu)^{1/2}$), and the interaction rate for the reactions which interconvert neutrons and protons decreases with increasing $\tau_{1/2}$ ($\Gamma_{wk} \propto \tau_{1/2}^{-1}$), increasing either N_ν or $\tau_{1/2}$ results in an earlier

freeze out and a larger value of (n/p) . Thus Y_p increases with increasing N_ν and $\tau_{1/2}$. Y_p also depends, although in a weaker way, on the time of onset of the nuclear interactions which convert neutrons to ${}^4\text{He}$. As the number density of nucleons increases, the various nuclear bottlenecks break at higher temperature where (n/p) has a larger value, thus resulting in the increase of Y_p with increasing η .

The primordial production of D and ${}^3\text{He}$ depends upon the efficiency of ${}^4\text{He}$ production. Nearly all D and ${}^3\text{He}$ is processed into ${}^4\text{He}$ and thus their abundance depends upon the nuclear interaction rate-expansion rate interplay. Higher nucleon abundances imply faster D and ${}^3\text{He}$ depletion, while a faster expansion rate leads to an earlier freeze-out of nuclear interactions, when the D and ${}^3\text{He}$ abundances are larger. Therefore D and ${}^3\text{He}$ decrease with increasing η or decreasing N_ν .

The yield of primordial ${}^7\text{Li}$ is determined by a competition between the production and destruction reactions. For low nucleon abundances ($\eta \leq 3 \times 10^{-10}$), ${}^7\text{Li}$ is produced via ${}^3\text{He}(t,\gamma){}^7\text{Li}$ and destroyed by ${}^7\text{Li}(p,\gamma){}^4\text{He}$. The final abundance is determined by the eventual freeze-out of the destruction reaction, hence the decrease of primordial ${}^7\text{Li}$ with increasing η at low η (an equivalent effect results from increasing N_ν and thus the expansion rate). For $\eta \geq 3 \times 10^{-10}$, ${}^7\text{Li}$ results from the decay of ${}^7\text{Be}$ (via electron capture), which is produced in the reaction ${}^4\text{He}({}^3\text{He},\gamma){}^7\text{Be}$. Increasing η results in a larger rate of ${}^7\text{Be}$ production, and thus the increase in ${}^7\text{Li}$ with increasing η for $\eta \geq 3 \times 10^{-10}$.

The production of D, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ is summarized for the standard model of nucleosynthesis in Figure 1. We shall use $\tau_{1/2} = 10.6$ min and $N_\nu = 3$ as our canonical values for these parameters.

III. Including Generic Particles

We have altered the code of Wagoner⁸ to take into account the presence of a generic particle species. We will describe the physics of those changes here. A generic species i , which is always in thermal equilibrium with the photons, has an energy density and pressure given by

$$\rho_i(T_\gamma) = \left(\frac{g_i T_\gamma^4}{2\pi^2} \right) \int_{y_i}^{\infty} \frac{\xi^2 (\xi^2 - y_i^2) d\xi}{e^{\xi + \theta_i}} \quad (5)$$

and

$$p_i(T_\gamma) = \left(\frac{g_i T_\gamma^4}{6\pi^2} \right) \int_{y_i}^{\infty} \frac{(\xi^2 - y_i^2)^{3/2} d\xi}{e^{\xi + \theta_i}} \quad (6)$$

where g_i is the number of spin states (e.g., $g_e = 2$, $g_\nu = 1$, ...), T_γ the photon temperature, $y_i = m_i/T_\gamma$, and θ_i is $+(-)1$ for fermions (bosons). Similar expression follow for a species j which is always in thermal equilibrium with the neutrinos (with $T_\gamma \rightarrow T_\nu$).

For $T \sim 10$ MeV, the entropy of the Universe is dominated by that of the photons, electrons, light neutrinos, and the generic particles. The entropy per comoving volume, $S = R^3(t)s$, ($R(t)$ is the FRW scale factor, and s the entropy density) can be written

$$S = \frac{R^3}{T_\gamma} \left[2(\rho_e + p_e) + 2N_\nu(\rho_\nu + p_\nu) + \rho_\gamma + p_\gamma + \sum_k (\rho_k + p_k) \right] \quad (7)$$

with the subscripts e , γ , and ν , referring to the contributions of the electrons, photons, and neutrinos, respectively. The factor of 2

results from consideration of particles and anti-particles, and the sum is over generic species which maintain good thermal contact with either photons or light neutrinos.

For $T \gtrsim 3 \text{ MeV}$,⁹ light neutrinos and photons maintain good thermal contact via neutral and charged current weak interactions and hence, $T_\gamma = T_\nu$. Until this decoupling we have,

$$S = \text{const.} \quad (T \gtrsim T_{\text{dec}} \sim 3 \text{ MeV}). \quad (8)$$

For $T \lesssim 3 \text{ MeV}$, the light neutrinos and any generic species j decouple from the electromagnetic plasma and the generic species i . Thereafter the electromagnetic and neutrino entropy per comoving volume are separately conserved:

$$S_\gamma = \frac{R^3}{T_\gamma} [2(\rho_e + p_e) + \rho_\gamma + p_\gamma + \sum_i (\rho_i + p_i)] = \text{const.} = S_\gamma(T_\gamma = T_{\text{dec}}), \quad (9a)$$

and

$$S_\nu = \frac{R^3}{T_\nu} [2N_\nu(\rho_\nu + p_\nu) + \sum_j (\rho_j + p_j)] = \text{const.} = S_\nu(T_\nu = T_{\text{dec}}), \quad (9b)$$

where the sums are now taken only over generic particles coupled to photons, in the case of S_γ , and neutrinos, in the case of S_ν .

Substituting expressions (5) and (6) into eqn. (9) we can write

$$S_\nu = \frac{4\pi^2}{45} I_\nu (y_j) (RT_\nu)^3 \quad (10a)$$

and

$$S_\gamma = \frac{4\pi^2}{45} I_\gamma (y_i) (RT_\gamma)^3 \quad (10b)$$

where

$$I_\nu (y_j) = \frac{15}{8\pi^4} \sum_j g_j F(y_j) + \frac{7}{8} N_\nu , \quad (11a)$$

$$I_\gamma (y_i) = \frac{15}{8\pi^4} \sum_i g_i F(y_i) + 1 , \quad (11b)$$

and the function $F(y)$ is given by

$$F(y) = \int_y^\infty \frac{(4\xi^2 - y^2)(\xi^2 - y^2)^{1/2} d\xi}{e^{\xi + \theta}} \quad (12)$$

which has the limiting form for small y (i.e., $T \gg m$) of $7\pi^4/30$ ($4\pi^4/15$) for fermions (bosons).

Eqns. (9 and 10) can be used to compute the neutrino-to-photon temperature ratio for $T_\gamma \leq T_{\text{dec}}$:

$$\left(\frac{T_\nu}{T_\gamma}\right)^3 = \left(\frac{I_\nu}{I_\gamma}\right) \Big|_{T_{\text{dec}}} / \left(\frac{I_\nu}{I_\gamma}\right) \Big|_T \quad (13)$$

In particular, if none of the generic particles have mass $\sim T \sim$ few MeV, I_γ and I_ν take on particularly simple forms:

$$I_\gamma(T) = 1 + 1/2 \sum_{m_i < T} g_{\text{eff } i} \quad (14a)$$

$$I_\gamma(T_{\text{dec}}) = \frac{11}{4} + \frac{1}{2} \sum_{m_i < T_{\text{dec}}} g_{\text{eff } i} \quad (14b)$$

$$I_\nu(T) = \frac{7}{8} N_\nu + \frac{1}{2} \sum_{m_j < T} g_{\text{eff } j} \quad (15a)$$

$$\text{and } I_\nu(T_{\text{dec}}) = \frac{7}{8} N_\nu + \frac{1}{2} \sum_{m_j < T_{\text{dec}}} g_{\text{eff } j} \quad (15b)$$

where as usual, $g_{\text{eff}} = g$ (for bosons) and $7/8 g$ (for fermions).

In the standard scenario (no new generic particles), the present ratio of the neutrino temperature to the photon temperature is given by

$$\left(\frac{T_\nu}{T_\gamma}\right)_{\text{today}}^3 = \left(\frac{I_\nu}{I_\gamma}\right)_{T_{\text{dec}}} \left(\frac{I_\gamma}{I_\nu}\right)_{\text{today}} = \left(\frac{7/8 N_\nu}{11/4}\right) \left(\frac{1}{7/8 N_\nu}\right) = \frac{4}{11} \quad (16)$$

the familiar result. Now consider the effect of an axion-like generic particle (i.e., a $g = 1$ boson coupled to photons). In this case

$$\left(\frac{T_\nu}{T_\gamma}\right)_{\text{today}}^3 = \left(\frac{7/8N_\nu}{11/4 + 1/2}\right) \left(\frac{1}{7/8N_\nu}\right) = \frac{4}{13} \quad (17)$$

A comment is in order at this point. In computing $(T_\nu/T_\gamma)^3$ for this example, we assumed that the entropy in the axion-like particle was transferred to the photons and thermalized by the present epoch. This requires that $m \geq 3^\circ\text{K}$ so that the entropy transfer occurs by the present epoch. If thermalization of the photons takes place as usual via the double Compton process,¹⁰ then the entropy transfer must occur at $t \leq 10^8$ sec, corresponding to $m \geq 100$ eV for an axion-like particle.

Finally consider the effect on (T_ν/T_γ) of a generic particle which couples to neutrinos. An example is the Majoron model of Gelmini and Roncadelli.¹¹ In this model there are two 'generic' particles - the massless Majoron ($g = 1$ boson) and a light Higgs boson (also $g = 1$). These two particles couple to the neutrinos with 'stronger than weak' interactions. In one version of the model, one of the three neutrino species is heavy, having a mass ≤ 0 (100 keV), while the other two are light (\ll keV). Due to the interactions mediated by the Majorons, neutrinos and the new particles remain coupled. Today, the species still present are the two light neutrinos and the Majoron - the other species having transferred their entropy to these particles. In this model the present neutrino-to-photon temperature ratio is then:

$$\left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{4}{11} \cdot \frac{1 + 21/8}{2 + 7/4} = \frac{58}{99} \quad (18)$$

Using eqns. (9-12) we can describe the dynamical evolution of the Universe as a function of the photon temperature. Since S_γ is conserved after decoupling, the scale factor is given by

$$R(T_\gamma) = \frac{(RT)_{\text{dec}}}{T_\gamma} \left[\frac{I_\gamma(T_{\text{dec}})}{I_\gamma(T)} \right]^{1/3}. \quad (19)$$

The neutrino temperature is given in terms of the photon temperature by eqn. (14). Knowing T_γ and T_ν we can calculate the energy density of the Universe and therefore the expansion rate H ,

$$H^2 = (\dot{R}/R)^2 = \frac{8\pi G}{3} \rho_{\text{TOT}} \quad (20)$$

where $\rho_{\text{TOT}} = \sum_i \rho_i(T_\gamma) + \sum_j \rho_j(T_\nu)$.

The age of the Universe of course simply follows as

$$t = \int \frac{dR}{\dot{R}}. \quad (21)$$

The weak interaction rates, which determine (n/p) are a function of both T_γ and T_ν through the phase space dependence of the initial and final states of the interactions $p\nu_e \leftrightarrow ne^+$, $p e^- \leftrightarrow n\bar{\nu}_e$, and $n \leftrightarrow p e^-\bar{\nu}_e$, and the number densities of e^\pm , ν_e and $\bar{\nu}_e$. We have supplemented Wagoner's code by numerically calculating these interaction rates,

including radiative and Coulomb corrections,¹² and allowing the ratio T_ν/T_γ to evolve according to eqn. (13).

IV. Nucleosynthesis with Generic Particles

By assumption, a given generic particle species maintains thermal equilibrium during the epoch of nucleosynthesis with either photons or light neutrinos. A generic species is thus completely specified by its mass, degrees of freedom, spin statistics, and whether it couples to photons or neutrinos. In figures 2-5, we show the ${}^4\text{He}$, D, D + ${}^3\text{He}$, and ${}^7\text{Li}$ yields for primordial nucleosynthesis with generic particles coupled either to photons or light neutrinos, having $g = 1$ and 2 , and in the mass range $10^{-2} \lesssim m/m_e \lesssim 10^2$. All calculations were done with $\eta = 3 \times 10^{-10}$, $\tau_{1/2} = 10.6$ min., and with $N_\nu = 3$. [Following Yang, et al.,¹ we display D + ${}^3\text{He}$ and D rather than D and ${}^3\text{He}$ separately. These two primordial abundances prove to be more useful than D and ${}^3\text{He}$ in using primordial nucleosynthesis as a probe of the early Universe.] Horizontal lines indicate standard model yields for the indicated number of light neutrino species. We have also computed the yields of primordial nucleosynthesis in the presence of generic particles for other values of η and $\tau_{1/2}$, and the results are qualitatively similar. That is, the equivalent number of additional neutrino species required to produce the same effect as a generic particle of a given mass and g is insensitive to η and $\tau_{1/2}$.

The $g = 2$ generic particle is a fermion and thus one would naively expect that it would behave like a single additional neutrino species. The $g = 1$ generic particle is a boson and one would expect that it would behave as $4/7$ of an additional neutrino species. In general, our

results show that the naive estimate that a particle's effect on primordial nucleosynthetic yields is the same as the equivalent number of light neutrinos can be incorrect. For example, a $g = 2$ fermion coupled to light neutrinos acts like an extra neutrino species only for $m \lesssim m_e$.

The dependence of Y , $D + {}^3\text{He}$, and ${}^7\text{Li}$ on the generic particle mass are most easily understood by considering three mass regions:

- (I) $m \geq 30 \text{ MeV}$
- (II) $0.5 \text{ MeV} \lesssim m \lesssim 10 \text{ MeV}$
- (III) $m \lesssim 50 \text{ keV}$

Region I - We recover the standard model ($N_\nu = 3$) for $m \geq 30 \text{ MeV}$ since particles of this mass range are present only in trivial abundances at the time of primordial nucleosynthesis, and the entropy transfer from the generic species takes place well before nucleosynthesis.

Region III - Generic particles coupled to photons with masses $\lesssim 50 \text{ keV}$ increase ${}^4\text{He}$ and ${}^7\text{Li}$ production and decrease $D + {}^3\text{He}$ production relative to an additional light neutrino species for $g = 2$, or $4/7$ of a neutrino species for $g = 1$. The reason for this is simple: they transfer their entropy after nucleosynthesis, necessitating a larger value of η during nucleosynthesis, thereby increasing the yield of ${}^4\text{He}$ and ${}^7\text{Li}$ and decreasing the yield $D + {}^3\text{He}$.¹³ Generic particles coupled to neutrinos transfer their entropy after photons and neutrinos decouple and hence do not alter η . Thus such particles with masses below 50 keV

act only to increase the energy density of the Universe. Therefore, in Region III, the usual formula of eqn. (1) is valid for generic particles coupled to neutrinos, i.e., the $g = 1$ boson is characterized by $\Delta N_\nu = 4/7$ and the $g = 2$ fermion by $\Delta N_\nu = 1$.

Region II - In this region, things are a bit more complicated. Our results are most easily understood in terms of the energy density of the Universe when generic particles are included, relative to models without generic particles. In what follows, we show that it is possible to reduce the energy density of the Universe relative to that of the standard 3 light neutrino model by including a massive generic particle coupled to photons and that it is also possible to increase the relative energy density of the Universe by replacing a light neutrino with a massive generic particle coupled to neutrinos. Therefore, in this mass region, one expects the effect of a massive generic particle coupled to photons to be less than 3 light neutrino species, and that the effect of a generic particle coupled to neutrinos to be more than 4 neutrino species (for a $g = 2$ fermion), and more than $3 + 4/7$ of a neutrino species (for the $g = 1$ boson). In both cases the expected effect is observed.

We need to consider the energy density as a function of T_γ to understand the effect of generic particles in this mass range. The energy density of the Universe containing a generic particle coupled to photons can be written:

$$\rho(T_\gamma) = a_1 T_\gamma^4 + a_2 (T_\nu/T_\gamma)^4 T_\gamma^4 + f(y) T_\gamma^4 \quad (22)$$

where the first term accounts for electrons and the photons, the second

for light neutrinos, and the third for the generic particles with $y = m/T_\gamma$ and $f(y)$ defined as T_γ^{-4} times eqn. (5). On the other hand, the energy density of the standard model is just:

$$\rho_{\text{STD}}(T_\gamma) = a_1 T_\gamma^4 + a_2 (T_\nu/T_\gamma)_{\text{STD}}^4 T_\gamma^4 \quad (23)$$

For a given T_γ we can have $\rho < \rho_{\text{STD}}$, even though we have an additional particle, provided that

$$a_2 (T_\nu/T_\gamma)^4 + f(y) < a_2 (T_\nu/T_\gamma)_{\text{STD}}^4 \quad (24)$$

In order for this to occur

$$f(y) < a_2 [(T_\nu/T_\gamma)_{\text{STD}}^4 - (T_\nu/T_\gamma)^4] \quad (25)$$

It is possible for particles in the mass range 0.5 MeV - 10 MeV to satisfy this inequality leading to $\rho < \rho_{\text{STD}}$, which causes an underproduction of ${}^4\text{He}$, D, and D + ${}^3\text{He}$ relative to the standard model ($N_\nu = 3$). That is, adding a generic particle in this mass range actually has the effect of reducing N_ν . The effect is apparently not large enough to show up in ${}^7\text{Li}$ production.

Now consider the effect of a generic particle which couples to neutrinos on the energy density. Again we write

$$\rho = a_1 T_\gamma^4 + a_2 (T_\nu/T_\gamma)^4 + f(y) (T_\nu/T_\gamma)^4 T_\gamma^4 \quad (26)$$

with $y = m/T_\nu$. Consider the energy density of the standard model with an

extra light neutrino species, i.e., $N_\nu = 4 (a_2 \rightarrow a_2 + 7/8$ in eqn. (22). The criteria for $\rho > \rho_{\text{STD}+1}$ is just

$$[a_2+f(y)]/(a_2+7/8) > (T_\nu/T_\gamma)^4/(T_\nu/T_\gamma)_{\text{STD}+1}^4 \quad (27)$$

For a $g = 2$ fermion, $0 \leq f(y) \leq 7/8$, and there exists a mass range of generic particles which heat the neutrinos by annihilation in such a way that $\rho > \rho_{\text{STD}+1}$. That is, adding an additional generic particle in this mass range can actually have the effect of increasing N_ν by more than one unit for the $g = 2$ fermion and more than $4/7$ for the $g = 1$ boson. The effect on Y_p is not as large as it might be because there is a compensating effect - the entropy release by the generic particle increases T_ν/T_γ , which increases the weak rates which interconvert neutrons and protons thereby lowering the neutron-to-proton ratio somewhat. For these generic particles, we observe overproduction of ${}^4\text{He}$, $\text{D} + {}^3\text{He}$, and ${}^7\text{Li}$, relative to their standard model plus an additional light neutrino counterpart.

In order to use the results of generic particle primordial nucleosynthesis to place constraints on a given particle physics model, it is necessary to consider a few points. To qualify as a generic particle, the interactions of a new particle with light neutrinos or photons must be fast enough to keep the particle in thermal equilibrium throughout primordial nucleosynthesis. If this is the case, we then use the fact that Yang et al.¹ have shown that the standard model of big-bang nucleosynthesis with 4 light neutrinos is barely compatible with the observations of the present abundances of D , ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$, and therefore, if the effect of adding the generic particle is greater

than the effect of an additional neutrino species in the standard model, that generic particle is 'ruled out' by primordial nucleosynthesis.

V. Concluding Remarks

We have modified the standard scenario of primordial nucleosynthesis to accommodate a generic particle which is in good thermal contact with either the photons or the neutrinos during primordial nucleosynthesis. We show that the addition of such a particle alters the usual prediction for the ratio of the relic neutrino and photon temperatures. In addition we have shown that the effect of such a species on primordial nucleosynthesis is in general very different than that of an additional particle species which decoupled before nucleosynthesis (that is, it is not well approximated by an equivalent number of light neutrino species).

VI. Acknowledgements

TPW would like to thank G. Steigman for useful comments and discussions. This work was supported in part by the DOE (at Chicago and Fermilab), NASA (at Fermilab), and MST's Alfred P. Sloan Fellowship.

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9. The actual neutrino decoupling temperature is slightly different for each species. See D. A. Dicus, E. W. Kolb, A. M. Gleeson, E. C. G. Sudarshan, V. L. Teplitz, and M. S. Turner, *Phys. Rev.* **D26**, 2694(1982).
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11. See Ref. 5.
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13. As mentioned in section III, particles of $m \leq 100$ eV which couple to photons transfer their entropy to photons too late for thermalization to occur by the present epoch, and thus n would be unaltered. Therefore, the effect of $m < 100$ eV generic particles coupled to photons throughout nucleosynthesis is only due to their energy density, and thus they produce the effect of an equivalent number of neutrino species. The curves of figures 2-5 should coincide with the analogous curves for generic particles coupled to neutrinos below $m/m_e \leq 10^{-3}$. Of course, the decays of such particles will distort the microwave background and therefore are cosmologically unacceptable (see Ref. 10).
14. In order to determine whether or not a generic species maintains thermal equilibrium with photons or light neutrinos we should solve the rate equation $\dot{n} = -3Hn + \langle\sigma v\rangle(n_{eq}^2 - n^2)$ where n is the number density of the species and n_{eq} is the equilibrium number density

(see, e.g., S. Wolfram, Nucl. Phys. **B82**, 65(1979); G. Steigman, Ann. Rev. Nucl. Part. Sci. **29**, 313(1979); J. Berenstein, L. Brown, and G. Feinberg, Phys. Rev. **D32**, 3261(1985); R. Scherrer and M. Turner, Phys. Rev. **D33**, 1585 (1986). Our approximate method will suffice to illustrate the point.

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Figure Captions

Figure 1: The primordial production of D, D + ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ as a function of η , the baryon-to-photon ratio, for $N_\nu = 3$, and $\tau_{1/2} = 10.6$ min (from Yang, et al., Ref. 1).

Figure 2,3,4,5: The primordial production of (D, D + ${}^3\text{He}$, ${}^4\text{He}$, ${}^7\text{Li}$), for $\eta = 3 \times 10^{-10}$, $\tau_{1/2} = 10.6$ min, $N_\nu = 3$, and the indicated generic species, as a function of the mass of the generic species (in units of the electron mass). Solid curves are for a generic species coupled to photons; dashed-dot curves are for a generic species coupled to neutrinos. The $g = 2$ curves are for a fermionic species, and the $g = 1$ curves are for a bosonic species. For reference, the horizontal lines indicate the production for the indicated number of light neutrino species and no generic particles.

APPENDIX

The feature which distinguishes generic particles from that of decoupled particles previously considered is that they remain in good thermal contact with either photons or light neutrinos throughout the epoch of primordial nucleosynthesis. In order for this to be so, the interactions between a generic particle and photons or light neutrinos must occur at a rate faster than the expansion rate of the Universe for $T \geq 100$ keV.¹⁴ That is,

$$\Gamma_{\text{eq}} > H \quad T \geq 100 \text{ keV} \quad (\text{A.1})$$

where $H = \dot{R}/R$ can be written

$$H \approx \frac{1.7}{m_{\text{Pl}}} g_*^{1/2} T^2, \quad (\text{A.2})$$

where $m_{\text{Pl}} = 1.22 \times 10^{19} \text{ GeV}$ and $g_* = \frac{7}{8} \sum_{\text{Fermi}} g_k (T_k/T_\gamma)^4 + \sum_{\text{Bose}} g_k (T_k/T_\gamma)^4$, is 10.75 at $T \sim \text{MeV}$ and 3.4 for $T \leq \text{few } 100 \text{ keV}$ (assuming the standard model with 3 light neutrino species).

First consider criteria (A.1) for the heavy neutrinos of the Majoron model of Gelmini and Roncadelli⁵. Heavy and light neutrinos maintain thermal equilibrium via interactions which are mediated by the massless Goldstone boson of the theory, the Majoron. In particular we consider the process $\nu_H \bar{\nu}_H \rightarrow (M^0) \rightarrow \nu_L \bar{\nu}_L$ which has a cross section

$$\sigma = \frac{g_H^2 g_L^2}{32\pi} \left(\frac{1}{\beta s} \right) \quad (\text{A.3})$$

where $g_{H(L)}$ is the heavy (light) neutrino-Majoron coupling, \sqrt{s} the c.m. energy, and β the relative velocity. The equilibrium number density of heavy neutrinos takes two limiting forms:

$$n_{\nu_H} \sim \begin{cases} \frac{3\zeta(3)}{2\pi^2} T_\nu^3 & (T_\nu \gg m_\nu) \\ 2\left(\frac{m_\nu T_\nu}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\nu}{T_\nu}\right) & (T_\nu \ll m_\nu) \end{cases} \quad (\text{A.4})$$

The rate which controls the equilibrium abundance of heavy neutrinos is the annihilation rate of $\nu_H \bar{\nu}_H \rightarrow \nu_L \bar{\nu}_L$ ¹⁵, $\Gamma_{\text{ann}} = n_{\nu_H} \sigma \beta$, and using eqn. (A.4)

$$\Gamma_{\text{ann}} = \frac{g_H^2 g_L^2}{32\pi} \begin{cases} \frac{3\zeta(3)}{4\pi^2} T_\nu & (T_\nu \gg m_\nu) \\ \frac{1}{2} \frac{(m_\nu T_\nu)^{3/2}}{2\pi} \frac{\exp(-m_\nu/T_\nu)}{m_\nu^2} & (T_\nu \ll m_\nu) \end{cases} \quad (\text{A.5})$$

and the heavy neutrino-Majoron-light neutrino system remains tightly coupled throughout primordial nucleosynthesis provided $\Gamma_{\text{ann}} \geq H$.

In order to use the results of generic particle nucleosynthesis, we need the heavy neutrinos and Majorons to remain tightly coupled to light neutrinos down to $T \sim 100$ keV. Using the fact that neutrino masses in the Majoron model are given by $m_{\nu_i} = g_i v_T$ ($i = H, L$), where v_T is the vacuum expectation value of the Higgs triplet, the constraint $\Gamma_{\text{ann}} \geq H$ for $T \geq 0.1$ MeV yields the following conditions on the couplings g_H and g_L

$$\left. \begin{array}{l} g_H^2 g_L^2 \\ \\ \frac{g_H^2 g_L^2}{(g_H v_T)^{1/2}} e^{-10 g_H v_T} \end{array} \right\} \geq 10^{-19} \quad (A.6)$$

$(g_H < 0.1 \text{MeV}/v_T)$
 $(g_H > 0.1 \text{MeV}/v_T)$

which can easily be met for reasonable values of g_i .

We point out that in models such as this where particles acquire mass through the spontaneous breaking of a symmetry at a critical temperature $T_c \sim v_T$, the analysis of this appendix only applies if $v_T \geq 100$ keV. That is, the phase transition which breaks the symmetry must occur at an energy scale larger than that associated with primordial nucleosynthesis. For $v_T \leq 100$ keV, all the generic particles are massless during nucleosynthesis, and one need only count the equivalent number of light neutrino species (in the GR model, 3 light neutrinos and 2 scalar bosons) present during primordial nucleosynthesis.¹⁶ For the Majoron model, limits from stellar evolution seem to imply $v_T \leq O(100 \text{ keV})$ ¹⁷ and so special care needs to be taken in evaluating the effects of Majorons on big-bang nucleosynthesis.¹⁸

Thus, for the right choice of couplings, the heavy neutrinos of the Majoron model can be considered as generic particles, provided the vev of the triplet is such that the symmetry breaking giving masses to the neutrino and Higgs occurs before the epoch of nucleosynthesis.

Next we consider when the axion, introduced to solve the strong CP problem⁶, qualifies as a generic particle. At energies of order an MeV, axions can maintain thermal contact with photons via Compton-like processes with electrons, $Ae \leftrightarrow Ye$, or by the 2-photon process $A \leftrightarrow 2\gamma$. We will show that for the standard axion-electron coupling, the axion does not stay in equilibrium with photons during primordial nucleosynthesis unless it is quite heavy, but that if this coupling can be suppressed, axion-like particles can be considered as generic particle candidates, since they can maintain equilibrium with photons via $A \leftrightarrow 2\gamma$ processes.

For the $A \leftrightarrow 2\gamma$ interaction, the condition $\Gamma \geq H$ can be expressed

$$\gamma\tau(A \rightarrow 2\gamma) \lesssim 1/H \quad (\text{A.7})$$

where $\gamma = E_A/m_A$ and $\tau(A \rightarrow 2\gamma)$ is the axion lifetime for decay into 2γ . Using current algebra methods¹⁹, the $A \rightarrow 2\gamma$ lifetime and the axion mass have been computed:

$$\tau(A \rightarrow 2\gamma) = \left(\frac{m_\pi}{m_A}\right)^5 \frac{1}{Z} \tau(\pi^0 \rightarrow 2\gamma) \quad (\text{A.8})$$

and

$$m_A = N \frac{2\sqrt{Z}}{1+Z} \frac{f_\pi m_\pi}{f_A}, \quad (\text{A.9})$$

where f_A is the Peccei-Quinn symmetry breaking scale, $f_\pi \sim 95$ MeV is the

pion decay constant, m_π the pion mass, N the number of generations, $z = m_u/m_d \sim 0.6$, and $\tau(\pi^0 \rightarrow 2\gamma) = 8 \times 10^{-17}$ s. With $N=3$ we can write the $A \rightarrow 2\gamma$ lifetime in terms of f_A as

$$\tau(A \rightarrow 2\gamma) \approx 10^{-1} \left(\frac{f_A}{250 \text{ GeV}} \right)^5 \text{ sec} \quad (\text{A.10})$$

The constraint that the axion lifetime be shorter than $1/H$ requires that $f_A \lesssim 400$ GeV.

The cross section for $Ae \rightarrow \gamma e$ is

$$\sigma |v| = \left\{ \frac{\alpha g^2}{8m_e E_A} \frac{2m_A^2}{s} + \frac{4m_e^2 m_A^2}{s(s-m_e^2)} + \frac{4m_A^2}{s-m_e^2} - \frac{4(s-m_e^2)}{s} + \frac{(s+m_e^2)(s+m_A^2-m_e^2)}{s^2} + 2 \frac{W^2+m_A^4}{W(s-m_e^2)} \ln \left(\frac{s+m_e^2-m_A^2+W}{s+m_e^2-m_A^2-W} \right) \right\} \quad (\text{A.11})$$

where $|v|$ is the axion velocity and E_A the axion energy, and $W^2 = s^2 + m_A^4 + m_e^4 - 2s(m_e^2 + m_A^2) - 2m_e^2 m_A^2$. The standard axion-electron coupling, g , is given by $g = m_e/f_A$. The constraint that axions remain in thermal contact with electrons via Compton-like scatterings then requires $f_A \lesssim 10^4$ GeV. However, limits obtained on the axion-electron coupling based upon stellar evolution theory require $f_A \geq 10^8$ GeV or $f_A \lesssim 200$ GeV so that axion emission not dominate stellar energy loss.²⁰ We then conclude that the standard axion cannot be considered a generic particle, unless it is heavy enough that its production in stars is Boltzmann-suppressed. However an axion which does not couple to electrons in the standard way need only satisfy $f_A \lesssim 400$ GeV to qualify as a generic particle.

We note that recent evidence of correlated $e^+ - e^-$ emission in heavy-ion collisions²¹ might be explained by the existence of a 'pseudo-axion' having a mass of 1.68 MeV and decaying to e^+e^- with a lifetime of $\sim 10^{-12}$ sec²². Such an axion would remain in thermal equilibrium with photons throughout primordial nucleosynthesis. The effect of this 'pseudo-axion' would be to reduce the amount of ${}^4\text{He}$ produced in the big-bang by about $\Delta Y \approx 0.002$ (see the solid $g=1$ curve of figure 4).

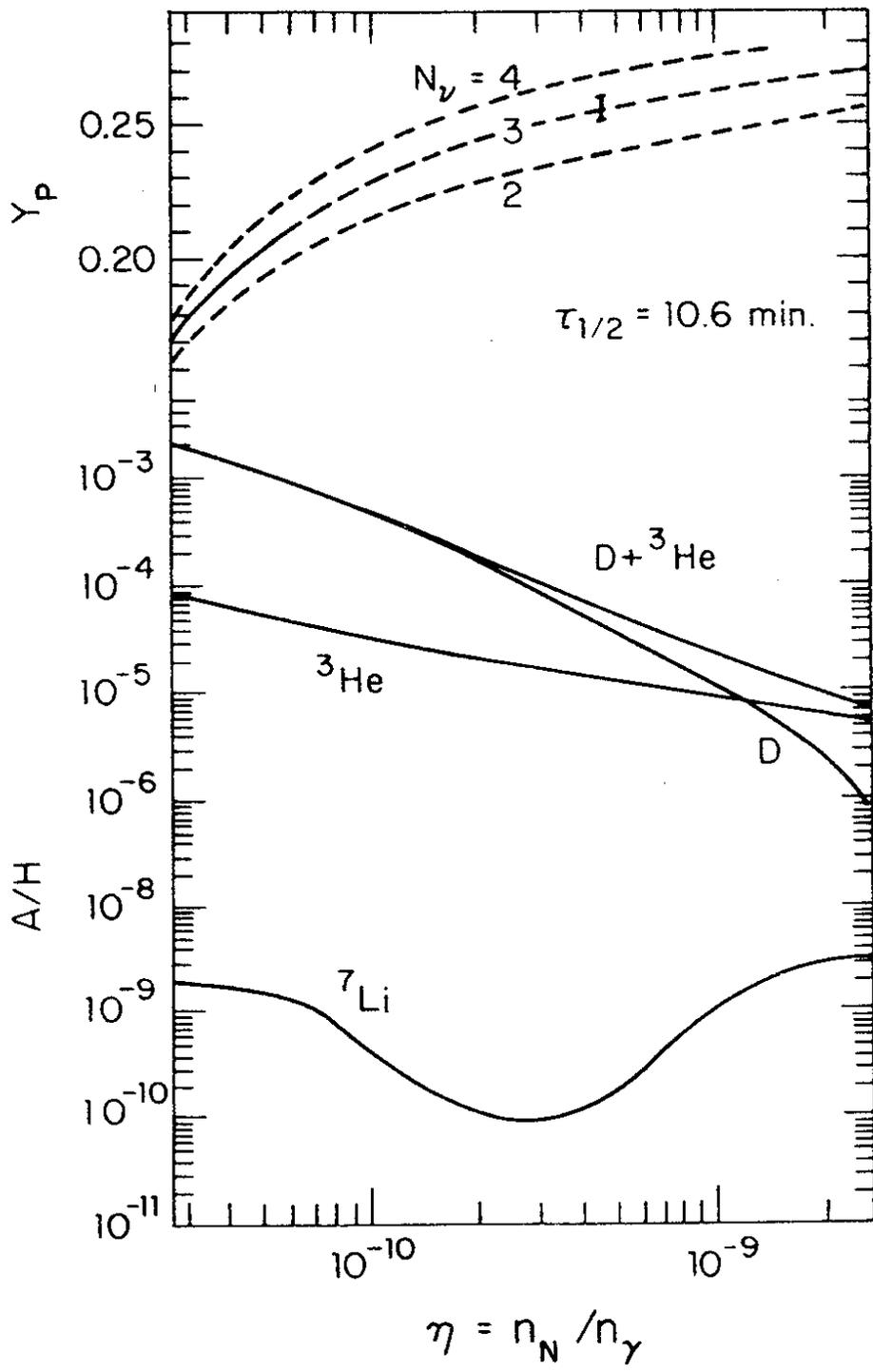


Fig. 1

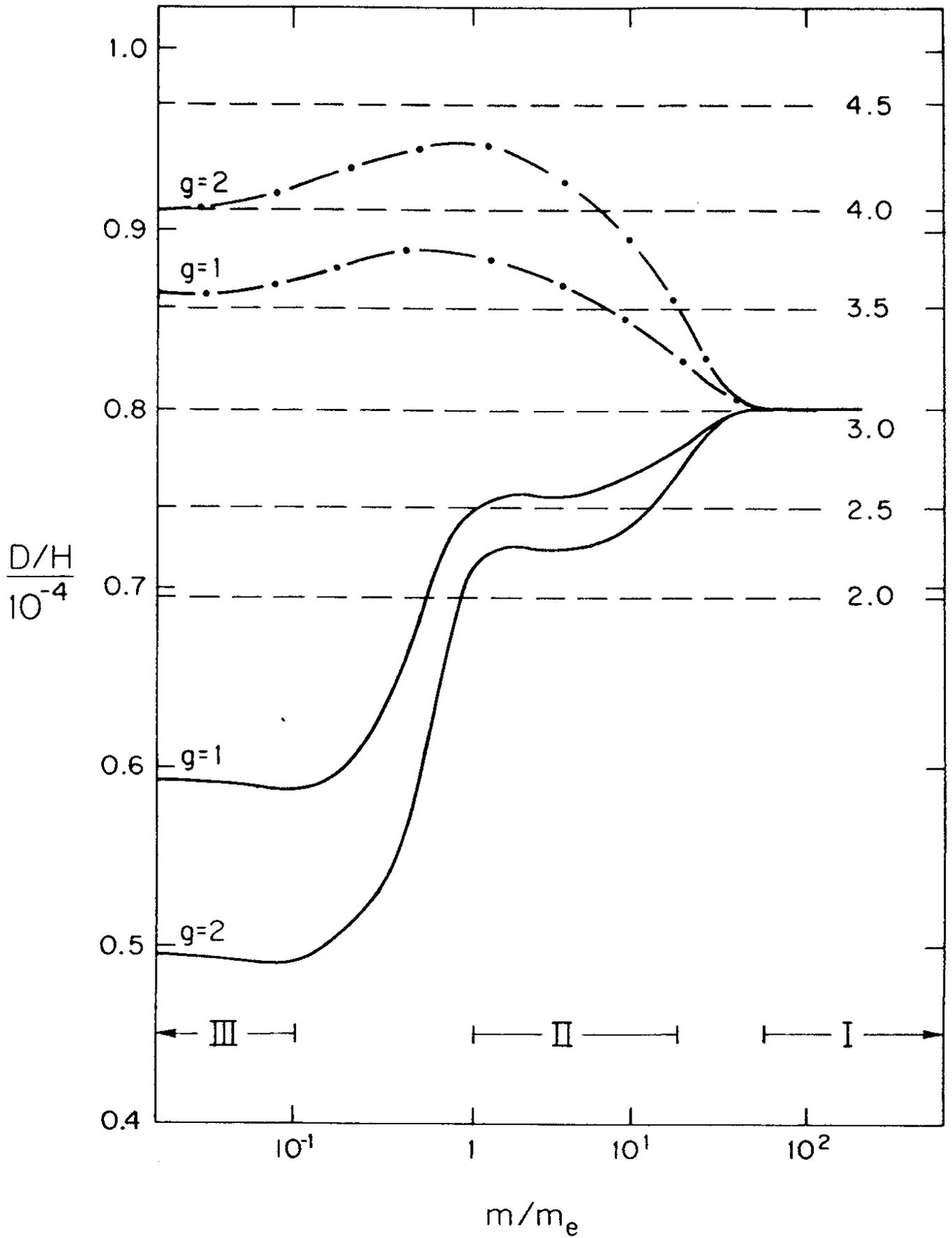


Fig.2

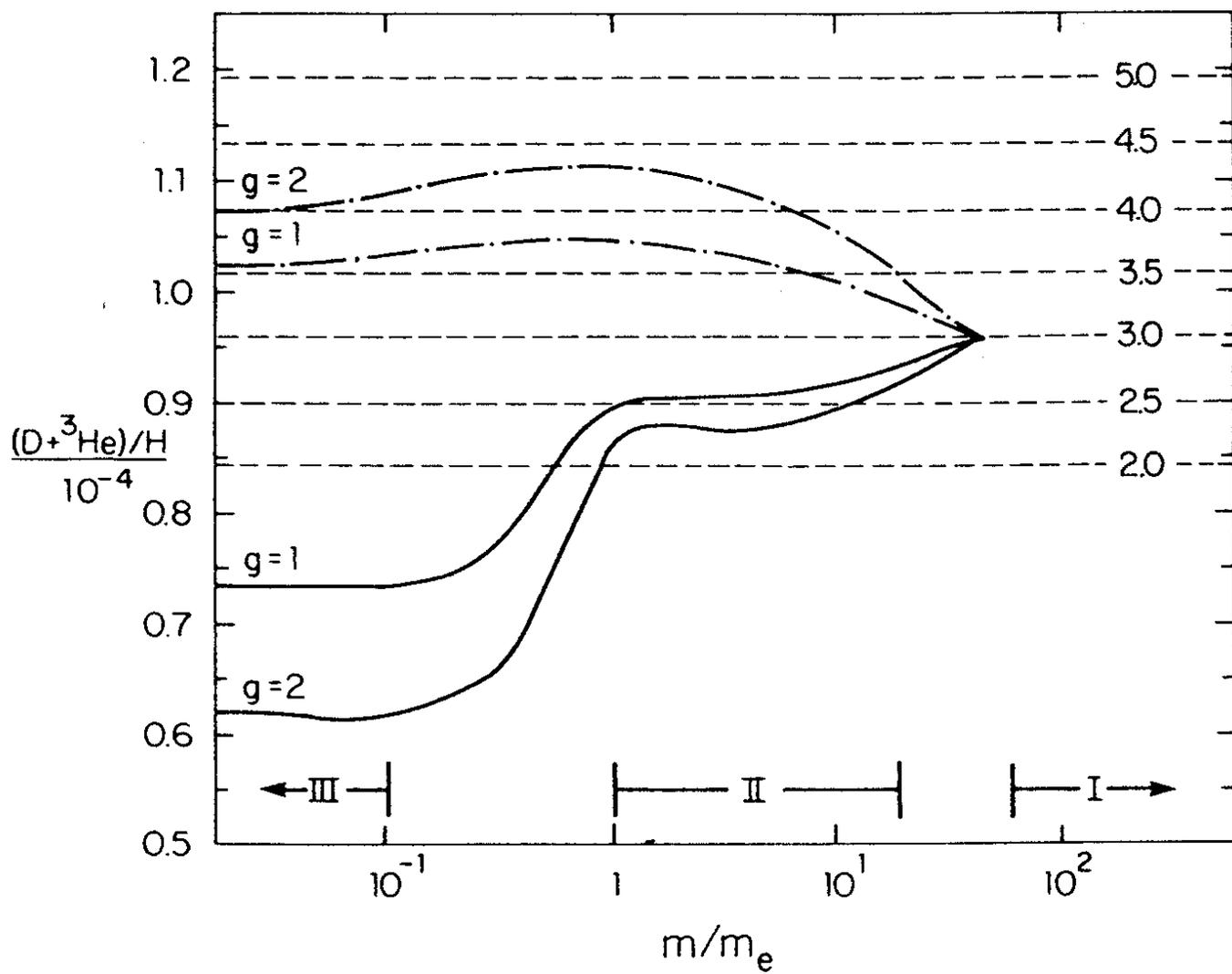


Fig.3

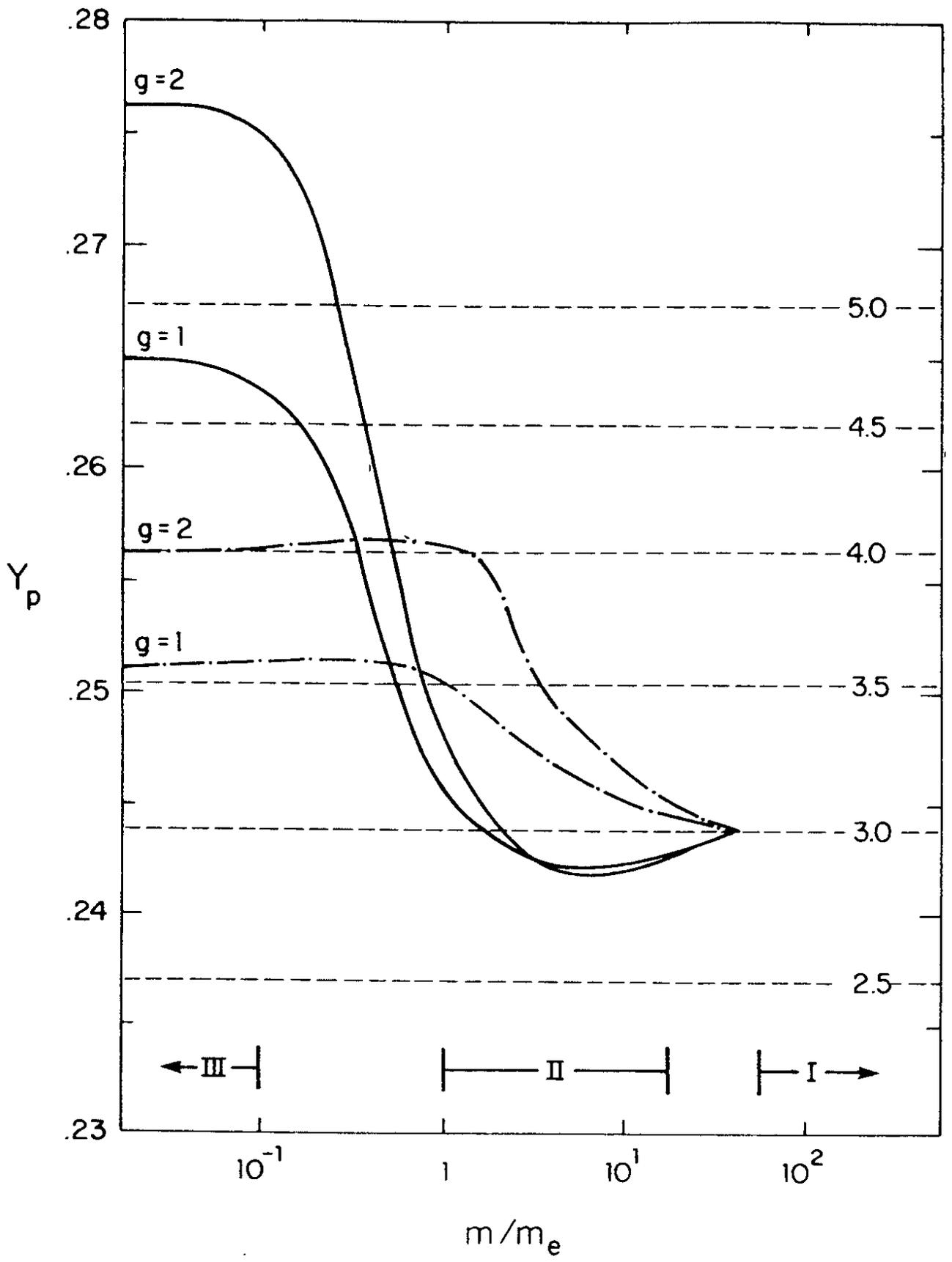


Fig. 4

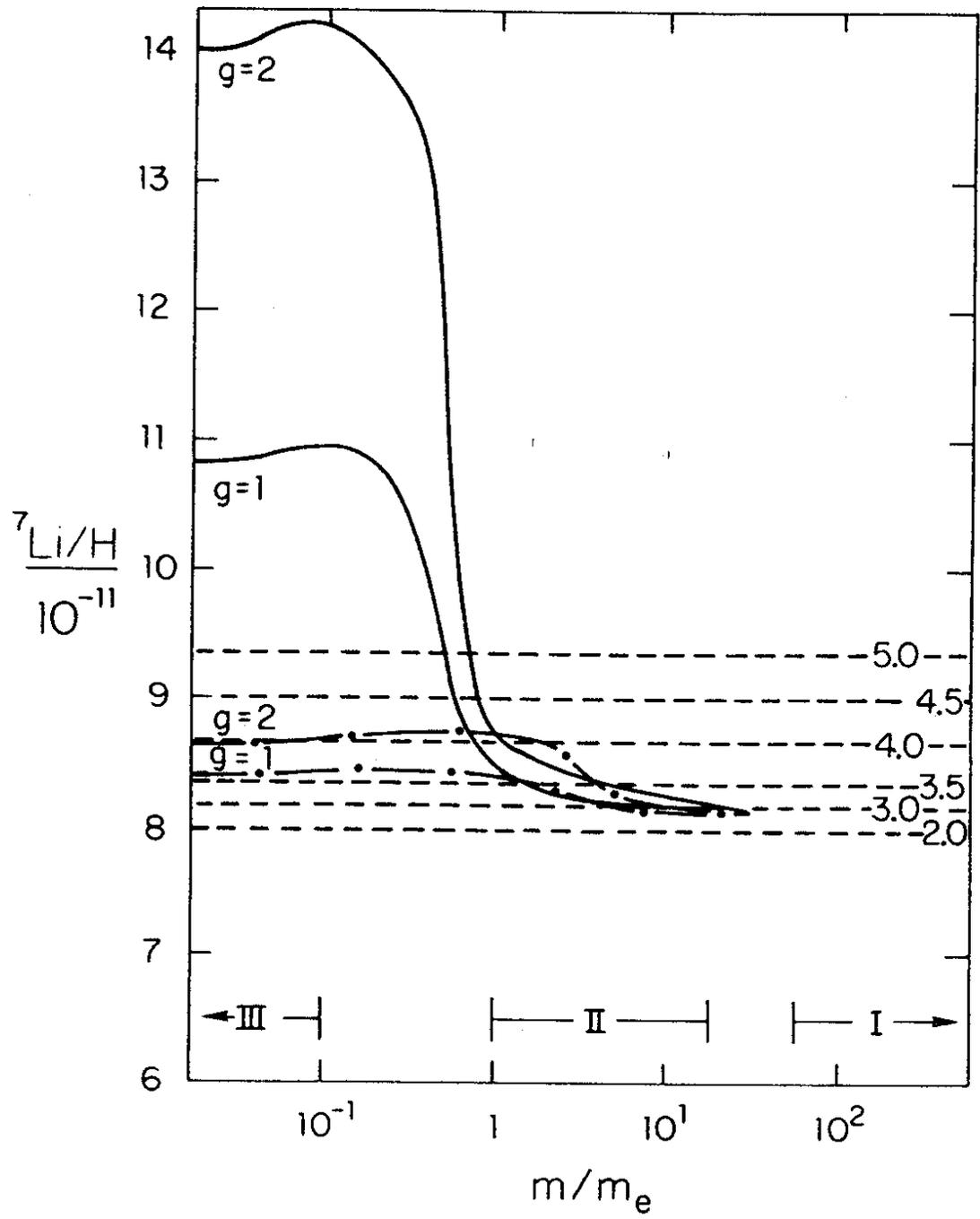


Fig. 5