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TOPOLOGICAL MASS TERMS ON AXION DOMAIN WALLS

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Abstract

Photons and electrons interacting with axion domain walls are considered in a simple model. A topological mass term for the photon and a parity-violating mass term for the electron arise naturally on axion domain walls. The quantization of topological mass and the interaction between domain walls and magnetic monopoles, along with other features, are discussed.

As a solution for the strong CP problem [1,2,3], axions have been studied and shown to have interesting properties [4]. It has been suggested that superconducting axion stings [5] can be formed around the GUT phase transtion time in the early universe. These cosmic strings will be connected by the domain walls [6] formed during the QCD phase transtion because of the instanton effects. The interaction between magnetic monopoles and axion domain walls gives rise to peculiar phenomena such as the exchange of electric charge between monopoles and walls [7].

Independent of the study of axions, there has been recent interest in three dimensional electrodynamics with a topological mass term [8,9]. In this theory magnetic flux induces fractional electric charge, fermion number, and angular momentum [10,11,12]. Consistent with the Bohm–Aharonov effect, fractional angular momentum produces anomalous statistics [13]. There have been attempts to find physical realizations of this theory in condensed matter [14] and high temperature physics [15].

In this paper I show that axion domain walls interacting with photons and fermions exhibit some properties of three dimensional electrodynamics. I extend the analysis of a similar model by Sikivie, and Huang and Sikivie [7] to show that it has a simple physical interpretation in terms of three dimensional electrodynamics. On an axion domain wall the axion-photon interaction term gives rise to a topological mass term for the photon. Further, there are electron and positron bound states with only one spin direction, resulting in a parity violating mass term for the fermion living on the domain wall. Because the model arises from a four dimensional theory, its features are similar but not identical to those of three dimensional electrodynamics.

The theory considered contains an axion field, $v\theta$, interacting with a photon field, A_μ, and an electron field, ψ. This is a simplified version of more realistic axion models [4,7]. The dynamics of the theory is defined by the Lagrange density

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{e^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ &+\bar{\psi}i\gamma^\mu\cdot(\partial_\mu - ieA_\mu)\psi - m\bar{\psi}e^{i\gamma_5\theta}\psi \\ &+\frac{v^2}{2}\partial^\mu\theta\partial_\mu\theta - v^2m_a^2(1-\cos\theta) \qquad (1)\end{aligned}$$

where e, m, v, and m_a are free parameters of the theory. The first and third terms in this expression are the standard spinor electrodynamics lagrangian. The second and fourth terms are the usual interaction terms between the axion and the photon and electron fields. The axion-photon interaction comes from the anomaly of the quasisymmetry U_{PQ} of Peccei and Quinn [1]. The last two terms are the axion lagrangian. The U_{PQ} symmetry is broken by the last term. With a real axion model in our mind, we will consider the energy scales, including m_a and m, much less than v.

The field equations are

$$\vec{\nabla}\cdot[\vec{E} - \frac{\alpha}{\pi}\theta\vec{B}] = \rho_e \qquad (2)$$

$$\vec{\nabla}\times[\vec{B} + \frac{\alpha}{\pi}\theta\vec{E}] - \frac{\partial}{\partial t}[\vec{E} - \frac{\alpha}{\pi}\theta\vec{B}] = \vec{j}_e \qquad (3)$$

$$\vec{\nabla}\cdot\vec{B} = \rho_m \qquad (4)$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial}{\partial t}\vec{B} = -\vec{j}_m \qquad (5)$$

$$i\gamma^\mu D_\mu\psi - me^{i\gamma_5\theta}\psi = 0 \qquad (6)$$

$$v^2\Box\theta + v^2m_a^2\sin\theta = 0 \qquad (7)$$

where $j_e^\mu = -e\bar{\psi}\gamma^\mu\psi$ is the electric current and, if there are any magnetic monopoles. j_m^μ is the magnetic current. The photon and electron contributions to the axion field equation are negligible.

In the model, the axion field θ has many stable vacua, $\theta_v = 2\pi n$, with integer n. Thus, there are domain walls as θ varies from one vacuum to another vacuum along one spatial direction. I consider a domain wall which lies on the x-y plane. θ goes from 0 to 2π as z goes from $-\infty$ to $+\infty$. It is easy to obtain a domain wall solution to Eq.7,

$$\theta(z) = \pi + 2\sin^{-1}[\tanh m_a z] \tag{8}$$

For this domain wall the energy density per unit area is $8\lambda v^3$ and the width is of order m_a^{-1}.

I begin with the interaction between photons and axions. As $F\tilde{F}$ is a total derivative, the photon-axion interaction term in Eq.1 can be rewritten by a partial integration. After integration over the z coordinate, the interaction term becomes

$$\int dz \frac{\alpha}{4\pi}\theta F\tilde{F} = \frac{2\alpha}{4}\epsilon^{abc}F_{ab}A_c \tag{9}$$

where a, b, and c denote 0,1, and 2. Here I assume that the vector field, A_a, varies slowly over the width of the domain wall. This is the standard topological mass term, with the mass equal to 2α in 2+1 dimensional electrodyanamics. Since the interaction of photons and axions is produced by the axial anomaly, the topological mass term in Eq.9 is not renormalized. (If there are N families of electrons, the coefficient of the $F\tilde{F}$ term is $N\theta$ instead of θ, with the change of θ across the domain wall= $\frac{2\pi}{N}$. Thus the topological mass is still 2α.) This quantization of the topological mass term, which can be seen to occur even for a nonabelian gauge group, is consistent with the Dirac quantization of magnetic charges [7]. These phenomena of nonrenomalization and quantization occur in 2+1 dimensional gauge theories for related reasons [16,17,18]. There is no photon bound state on the wall [7]. But also there is no free photon with topological mass on the wall in the 2+1 dimensional sense because the z dependence of B^z leads non-zero $B^{x,y}$ via Eq.4.

From Eq.2, the induced charge on the domain wall is

$$Q^{3+1}_{induced} \equiv \int dx^3 \vec{\nabla}\cdot\vec{E} = 2\alpha\Phi \tag{10}$$

for a magnetic flux Φ through the domain wall. The sign of $Q^{3+1}_{induced}$ is opposite to the case of 2+1 dimensional electrodynamics [8,9]. The reason is that $-\partial_z E^z$ plays the 2+1 dimensional charge density. From Eq.3 the induced current on the domain wall is

$$\vec{J}^{3+1}_{induced} \equiv \int dz \vec{\nabla} \times \vec{B} = -2\alpha \hat{z} \times \vec{E} \tag{11}$$

Because the current is perpendicular to the electric field, this is similar to the quantum Hall effect [14,19]. The induced charge and current can be summarized as the Witten electric current [20], $j^{\mu}_{W} = -\frac{\alpha}{\pi}\partial_{\nu}(\theta \tilde{F}^{\mu\nu})$, which is conserved.

In the thin wall approximation, on the wall the boundary conditions are

$$\begin{aligned} \Delta B^z &= \Delta E^{x,y} = 0 \\ \Delta E^z &= 2\alpha B^z, \;\; \Delta B^{x,y} = -2\alpha E^{x,y} \end{aligned} \tag{12}$$

The fields by a dyon at $\vec{x}_0$ with electric and magnetic charge (q,g) can be found by the image charge method in Ref.[7]. From the fields, I obtain the magnetic flux and the field angular momentum,

$$\Phi = \frac{1}{2}\frac{\alpha q \mp g}{1+\alpha^2} \tag{13}$$

$$M^z = \frac{1}{8\pi}\frac{\alpha}{1+\alpha^2}[q^2 - g^2 \pm 2\alpha q g] \tag{14}$$

where the upper and lower sign are for $z_0 < 0$ and $z_0 > 0$, respectively. The induced charge and angular momentum are fractional. This is not contradictory to the known principles, like the charge quantization or the angular momentum quantization. The rotational symmetries along the x and y directions are broken and there is no reason for M^z to be quantized. This may lead to anomalous statistics [11,13] and will not be pursued here. Note that the ususal electric charge quantization of a particle without magnetic charge in the presence of a magnetic monopole is a result of the angular momentum quantization of the charge and a magnetic monopole. In the similar reason the electric charge of a dyon need not be quantized. If the wall has a finite size with the axion string as its boundary, the induced charge comes from the string and there is no fractional electric charge or angular momentum in the whole system.

I now consider briefly a magnetic monopole of charge (0,g) passing the domain wall from $z < 0$ to $z > 0$ [7]. By the magnetic charge quantization condition, $eg = 2\pi$ for the minimal magnetic charge. As it passes the domain wall, it acquires electric charge $2\alpha g = e$ by the Witten effect [20]. On the other hand, the total flux change is $-g$ by Eq.13. Thus the change in the induced charge on the domain wall is $-e$ by Eq.10. This shows the electric charge conservation. Similarly, one can show that the angular momentum (14) is conserved because the monopole aquires electric charge.

I turn now to a discussion of the interaction between the electron and the domain wall. Consider a classical electron of charge $-e < 0$ near the domain wall. If the electron magnetric moment is in the $+\hat{z}$ direction (so the spin is in the $-\hat{z}$ direction), there is a magnetic flux on the wall. The sign of the electric charge induced by this magnetic flux will be positive by Eq.10, and so there is an attraction between the electron and the wall. With the same spin direction, there is also an attraction between the positron and the wall. Thus we expect that there are bound states of electrons and positrons with a single spin direction.

The real interaction between the electron and the wall is more complicated because of the presence of image charges [7]. However, the electromagnetic interaction between the wall and the electron is at most of order α^2 and can be shown to have negligible effect on the bound state energy compared to the direct interaction between the axion wall and the electron, unless $m_a < \alpha^6 m$. For the invisible axion model [3], $m_a < \alpha^6 m$. The cosmic background temperature $3K^0$ is larger than the axion mass and the electron bound states will be wiped out.

I want to see if there are bound states on the domain wall. I natually expect that the bound states with the lowest energy are independent of x and y coordinates. After changing the variable by $\psi = e^{-\frac{i}{2}\gamma_5\theta - i\omega t}\psi_0(z)$, the Dirac equation (6) becomes [7]

$$[\omega - m\gamma^0 + i\gamma^0\gamma^z\partial_z + S^z\partial_z\theta]\psi_0(z) = 0 \tag{15}$$

where $S^z = \frac{i}{4}[\gamma^1, \gamma^2]$ is the z-component of the spin matrix. In the Dirac representation of γ^μ's, Eq.15 becomes

$$\begin{bmatrix} \omega - m + \frac{\sigma_z}{2}\partial_z\theta & \sigma_z i\partial_z \\ \sigma_z i\partial_z & \omega + m + \frac{\sigma_z}{2}\partial_z\theta \end{bmatrix} \begin{pmatrix} f(z) \\ g(z) \end{pmatrix} = 0 \tag{16}$$

where $f(z)$ and $g(z)$ are the first and last two components of ψ_0, respectively. The diffential operator commutes with $S^z = \sigma_z \otimes 1_{2\times2} = diag(1, -1, 1, -1)$. Thus, one can classify the eigenfunctions of Eq.18 by $(\pm, \pm)$, where the first and second denote the eigenvalues of the matrix σ_z for $f(z)$ and $g(z)$, respectively.

It can be shown generally that there is no bound state with $(+, -)$ and $(-, +)$ eigenvalues. In the thin wall limit, $m_a \gg m$, $\partial_z\theta = 2\pi\delta(z)$, and the equation can be solved exactly. For the $(+, +)$ ($(-, -)$) case, there is only positron (electron) bound state of $\langle S^z \rangle = -\frac{1}{2}$. The width of the wave function is of order, m^{-1}. The energy of the bound state is

$$|\omega| = \frac{\pi^2 - 4}{\pi^2 + 4} m \quad , m_a \gg m \tag{17}$$

When $m_a \ll m$, I use the nonrelativistic approximation. The lowest energy states have the same spin structure as before. The width becomes now $(mm_a)^{-1/2}$. The energy eigenvalue in the quadratic approximation of the potential near the bottom is

$$|\omega| = [1 - \frac{m_a}{m} + (\frac{m_a}{m})^{3/2}]m \quad , m_a \ll m \tag{18}$$

As the bound states of electrons and positrons have the single spin direction, they are essentially fermions with a parity violating mass in 2+1 dimensional electrodynamics [8,9]. Note that $\langle S^z \rangle$ is negative and coincides with that obtained from the magnetic dipole argument before. The effect of the existence of bound states of fermions on the interaction between photons and domain walls will be higher order in α.

Reconsider a monopole passing through the domain wall. In some models there are degenerate magnetic monopoles with fermion number $\pm\frac{1}{2}$ [21]. If the monopole has a fermion number minus one half at the beginning, it can create a positron bound to the wall, with the monopole fermion number equal to one half, rather than acquiring electric charge by the Witten effect. As the dyon excited energy is much larger than the electron mass in most models with monopoles, this process is energetically more favorable. The angular momentum and electric charge can be shown to be conserved.

Finally, consider the case where the domain wall is bounded by the axion superconducting string [5,6]. There are chiral zero modes on the string, resulting in the chiral anormaly. This means that the current is not conserved at the string. Callan and Harvey [22] showed that the photon and string interaction by $\theta F\tilde{F}$ in 3+1 dimensions or $\theta\tilde{F}A$ in 2+1 dimensions leads to current nonconservation which will compensate that by the chiral anomaly. Around the QCD phase transition the angle width of the wall changes continuously from 2π to 0. Thus, the axion string with a domain wall attached provides a unified view for their argument.

I have shown that a model of the axion domain wall interacting with photons and electrons provides an effective theory that looks like 2+1 dimensional electrodynamics. The model combines some fascinating features of the axion physics and 2+1 dimensional electrodynamics. Even though at present time it appears unlikely that an axion domain wall will be observed [22], it provides a good laboratory for field theory. Further details of the relation between axion domain walls and 2+1 dimensional gauge theories, including the strong and weak interactions, are worth considering.

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