Fermi National Accelerator Laboratory

FERMILAB-Pub-86/150-A
October 1986

Thermal Production of Not So Invisible Axions in the Early Universe

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Abstract

We find that for Peccei-Quinn symmetry-breaking scales $\lesssim 2 \times 10^8 GeV$ (corresponding to axion masses $\gtrsim 3 \times 10^{-2} eV$) thermal production of axions in the early Universe (via the Primakoff and photoproduction processes) dominates coherent production by a factor of about $1200\,(m_a/eV)^{2.175}$. The photon luminosity from the decays of these relic axions leads to a model-independent upper limit to the axion mass of order $2 - 5eV$. If the axion mass saturates this bound, relic axion decays may well be detectable.

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Perhaps the only blemish on QuantumChromoDynamics (QCD) is the strong CP problem, namely, the fact that QCD instanton effects violate CP symmetry[1]. To date the most attractive solution appears to be the axion[2-4]. The axion is the almost massless, psuedo Nambu-Goldstone boson associated the spontaneously-broken Peccei-Quinn (PQ) quasi-symmetry. Its mass and lifetime (axion $\to 2\gamma$) are determined by the scale of PQ symmetry breaking ($\equiv f$)

$$m_a \simeq 0.6eV/f_7 \tag{1}$$

$$\tau_a \simeq 3.9 \times 10^{24} sec\ m_{eV}^5 \tag{2}$$

where $f_7 \equiv f/10^7 GeV$ and $m_{eV} \equiv m_a/eV$. Here and throughout we use units such that $\hbar = k_B = c = 1$. The couplings of the axion to the other fields in the theory are model dependent, and we refer the reader to refs. 5 and 6 for a detailed discussion of this issue.

The effect of axion production on stellar evolution results in an upper bound to the axion mass[7]: about $20eV$ for axions which do not directly couple to electrons, and about $10^{-2}eV$ for axions which do couple directly to electrons. Coherent axion production (due to the initial misalignment of the vacuum angle $\theta = a/f$; a = axion field) leads to a lower limit to the axion mass based upon their contribution to the mass density of the Universe: $m_a \gtrsim 10^{-5}eV$ (in models which undergo inflation after or during PQ symmetry breaking this bound depends upon the initial misalignment angle, θ_i; see ref. 8). The *axion window* then, is between about $10^{-5}eV$ and $20eV$.

In this paper we will consider the thermal production of axions in the early Universe via the Primakoff and photoproduction processes (see Fig. 1). We will show that for axion masses greater than about $3 \times 10^{-2}eV$ thermally-produced axions dominate the relic axion population. Recently, Kephart and Weiler[9] have pointed out that because of their large relic abundance and lifetimes which are cosmologically well-matched, invisible axions with masses in the eV range can produce a large photon luminosity due to their decays and may not be so invisible after all. By considering the diffuse photon background produced by the decays of unclustered relic axions and the line radiation from decaying axions which cluster, we obtain the bound:

$$m_a \lesssim 2 - 5eV$$

independent of whether axions couple to electrons or not. If the axion mass comes close to saturating this bound relic axions may well be detectable.

First, let us review coherent axion production. When PQ symmetry breaking takes place ($T \sim f$) the vacuum angle is left undetermined due to the masslessness of the axion at high temperatures ($T \gg \Lambda_{QCD}$). At low temperatures ($T \ll \Lambda_{QCD}$) the axion develops a mass due to instanton effects and a preferred vacuum angle is picked out–the one which minimizes the vacuum energy. In general the initial vacuum angle is not aligned with this, and so the angle begins to relax to this value. In so doing it oscillates about the preferred vacuum angle. These oscillations correspond to a very cold, NR condensate of axions. Their contribution to the energy density of the Universe today has been calculated to be[8,10]

$$\Omega_{coherent} h^2/T_{2.7}{}^3 \simeq 1.1 \times 10^{-6} m_{eV}^{-1.175} \tag{3}$$

where $\Omega_a \equiv \rho_a/\rho_c$ is the fraction of critical density contributed by axions, $\rho_c = 1.05 \times 10^4 h^2 eV cm^{-3}$, h is the present value of the Hubble parameter H_0 in units of $100 kms^{-1} Mpc^{-1}$, and $T_{2.7}$ is the present microwave temperature in units of $2.7K$. Eqn(3) is valid for axion masses greater than about $2 \times 10^{-11} eV$ and was derived assuming that inflation did not take place after or during PQ symmetry breaking (if it did, then the relic axion abundance depends upon the initial misalignment angle squared and in general is less than this). For a detailed discussion of the derivation of Eqn(3) and the uncertainties involved, see ref. 8, and for further discussion of coherent axion production see refs. 10.

Now consider the thermal production of axions in the early Universe. For simplicity we will only consider the interactions of the heaviest quark (Q, mass M) which couples to the axion. As we shall see its interactions are the dominant production mechanism; the interactions of the other quarks which couple to the axion only serve to slightly *increase* the axion production rate. We are intentionally ignoring the coupling of the axion to electrons as the limit we derive is most relevant for models where the axion does not couple directly to electrons; in any case, axion production via electrons is always subdominant.

At low temperatures ($T \lesssim M$) the dominant axion production mechanism involving the heavy quark Q is the Primakoff process (see Fig.1):

$$q + \gamma \rightarrow q + a$$

where q is any light (mass $\lesssim T$), charged fermion. At energies below M, the heavy quark loop can be shrunk to a point and the low energy cross section is:

$$< \sigma v > \simeq \alpha^3/f^2 \tag{4}$$

where $<>$ indicates thermally averaging (for a detailed discussion of the Primakoff process and thermal averaging, see Raffelt in ref. 7). [Note that at very high temperatures, $T \gg \Lambda_{QCD}$, the photons in this diagram can be replaced by gluons and α by α_s, thereby greatly increasing the production rate.] The rate of axion production then is just given by $\Gamma_p \simeq n_q < \sigma v >$, where n_q is the number density of light, charged particles. Taking $n_q \simeq g_* T^3/\pi^2$, i.e., equal to the number density of relativistic particles (g_* as usual counts the effective number of relativistic degrees of freedom), it follows that

$$\Gamma_p \simeq g_* \alpha^3 T^3/(\pi^2 f^2) \tag{5}$$

At high temperatures ($T \gtrsim M$), the heavy quark loop can no longer be shrunk to a point, and the dominant process is photoproduction off the thermal bath of heavy quarks:

$$Q + \gamma \rightarrow Q + a$$

[At low temperatures this process is not important because the ambient number density of heavy quarks is suppressed by a Boltzmann factor.] The thermally-averaged cross section for this process is

$$< \sigma v > \simeq \alpha (M/f)^2 T^{-2} \tag{6}$$

The axion production rate then goes like

$$\Gamma_Q \simeq \alpha T M^2/(\pi^2 f^2) \tag{7}$$

Whether or not axions are in thermal equilibrium depends upon the magnitude of their production rate relative to the expansion rate of the Universe H. During its early history ($t \lesssim 10^{10} sec$ and $T \gtrsim 10 eV$) the Universe was radiation-dominated and

$$H = 1.67 g_*^{1/2} T^2/m_{pl}$$

Whenever $\Gamma \gtrsim H$ axions should be in thermal equilibrium and have a number density

$$n_a = \varsigma(3) T^3/\pi^2$$

where $\varsigma(3) = 1.20206....$ On the other hand, whenever $\Gamma \lesssim H$ axions should be decoupled and have a number density per comoving volume which remains constant ($n_a R^3 = const$; here $R(t)$ is the FRW scale factor).

Figure 2 shows a schematic plot of Γ/H as a function of T. For high temperatures ($T \gtrsim M$) Γ/H varies as T^{-1} and for low temperatures as T, achieving its maximum at a temperature of about $T \simeq M$. The value of Γ/H at the maximum is about

$$(\Gamma/H)|_{T=M} \simeq 10^4 M_{30}/f_7^2$$

where $M_{30} \equiv M/30GeV$. For $f \lesssim 10^9 GeV M_{30}^{1/2}$ there is a period when the axion should have been in thermal equilibrium (see Fig. 2). We know that the top quark is more massive than about $30GeV$ and in some axion models[3] the heavy quark is very massive ($M \gtrsim 10^{14} GeV$), so it seems safe to assume that for $f \lesssim 10^9 GeV$ axions were once in thermal equilibrium. Now let us calculate their final decoupling or freeze out temperature; roughly speaking that occurs when $\Gamma_p/H \simeq 1$:

$$T_d \simeq 20GeV f_7^2 \simeq 10GeV m_{eV}^{-2} \tag{8}$$

For $T \lesssim T_d$ the number density of axions just decreases as R^{-3}. Assuming that the entropy per comoving volume ($\equiv S \propto sR^3$; where the entropy density $s = 2\pi^2 g_* T^3/45$) remains constant (i.e., no significant entropy production), we calculate the number density of axions today by using the constancy of $n_a/s = 0.278/g_{*d}$ and $s_{today} \simeq 7.04 n_\gamma \simeq 2810 T_{2.7}^3 cm^{-3}$:

$$n_a = (n_a/s)s_{today} \simeq 13(60/g_{*d})T_{2.7}^3 cm^{-3} \tag{9}$$

$$\Omega_{thermal} h^2/T_{2.7}^3 \simeq 7.7 \times 10^{-4} f_7^{-1}(60/g_{*d}) \simeq 1.3 \times 10^{-3} m_{eV}(60/g_{*d}) \tag{10}$$

Here $g_{*d} \equiv g_*(T_d)$; for T_d of order $1 - 100GeV$, $g_{*d} \simeq 60$, while for $T_d \lesssim few100MeV$, $g_{*d} \simeq 10$. Note that $\Omega_{thermal} \simeq 1.$ is achieved for $m_a \simeq 130h^2\, eV$; however, since axions more massive than about $25eV$ decay in less than the age of the Universe, $\Omega_{thermal} \simeq 1.0$ is likely precluded.

Comparing Eqns(3) and (10) we see that thermal production dominates coherent production for $f \lesssim 2 \times 10^8 GeV$ (or $m_a \gtrsim 3 \times 10^{-2} eV$). The ratio of the thermally-produced axions to the coherently produced axions is

$$\Omega_{thermal}/\Omega_{coherent} \simeq 1200(60/g_{*d})m_{eV}^{2.175} \tag{11}$$

The contribution of coherently and thermally produced axions to the present energy density of the Universe is summarized in Fig. 3.

The coherently-produced axions come into existence when the vacuum angle starts to oscillate, $T_{osc} \simeq 6GeV f_7^{-0.175}$ (ref. 8). For $f \gtrsim 10^7 GeV$ axions have already decoupled, and so the thermal and coherent populations exist separately. On the other hand, for $f \lesssim 10^6 GeV$, axions are still in thermal equilibrium, and the coherently-produced population should thermalize, leaving a single population in the end.

Once the thermally-produced axions decouple they expand freely with axion momenta redshifting $\propto R^{-1}$. As long as they are relativistic they will maintain their thermal distribution albeit with a temperature which varies as R^{-1}. When their temperature reaches of order $m_a/3$, the axions become non-relativistic and thereafter axion velocities decrease as R^{-1}. Today then, they should be characterized by a velocity dispersion of order

$$< v_a^2 >^{1/2} \simeq 2.7 \times 10^{-4}(60/g_{*d})^{1/3} T_{2.7} m_{eV}^{-1} \tag{12}$$

For axion masses less that about $25eV$ axion lifetimes are greater than the age of the Universe, so that most of the relic axions are still with us today. We will now estimate the integrated photon luminosity of relic axions. In order to do so we must make some assumptions about where axions might be today. We consider two plausible possibilites: (1) that they are unclustered; (2) that they cluster and account for a fraction of the *dark matter* in galaxies (although they aren't so dark!).

(1) **Unclustered axions**–As we shall see this is the most conservative assumption regarding their detectability. In this case it is straightforward to compute the integrated photon intensity (assuming for simplicity the $\Omega = 1$ flat FRW model):

$$I = \frac{n_a m_a}{4\pi \tau_a \lambda_a H_0} (\lambda_a/\lambda)^{7/2} \tag{13a}$$

$$I \simeq 2.4 \times 10^{-23} erg cm^{-2} sec^{-1} arcsec^{-2} \mathring{A}^{-1} (\lambda_a/\lambda)^{7/2} h^{-1} (10/g_{*d}) m_{eV}^7 \tag{13b}$$

where $\lambda_a = 24800\mathring{A}/m_{eV}$ is the rest frame wavelength of a decay photon. [For reference $1sr \simeq 4.3 \times 10^{10}\ arcsec^2$.] In deriving Eqn(13) we have ignored the very tiny velocity dispersion of the relic axions. The diffuse photon background (or night sky) in the optical (corresponding to axion masses in the eV range) has an intensity of about[11]

$$I_{diffuse} \simeq 10^{-18} erg cm^{-2} sec^{-1} arcsec^{-2} \mathring{A}^{-1} \tag{14}$$

which implies an upper limit to the axion mass of about $5eV$ (corresponding to a lower limit to f of about $10^6 GeV$).

(2) **Clustered axions**–If structure formation in the Universe proceeds via primeval density perturbations which grow through the Jeans instability (as is generally believed), then axions should participate in structure formation. One would naively expect that the ratio of axions to baryons ($\equiv r$) in a structure which has not undergone significant dissipation (such as the halo of a galaxy, or a cluster of galaxies) would be about Ω_a/Ω_b, where the fraction of critical density contributed by baryons is constrained by primordial nucleosynthesis to be: $0.014 \lesssim \Omega_b h^2 \lesssim 0.035$ (ref. 12). Taking $\Omega_b h^2 \simeq 0.02$ and considering only the thermally-produced axions we find that

$$\Omega_a/\Omega_b \simeq 0.24 f_7^{-1} (10/g_{*d}) \simeq 0.4 (10/g_{*d}) m_{eV}$$

However, there are other considerations. Is the axion Jeans mass sufficiently small at the time of galaxy formation so that they can participate in the collapse which results in the formation of a galaxy (equivalently, is their velocity dispersion smaller than the gravitational velocity dispersion of a galaxy, $< v_{gal}^2 >^{1/2} \simeq 10^{-3}$)? Is there enough phase space in a galaxy for the axions to occupy? For axions in the multi-eV mass range and galaxy formation at redshifts $z \lesssim 10$ the answer to the first question is very likely yes. The second question is a bit more subtle. Once axions decouple their phase space number density is microscopically conserved. For a thermal distribution of bosons *or* fermions the phase space occupancy is of order unity. Thus the arguments Tremaine and Gunn[13] originally applied to neutrinos apply here too. Modelling structures as isothermal spheres (with core radius a and velocity dispersion σ) we find that there is enough phase space for an axion mass of about

$$M_a \simeq 10^7 M_\odot m_{eV}^4 a_{30}^3 \sigma_{-3}^3 g \tag{15}$$

where g is the initial phase space occupancy, $a_{30} \equiv a/30kpc$, and $\sigma_{-3} \equiv \sigma/10^{-3}c$. The baryonic mass of a typical spiral galaxy is about $10^{11} M_\odot$ and for the halo $a_{30} \simeq \sigma_{-3} \simeq 1$, so that for a typical galaxy, the maximum value of r permitted by phase space considerations is

$$r_{max} \simeq 10^{-4} m_{eV}^4 \tag{16}$$

which is less than Ω_a/Ω_b for axion masses in the eV range. For spiral galaxies then, one would expect the ratio of axions to baryons in the halo to be of order r_{max}. [Because of their enormous initial phase space occupancy ($g \gtrsim 10^{48} m_{eV}^{2.7}$–truly a Bose condensate) this argument is irrelevant for the coherently-produced axions.]

Using an isothermal model sphere model for the halo of our galaxy with

$$\rho_{halo}(r) = \rho_\odot (R^2 + a^2)/(r^2 + a^2)$$

where r is the distance from the galactic center, $R \simeq 9\,kpc$ is our distance from the galactic center, $\rho_\odot \simeq 5 \times 10^{-25} gcm^{-3}$ is the halo density near the solar system, and a is the core radius, and assuming that a fraction of the halo r_{max} is axions it straightforword to calculate the photon intensity from the galactic halo:

$$I_{halo} = 2.4 \times 10^{-23} ergcm^{-2} sec^{-1} arcsec^{-2} \mathring{A}^{-1} m_{eV}^{10} J(\theta, x) \tag{17a}$$

$$J(\theta, x) = \frac{1 + x^2}{x^2 + sin^2\theta}[\frac{\pi}{2} + tan^{-1}\{cos\theta/(x^2 + sin^2\theta)^{1/2}\}] \tag{17b}$$

where $x \equiv a/R$ which is of order unity, and θ is the angle between the direction of observation and the galactic center. The axion produced radiation is in a line of width $\Delta\lambda \simeq 10^{-3}\lambda_a \simeq 24.8\mathring{A}/m_{eV}$ at wavelength $\lambda_a \simeq 24800\mathring{A}/m_{eV}$. For $m_a \gtrsim 3eV$ the axion-produced line should stand out above the night sky. Thus the predicted glow of our own axionic halo implies an upper bound to the axion mass of about $3eV$. Should the axion mass saturate this bound, the angular dependence of the axionic halo glow provides a unique signature for its detection. In fact, if one allowed for a more general form for the halo density, the angular dependence could be used to probe the distribution of matter in the galactic halo[14].

Now consider the photon luminosity produced by axion decays in the halo of a distant galaxy whose image fills the aperture of the detector. Again taking the fraction of axions in the halo to be r_{max} and assuming the baryonic mass in the halo to be of order $M_{11}10^{11}M_{\odot}$ it is straightforward to derive the photon intensity:

$$I \simeq 3.6 \times 10^{-24} erg cm^{-2} sec^{-1} arcsec^{-2} \mathring{A}^{-1} m_{eV}^{10} M_{11} \tag{18}$$

which also results in a limit of order $3eV$.

For clusters of galaxies (mass of the order $M_{13}10^{13}M_{\odot}$, $a_{30} \simeq 10$ and $\sigma_{-3} \simeq 10$) axions in the eV mass range should contribute a fraction Ω_a/Ω_b of the cluster mass as r_{max} is greater than Ω_a/Ω_b (i.e., there is ample phase space to have $r = \Omega_a/\Omega_b$). The photon intensity from cluster axions in a detector whose aperture they fill should be about

$$I \simeq 3 \times 10^{-20} erg cm^{-2} sec^{-1} arcsec^{-2} \mathring{A}^{-1} (10/g_{*d}) M_{13} m_{eV}^{7} \tag{19}$$

in a line of width $\Delta\lambda \simeq 10^{-2}\lambda_a \simeq 248\mathring{A}/m_{eV}$. The detectability/mass bound here is of order $2eV$.

In sum, we have shown that for axion masses greater than about $3 \times 10^{-2}eV$ ($f \lesssim 2\times 10^{8}GeV$) the relic axion population is dominated by thermally-produced axions, and for masses greater than about $6eV$ the coherently-produced axions are themselves thermalized. The dominant production mechanism is the Primakoff process. Consideration of the diffuse photon background produced by the decays of unclustered relic axions results in the mass limit: $m_a \lesssim 5eV$. A similar limit follows from axion-produced photon emission from galactic halos. A limit of $\lesssim 2eV$ follows from considering the photon luminosity due to axion decays in clusters of galaxies. Conversely, an axion of mass in the 1-5 eV range might be detected by the photons produced by the decays of relic axions.

We thank William Bardeen, Chris Hill, Tom Kephart, Rich Kron, Tom Weiler, and Don York for valuable discussions. This work was supported in part by the NASA and DOE at Fermilab and by the Alfred P. Sloan Foundation.

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Figure Captions

Fig. 1 -Feynman diagrams for axion production via the Primakoff process ($q + \gamma \rightarrow q + a$; the particle in the loop is the heavy quark Q) and photoproduction on the heavy quark ($Q + \gamma \rightarrow \gamma + a$).

Fig. 2 -Schematic plot of Γ/H as a function of temperature T (Γ = axion production rate). For $f \lesssim 10^9 GeV M_{30}^{1/2}$ there is an epoch when axions were in thermal equilibrium.

Fig. 3 -Thermal and coherent contributions to the fraction of critical density contributed by axions as a function of axion mass. Note for axion masses $\gtrsim 6eV$ the coherently-produced axions are thermalized and so only for $m_a \lesssim 6eV$ are there two axion populations. The kink in the curve for thermal production results because g_{*d} changes abruptly from about 60 to about 10 for $m_a \simeq 10eV$.

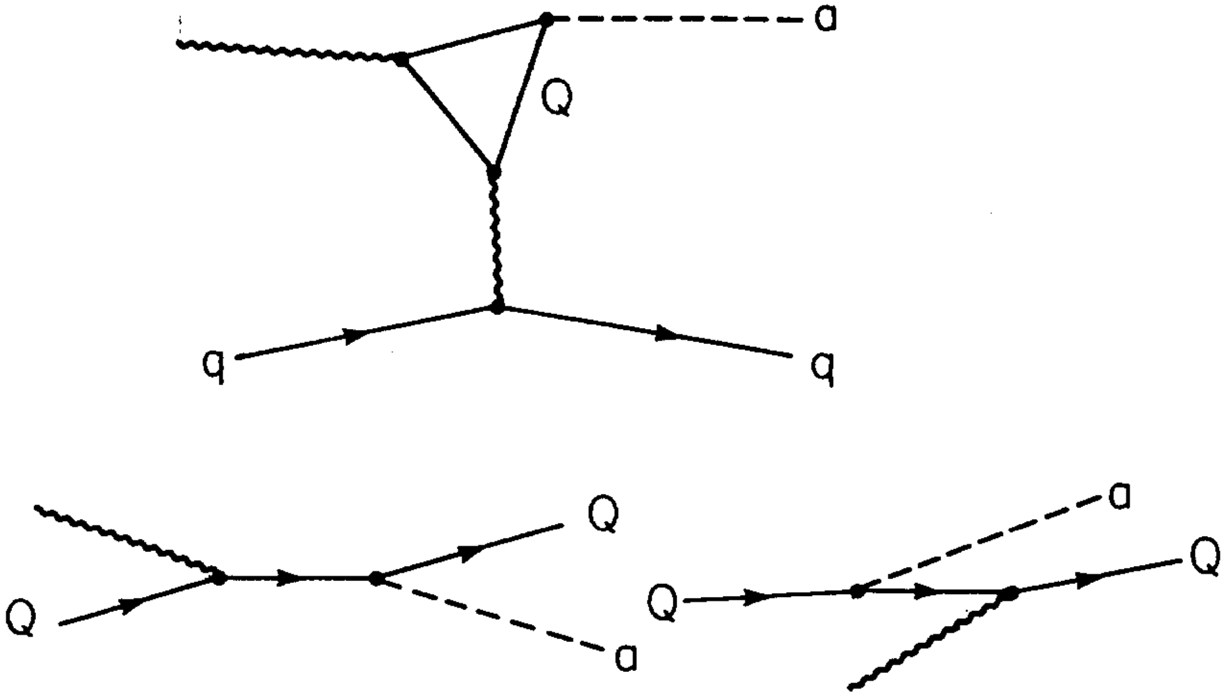

Figure 1

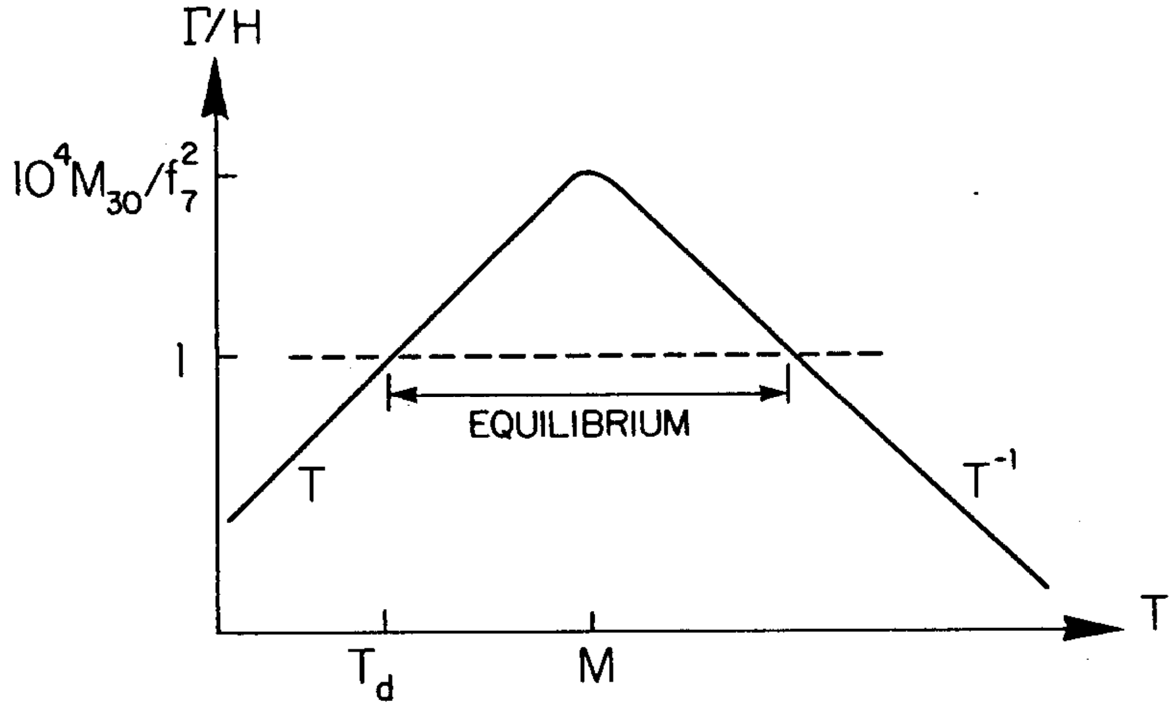

Figure 2

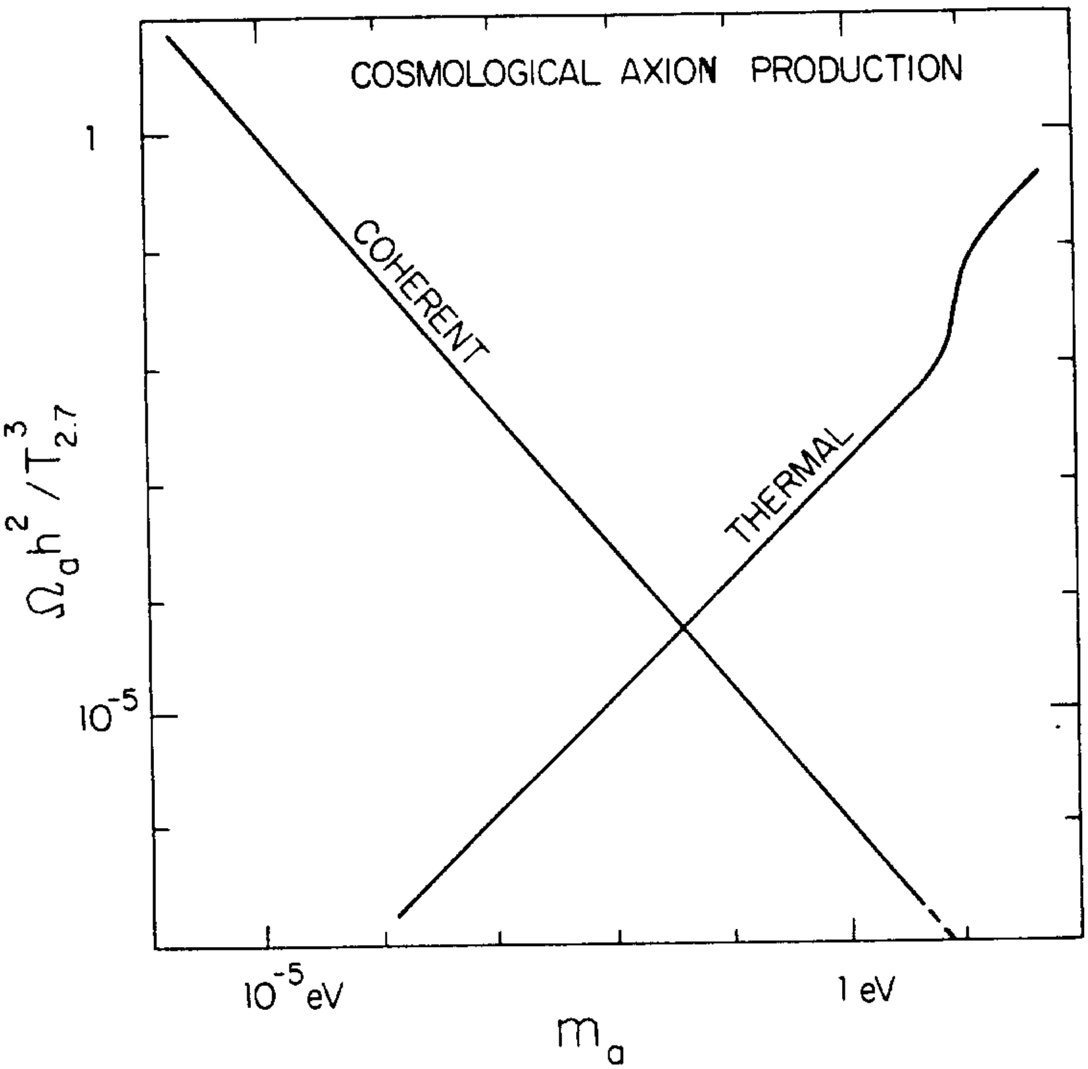
COSMOLOGICAL AXION PRODUCTION
1
10^{-5}
$\Omega_a h^2 / T_{2.7}^3$
COHERENT
THERMAL
10^{-5} eV
1 eV
m_a

Figure 3