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Old Inflation is not Prevented by Large Amounts of Anisotropy

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Abstract

We re-examine Barrow and Turner's claim that 'Old Inflation' can be prevented by large initial anisotropy in light of claims to the contrary. While we find that the claims to the contrary are based upon arguments which are either incorrect or not applicable, we also find an error in the paper of Barrow and Turner while invalidates their original conclusion. Old Inflation, like New Inflation, is not prevented by large initial anisotropy.

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(submitted to Matters Arising in *Nature*)

Barrow and Turner[1] have studied the effects of anisotropy on Guth's original model of inflation[2]. They concluded that anisotropy, if large, could prevent inflation from occurring. Though Guth's 'old inflation' suffers from the fatal 'graceful exit' problem (models which undergo sufficient inflation never again become radiation-dominated[3]), and has been supplanted by Linde's and Albrecht and Steinhardt's 'new inflation'[4], the work of Barrow and Turner is still frequently referred to. Furthermore, a number of authors[5] have argued that their conclusions are incorrect, although we find the arguments of these authors to be invalid or inapplicable. For these reasons, we felt it necessary to reexamine the paper by Barrow and Turner. In doing so, we have uncovered an error in their paper and find their basic conclusion to be wrong; large amounts of initial anisotropy do not, in general, prevent old inflation.

Guth's original model of inflation involves a first order phase transition associated with spontaneous symmetry breaking (SSB) of a Grand Unified Theory (GUT). SSB is effected when the scalar field ϕ acquires a vacuum expectation value, $\phi = \sigma$. At high temperatures ($T \gtrsim T_c$, where T_c is the critical temperature for the phase transition) the finite temperature effective potential, $U_T(\phi)$, is minimized for $\phi = 0$, while at low temperatures ($T \lesssim T_c$) $U_T(\phi)$ achieves its minimum at $\phi = \sigma$, which signals SSB. For a first order phase transition the symmetric state ($\phi = 0$) is still a local minimum of U_T for $T < T_c$, and is metastable for $T \lesssim T_c$ due to the potential barrier between $\phi = 0$ and $\phi = \sigma$. Because of this the Universe can exist in the metastable, symmetric state for $T \lesssim T_c$. While it does there is an enormous vacuum energy associated with the metastable (or false vacuum state), $U(\phi = 0) = 0(T_c^4)$ (where for $T << T_c, U_T \equiv U$ is temperature independent), and this vacuum energy drives an exponential expansion of the Universe (referred to as inflation). Because of the potential barrier between the false and true vacuum states, the transition to the true vacuum must occur by the nucleation of bubbles of the true vacuum state which then expand at the speed of light[6].

In the context of the Friedmann-Robertson-Walker cosmology Guth has calculated the probability that a given point in space remains in the symmetric phase and finds that it is $e^{-F(t)}$ where

$$F(t) = \int \lambda(t_1) R^3(t_1) V(t,t_1) dt_1$$

$$V(t,t_1) = \frac{4\pi}{3}\left[\int_{t_1}^{t} \frac{d\theta}{R(\theta)}\right]^3,$$

$R(t)$ is the Friedmann-Robertson-Walker scale factor, and $\lambda(t)$ is the nucleation rate which is taken to be constant once the phase transition has begun:

$$\lambda = \begin{cases} 0, & T < T_c \\ \lambda_o, & T \geq T_c. \end{cases}$$

[Throughout we use units where $\hbar = k = c = 1$ and $G \equiv m_{pl}^{-2}$.] Inflation begins when $T \simeq T_c$, at $t \equiv t_{GUT} \simeq 1/3\chi$, where $\chi^2 = 8\pi U(0)/3m_{pl}^2 = O\left(T_c^4/m_{pl}^2\right)$. During inflation, $R \sim e^{\chi t}$, and

$$F(t) = \frac{4\pi\lambda_o t}{3\chi^3}.$$

When $F(t)$ becomes of order unity, most of space is in the true vacuum phase, and the inflationary phase is over. A minimum of about 60 e-folds of inflation are required in order to solve the horizon and flatness problems[2]. Since inflation ends at $t = t_*$ where $F(t_*) = 1$, this implies that for inflation to be successful the nucleation rate λ_o must be $< 3\chi^4/4\pi N \simeq \chi^4/80\pi$ where the number of e-folds of inflation, N, is taken to be $\gtrsim 60$.

Now consider the effect on initial anisotropy on the inflationary process. Following Barrow and Turner, we assume that at early times, the energy density of the Universe is dominated by 'the anisotropy energy density,' Σ^2/R^6, and that the analogue Friedmann equation for the mean scale factor $R(t)$ can be written as (which is valid for all Bianchi I models)

$$\left(\frac{\dot{R}}{R}\right)^2 = \chi^2 + \frac{\Sigma^2}{R^6} + \frac{8\pi}{3}\frac{T^4}{m_{pl}^2}$$

where $\Sigma^2/R^6 \gg T^4/m_{pl}^2$, χ^2 represents the contribution of the vacuum energy, and the temperature $T \propto R^{-1}$. [Note that this equation is only valid so long as the Universe is still in the symmetric state, i.e., for $t \lesssim t_*$.] At early times ($t \lesssim t_V = 1/3\chi$) anisotropy dominates and $R \sim t^{1/3}$, while at late times ($t \gtrsim t_V = 1/3\chi$) the vacuum energy dominates and $R \sim e^{\chi t}$. Ignoring the T^4 term which is always subdominate, the evolution of the mean scale factor $R(t)$ is given by

$$R^3 = \frac{\Sigma}{\chi}\, sinh 3\chi t.$$

Note that the vacuum energy begins to dominate at $t \simeq 1/3\chi$, independent of the level of anisotropy. This fact, or equivalently, that Σ^2/R^6 is independent of Σ, has been used by a number of authors[5] to argue that anisotropy has no effect on the phase transition or inflationary process. These authors miss an essential point, the level of anisotropy does influence t_{GUT}, the time at which $T \simeq T_c$ and bubble nucleation can commence. In the presence of anisotropy

$$t_{GUT} = \gamma^{-1/2}/3\chi = \gamma^{-1/2} t_V$$

where $\gamma \equiv (\Sigma^2/R^6/\chi^2)|_{T=T_c}$, i.e., γ is the ratio of the anisotropy energy density to that in radiation when $T \simeq T_c$. The effect of larger anisotropy then, is to push t_{GUT} to earlier times, so that bubble nucleation can commence sooner.

During the anisotropy-dominated phase, $R \sim t^{1/3}$ and $F(t) = 9\pi\lambda_o t^4/80$, which implies that $t_* \simeq (80/9\pi\lambda_o)^{1/4}$. If $\lambda_o > 720\chi^4/\pi$, then $t_* < t_V$, and the transition to the true vacuum state will be complete before the vacuum energy comes to dominate, thereby preventing inflation. However, if this inequality is satisfied, then the previous constraint on the nucleation rate implies that sufficient inflation *would not* have occurred in the absence of anisotropy either. That is, anisotropy does not spoil inflation that would have otherwise been successful. It is also known that initial anisotropy does not adversely affect new inflation[7]. As Barrow and Turner pointed out and others have since studied in more detail[7], more than 60 or so e-folds of inflation will damp any initial anisotropy, regardless of amplitude, to an undetectable level today. Old inflation then also solves the anisotropy problem.

The error made by Barrow and Turner occurred just below Eqns(5,6) where they incorrectly interchanged t_{GUT} and t_V. In a radiation-dominated Friedmann Universe, both t_V and t_{GUT} are $O(m_{pl}/T_c^2)$, and as noted earlier t_V is independent of the level of anisotropy. However, as discussed above, in the presence of anisotropy $t_{GUT} \simeq \gamma^{-1/2} t_V$. This mistake led Barrow and Turner to conclude that by raising the initial level of anisotropy one could always make the Universe anisotropy-dominated at $t = t_*$, and thereby prevent inflation. Let us be more specific. While the Universe is anisotropy-dominated the ratio of anisotropy energy density to vacuum energy density decreases as t^{-2}, and so at $t = t_*$

$$[(\Sigma^2/R^6)/\chi^2]\,|_{t=t^*} = \gamma(t_{GUT}/t_*)^2 = (\pi\lambda_o/720)^{1/2}/\chi^2$$

which is independent of γ and only greater than unity if $\lambda_o > 720\chi^4/\pi$, the same condition found previously. By mistakenly using t_V for t_{GUT} in this equation Barrow and Turner found that the ratio of anisotropy to vacuum energy density at $t = t_*$ to be γ times the correct expression, and concluded that sufficient initial anisotropy could guarantee that the Universe was anisotropy-dominated at $t = t_*$.

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