Fermi National Accelerator Laboratory

FERMILAB-Pub-86/138-A
September 1986

COSMOLOGY AND EXTRA DIMENSIONS

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ABSTRACT. In the past few years the search for a consistent quantum theory of gravity and the quest for a unification of gravity with other forces have led to a great deal of interest in theories with extra spatial dimensions. These extra spatial dimensions are unseen because they are compact and small, presumably with typical dimensions of the Planck length, $l_{Pl} = 1.616 \times 10^{-33}$cm. If the "internal" dimensions are static and small compared to the large "external" dimensions the only role they would play in the dynamics of the expansion of the Universe is in determining the structure of the physical laws. However, if the big bang is extrapolated back to the Planck time, then the characteristic size of *both* internal and external dimensions were the same, and the internal dimensions may have had a more direct role in the dynamics of the evolution of the Universe. This chapter presents some speculations about the role of extra dimensions in cosmology.

1. MICROPHYSICS IN EXTRA DIMENSIONS

Theories that have been formulated in extra dimensions include Kaluza-Klein theories [1], supergravity theories [1], and superstring theories [2]. The exact motivation and goals of these approaches are quite different, but for many applications to cosmology they have several common features and they will be referred to simply as theories in extra dimensions. Among the common features of theories in extra dimensions are:

• *There are large spatial dimensions and small spatial dimensions*: If some of the dimensions are compact and smaller than the three large dimensions, it is possible to dimensionally reduce the system (integrate over the extra dimensions) and obtain an "effective" 3+1-dimensional theory. Present accelerators have probed matter at distances as small as 10^{-16}cm without finding evidence of extra dimensions. This is not surprising, as the extra dimensions are expected to have a size characteristic of the Planck length. The large dimensions may also be compact. If so, their characteristic size is greater than the Hubble distance, 10^{28}cm. This disparity of about 61 orders of magnitude is somewhat striking. This disparity is

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THEORY	α/α^0	G/G^0	G_F/G_F^0
Kaluza-Klein	$(b/b_0)^{-2}$	$(b/b_0)^{-D}$	$(b/b_0)^{-2}$
(D internal dimensions)			
Superstrings	$(b/b_0)^{-6}$	$(b/b_0)^{-6}$	$(b/b_0)^{-6}$
(6 internal dimensions)			

Table 1: Variation of fundamental constants with the size of the internal manifold

usually posed by the question "what makes the extra dimensions so small?" However, if gravity has anything to do with the size of dimensions, the only reasonable size *is* the Planck length, and a more appropriate question to ask is "what makes the observed dimensions so large?" One possible answer to the the last question is inflation. The possible connection between inflation and extra dimensions will be explored.

• *The effective low-energy theory depends upon the internal space*: In Kaluza-Klein theories the low-energy gauge group is determined by the continuous isometries of the internal manifold. In superstring theories, the structure of the internal space determines the number of generations of chiral fermions, whether there is low-energy supersymmetry, etc. If the internal space is distorted in any way the effective low-energy physics could be very different.

• *The fundamental constants we observe are not truly fundamental*: In theories with extra dimensions the truly fundamental constants are constants in the higher dimensional theory. The constants that appear in the dimensionally reduced theory are the result of integration over the extra dimensions. If the volume of the extra dimensions would change, the value of the constants we observe in the dimensionally-reduced theory would change. Exactly how they would change depends upon the theory. In Kaluza-Klein theories, gauge symmetries arise from continuous isometries in the internal manifold, while in superstring theories the gauge symmetries are part of the fundamental theory. In all theories the gravitational constant is inversely proportional to the volume of the internal manifold. In the most general case there is not a single radius in the internal manifold. However, for the sake of simplicity it will be assumed that there is a single radius, b, which characterizes the internal manifold. The b dependence of some fundamental constants are given in Table 1. In Table 1, α^0 is the present value of the fine structure constant, G^0 is the present value of the gravitational constant, G_F^0 is the present value of Fermi's constant, and b_0 is the present value of b.

• *The internal dimensions are static*: If the internal dimensions change, fundamental constants change. Limits on the time variability of the fundamental constants can be converted to limits on the time variability of the extra dimensions. Limits on time rate of change of the fine structure constant (assuming

$\lvert\dot{\alpha}/\alpha\rvert$	METHOD	$\Delta\tau$
$5 \times 10^{-15}\text{yr}^{-1}$	^{187}Re/^{187}Os	$5 \times 10^{9}\text{yr}$
$1 \times 10^{-17}\text{yr}^{-1}$	Oklo reactor	$1.8 \times 10^{9}\text{yr}$
$13 \times 10^{-13}h\ \text{yr}^{-1}$	Radio galaxies	$2 \times 10^{9}h^{-1}\ \text{yr}$
$2 \times 10^{-14}h\ \text{yr}^{-1}$	QSO	$5 \times 10^{9}h^{-1}\ \text{yr}$
$15 \times 10^{-15}h\ \text{yr}^{-1}$	Primordial nucleosynthesis	$6.6 \times 10^{9}h^{-1}\ \text{yr}$

Table 2: Constraints on the time variation of the fine structure constant

that the change is a power law in cosmological time) are given in Table 2. The look-back time, $\Delta\tau$, is the maximum time over which the limit may be applied. For the look-back time, an $\Omega = 1$ cosmology was assumed, i.e., a present age of $(2/3)H_0^{-1} = 6.6 \times 10^9 h^{-1}$yr. Long look-back times are relevant if the change is not a power law in cosmological time. It is interesting to know how soon after the bang the internal space had essentially the size it has today. The limit with the longest look-back time is the limit from primordial nucleosynthesis.

Primordial nucleosynthesis is a sensitive probe of changes in α, since the neutron-proton mass difference $Q = m_n - m_p = 1.293$ MeV has an electromagnetic component. Although the details of the neutron-proton mass difference are not known, it is reasonable to assume that the electromagnetic contribution is the same size (but the opposite sign) as the entire difference. With this assumption $\alpha/\alpha^0 = Q/Q^0$, where Q^0 is the value today.

The neutron-proton ratio at freeze out given by Eq. 1.78 is $\exp(-Q/T_f)$, so n/p is very sensitive to small changes in Q. The primordial ^{4}He mass fraction as a function of b/b_0 is given in Fig. 1, assuming that α, G, and G_F depend on b/b_0 as in Table 1. The curve labeled "SS" is the superstring model ($D = 6$), and the curves marked "KK$_2$" and "KK$_7$" are Kaluza-Klein models with $D = 2$ and $D = 7$ internal dimensions. The allowed range of the primordial ^{4}He, $Y_P = X_4 = 0.24 \pm 0.01$. For the superstring model, the primordial helium is within acceptable limits only if at the time of primordial nucleosynthesis $1.005 \geq b/b_0 \geq 0.995$. The Kaluza-Klein models give the slightly less stringent result $1.01 \geq b/b_0 \geq 0.99$. In either case, by the time of primordial nucleosynthesis the internal dimensions had obtained a size very close to the size they have today [3].

• *The ground state geometry does not have all the symmetries of the theory*: It is generally assumed that the ground state geometry is of the form $M^4 \times B^D$, where M^4 is four-dimensional Minkowski space, [1] and B^D is some compact

[1] The assumption of M^4 is not quite correct in a cosmological context, and should be replaced by $R^1 \times S^3$ for the closed model, $R^1 \times Q^3$ for the open model.

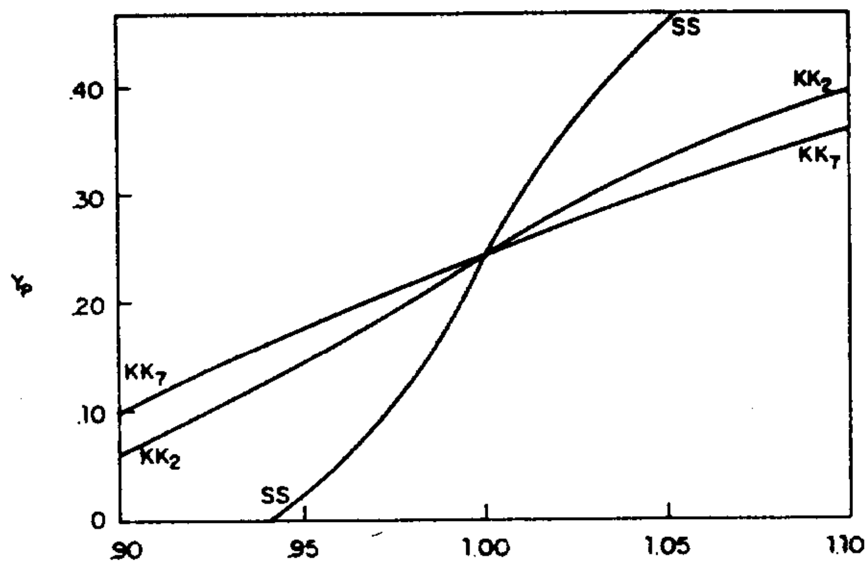

Figure 1: The primordial mass fraction as a function of b/b_0

$D-$dimensional space. The symmetries of the ground state are generally not as large as the symmetries of the theory, i.e., there is spontaneous symmetry breaking. One of the results of SSB is the existence of a massless (at least at the classical level) Nambu-Goldstone boson, which is sometimes called the dilaton.

• *The spectrum contains an infinite number of massive states*: If the radius of the internal space is b, then b^{-1} sets the scale for the massive states. The spectrum of the massive states depends upon the type of theory and the structure of the internal manifold. Since b is expected to be close to l_{Pl}, the massive states should have masses close to m_{Pl}.

2. STABILITY OF THE INTERNAL SPACE

All theories formulated in extra dimensions must contain some mechanism to keep the internal dimensions static. In the absence of such a mechanism, the extra dimensions would either contract or expand. The origin of the vacuum stress responsible for this is unknown. Here, some toy models are given, along with some possible cosmological effects.

In theories with extra dimensions new types of interactions may arise. For a starting point, consider the Chapline-Manton action [4], which is an $N = 1$ supergravity and an $N = 1$ super-Yang-Mills theory in 10 space-time dimensions. This theory is thought to be the field theory limit of a 10-dimensional superstring theory. It is not at all clear that the 10-dimensional field theory limit of the superstring ever makes sense. The 10-dimensional field theory description obtains only in the region between two similar energy scales. The first scale is determined by the string tension. It is the scale above which it is necessary to include the massive excitations of the string. Above this scale physics is "stringy" and any point-like field theory description is inadequate. The second scale is the compactification scale, which is determined by the radius of the internal space. At distances smaller than the compactification scale dimensional reduction no longer makes sense, the 3+1-dimensional description is inadequate, and the 10-dimensional theory must be used. The 10-dimensional field theory description makes sense at distance scales larger than the string tension scale, but smaller than the compactification scale. Since these two scales are expected to be the same order of magnitude, it is not clear if the 10-dimensional field theory description ever obtains. Nevertheless, it offers a convenient starting point for an exploration of cosmology in extra dimensions.

The Chapline-Manton Lagrangian contains the $N = 1$ supergravity multiplet $\{e_M^A;\ \psi_M;\ B_{MN};\ \lambda;\ \sigma\}$, where e_M^A is the vielbein, ψ_M is the Rarita-Schwinger field, B_{MN} is the Kalb-Ramond field, λ is the sub-gravitino, and σ is the dilaton, and the super Yang-Mills multiplet $\{G_{MN};\ \chi\}$, where G_{MN} is the Yang-Mills field strength and χ is the gluino field. The Lagrangian is [2]

$$\begin{aligned}
e^{-1}\ \mathcal{L}\ &= -\frac{1}{2}R - \frac{1}{2}\bar{\psi}_M\Gamma^{MPS}D_P\psi_S - \frac{3}{4}\exp(-\sigma)H_{MNP}H^{MNP} \\
&-\frac{1}{4}\partial_M\sigma\partial^M\sigma - \frac{3\sqrt{2}}{8}\bar{\psi}_M\ \not{\partial}\sigma\Gamma^M\lambda - \frac{1}{2}\bar{\lambda}\ \not{D}\lambda \\
&+\frac{\sqrt{2}}{16}\exp(-\sigma/2)H_{MNP}\left(\bar{\psi}_Q\Gamma^{QMNPR}\psi_R + 6\bar{\psi}^M\Gamma^N\psi^P\right. \\
&\left.-\sqrt{2}\bar{\psi}_R\Gamma^{MNP}\Gamma^R\lambda\right) - \frac{1}{2}\mathrm{Tr}\bar{\chi}\ \not{D}\chi - \frac{1}{4}\exp(-\sigma/2)\mathrm{Tr}G_{MN}G^{MN} \\
&-\frac{3}{4}\left(\mathrm{Tr}\bar{\chi}\Gamma_{MNP}\chi\right)^2 + \exp(-\sigma/2)H_{MNP}\mathrm{Tr}\bar{\chi}\Gamma^{MNP}\chi + \cdots
\end{aligned} \tag{2.1}$$

where $\Gamma^{MNP} = \Gamma^{[M}\Gamma^N\Gamma^{P]}$, and $H_{MNP} = \partial_{[M}B_{NP]}$. Four fermion couplings and other terms have been omitted.

The "Einstein equations" are straightforward to obtain:

$$\begin{aligned}
R_{MN}\ &=\ \frac{9}{2}\exp(-\sigma)\left(H_{MPQ}H_N^{\ PQ} - \frac{1}{12}g_{MN}H_{PQR}H^{PQR}\right) \\
&\quad -\exp(-\sigma/2)\left(\mathrm{Tr}G_{MP}G_N^{\ P} - \frac{1}{16}g_{MN}\mathrm{Tr}G_{PQ}G^{PQ}\right)
\end{aligned}$$

[2]The following notation will be used: D =number of extra dimensions; $M,\ N,\ P,\ Q,\ \ldots$ run from 0 to $D+3$; $\mu,\ \nu,\ \rho,\ \ldots$ are indices in the extra dimensions; and $m,\ n,\ p,\ q,\ \ldots$ are indices in the large spatial dimensions.

$$
\begin{aligned}
&-\frac{1}{2}\partial_M\sigma\partial^M\sigma-\frac{1}{8}\left(\mathrm{Tr}\bar{\chi}\Gamma_{PQR}\chi\right)\left(\bar{\lambda}\Gamma^{PQR}\lambda\right)g_{MN}\\
&-\frac{3}{16}\left(\mathrm{Tr}\bar{\chi}\Gamma_{PQR}\chi\right)^2 g_{MN}+\frac{9}{2}\exp(-\sigma/2)H_M{}^{PQ}\mathrm{Tr}\bar{\chi}\Gamma_{NPQ}\chi\\
&-\frac{3}{16}\exp(-\sigma/2)g_{MN}H_{PQR}\mathrm{Tr}\bar{\chi}\Gamma^{PQR}\chi+\cdots
\end{aligned}
\tag{2.2}
$$

The task at hand is to solve Eq. 2.2 to find the equations of evolution of the scale factor(s) in the expansion of the Universe toward the quasi-static ground state of the system where there are D static dimensions and 3 dynamic dimensions expanding as in a standard FRW cosmology.

In general it is necessary to choose background field configurations. For example consider the "bosonic" parts of the equations. What are the symmetries of the metric? What are the vacuum (background) values of H_{MNP}, of G_{MN}, of $\bar{\chi}\Gamma\chi$, of $\bar{\lambda}\Gamma\lambda$, of σ? In general, many (possibly infinitely many) solutions of the field equations are expected, even if there is but one ground state that describes the microphysics of our Universe. The immediate question to ask is what picks out the ground state and what is the evolution of the Universe to this ground state? Perhaps when the true string nature of the equations are taken into consideration there will be but one possible solution to the string equations even if there are many solutions to the field theory. Perhaps something in the evolution of the Universe prefers a unique or small number of possibilities. Such questions are reminiscent of the questions considered in inflation. If the conditions in some region of the Universe are such as to enter an inflationary phase, that region of the Universe will grow relative to a region that does not undergo inflation. It is possible to imagine that the Universe starts in a state with no particular background field configuration, but in a quantum state described by a wave function Ψ that describes the probability of a given configuration, Ψ(field configurations). If in some region of the Universe the wave function is peaked about a particular configuration that will inflate some spatial dimensions, that region will grow. All that is required to produce the Universe we observe is that there is some region that will lead to three spatial dimensions inflating (and some mechanism to keep D dimensions static). It may be that the theory is unique, but the ground state is not. It may be that somewhere outside of our horizon the Universe is quite different. There may be a different number of small versus large dimensions, or the internal space may have different topological properties leading to drastically different microphysics. Before this speculation is considered, it is necessary to understand the mechanism that leads to the stabilization of the internal space. This problem will be studied by considering individual contributions to the right-hand side of Eq. 2.2.

For simplicity, the metric will be taken to have the symmetry $R^1\times S^3\times S^D$

$$g_{MN} = \begin{pmatrix} 1 & & \\ & -a^2(t)\tilde{g}_{mn} & \\ & & -b^2(t)\tilde{g}_{\mu\nu} \end{pmatrix} \tag{2.3}$$

where $\tilde{g}_{mn}$ is the metric for S^3 of unit radius and $a(t)$ is the actual radius, and $\tilde{g}_{\mu\nu}$ is the metric for S^D of unit radius and $b(t)$ is the actual radius. The components of the Ricci tensor are

$$\begin{aligned} -R_{00} &= 3\frac{\ddot{a}}{a} + D\frac{\ddot{b}}{b} \\ -R_{mn} &= \left[\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{2}{a^2}\right] g_{mn} \\ -R_{\mu\nu} &= \left[\frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{D-1}{b^2}\right] g_{\mu\nu}. \end{aligned} \tag{2.4}$$

With the Einstein equations in the form

$$R_{MN} = 8\pi\bar{G}\left[T_{MN} - \frac{1}{D+2}g_{MN}T^P_P - \frac{1}{D+2}\frac{\Lambda}{8\pi\bar{G}}g_{MN}\right] \tag{2.5}$$

where $\bar{G}$ is the gravitational constant in $D+4$ dimensions, [3] and Λ is a possible cosmological constant in $D+4$ dimensions. All the terms on the right-hand side of Eq. 2.2 contribute to T_{MN} and Λ.

Symmetries of the stress tensor are usually chosen such that the only non-vanishing components of the stress tensor are

$$\begin{aligned} T_{00} &\equiv \rho \\ T_{mn} &\equiv -p_3 g_{mn} \\ T_{\mu\nu} &\equiv -p_D g_{\mu\nu} \end{aligned} \tag{2.6}$$

with $T^M_M = \rho - 3p_3 - Dp_D$. In terms of ρ, p_3, p_D, and $\rho_\Lambda = \Lambda/8\pi\bar{G}$ the Einstein equations are

$$\begin{aligned} 3\frac{\ddot{a}}{a} + D\frac{\ddot{b}}{b} &= -\frac{8\pi\bar{G}}{D+2}\left[(D+1)\rho + 3p_3 + Dp_D - \rho_\Lambda\right] \\ \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{2}{a^2} &= \frac{8\pi\bar{G}}{D+2}\left[\rho + (D-1)p_3 - Dp_D + \rho_\Lambda\right] \\ \frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{D-1}{b^2} &= \frac{8\pi\bar{G}}{D+2}\left[\rho - 3p_3 + 2p_D + \rho_\Lambda\right]. \end{aligned} \tag{2.7}$$

[3] $\bar{G}$ is related to Newton's constant G by $\bar{G} = GV_D^0$, where V_D^0 is the volume of the internal space today.

Some possible contributions to the right hand side will be considered in turn.

$\bullet R_{MN}$ =NOTHING: The simplest possible form for the right hand side is zero. For the moment abandon the choice of $R^1 \times S^3 \times S^D$, and consider a $D+3$ torus for the ground state geometry. The spatial coordinates can be chosen to take the values $0 \leq x^i \leq L$, where L is a parameter with dimension of length. The general cosmological solutions of the vacuum Einstein equations are the Kasner solutions. The Kasner metric is

$$ds^2 = dt^2 - \sum_{i=1}^{D+3} \left(\frac{t}{t_0}\right)^{2p_i} (dx^i)^2. \tag{2.8}$$

The Kasner metric is a solution to the vacuum Einstein equations provided the Kasner conditions are satisfied

$$\sum_{i=1}^{D+3} p_i = \sum_{i=1}^{D+3} p_i^2 = 1. \tag{2.9}$$

In order to satisfy the Kasner conditions at least one of the p_i must be negative. It is possible to have 3 spatial dimensions expanding in an isotropic manner and D dimensions contracting in an isotropic manner by the choice [5]

$$\begin{aligned} p_1 = p_2 = p_3 \quad &\equiv \quad p = \frac{3 + (3D^2 + 6D)^{1/2}}{3(D+3)} \\ p_4 = \ldots = p_{3+D} \quad &\equiv \quad q = \frac{D - (3D^2 + 6D)^{1/2}}{D(D+3)}. \end{aligned} \tag{2.10}$$

Note that $p > 0$ and $q < 0$. With this choice the metric may be written

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2 - b^2(t) d\vec{y}^2, \tag{2.11}$$

where x^i are coordinates of the 3 expanding dimensions, and y^i are coordinates of the D contracting dimensions. The two scale factors are given by $a(t) = (t/t_0)^p$, $b(t) = (t/t_0)^q$.

Somewhat more complicated classical cosmologies have been considered. The Kasner model can be regarded as an anisotropic generalization of the flat FRW cosmology, i.e., a Bianchi I cosmology. A generalization of the closed FRW model is the Bianchi IX model. The Bianchi IX vacuum solutions have the feature that the general approach to the singularity is "chaotic." [6] On approach to the initial singularity the scale factors in different spatial directions undergo a series of oscillations, contractions, and expansions. This feature is quite general, and independent of the state of the Universe after the singularity. The oscillation of the scale factors is well described by a sequence of Kasner models in which expanding and contracting dimensions are interchanged in "bounces." Such anisotropic behavior is predicted to be the general approach to the initial singularity. The question of whether such a chaotic approach to the initial singularity is present in more than three spatial dimensions has been considered. It has been shown that chaotic behavior obtains only for models with between 3 and 9 spatial dimensions [7]. The importance of this observation is clouded by the fact that at the approach to the singularity curvature may not dominate the right hand side of the Einstein equations, and near the singularity classical gravity may be a poor description.

The solutions above do not have solutions with a static internal space and if they are ever relevant, it is only for a limited time. The right-hand side must be more complicated than nothing. The next simplest thing to consider on the right-hand side is free scalar fields. Before discussing their effect on the evolution of the Universe it is necessary to discuss regularization in the background geometry.

The free energy of a non-interacting spinless boson of mass μ is given by [8]

$$F = T\frac{1}{2}\ln \mathrm{Det}\left(-\Box_{4+D} + \mu^2\right). \tag{2.12}$$

since finite temperature effects are of interest, the time is periodic with period of $1/2\pi T$, the relevant geometry is $S^1 \times S^3 \times S^D$, and the radii of the spheres are $1/2\pi T$, a, and b. The eigenvalues of $\Box$ on the compact space are discrete, and are given by the triple sum (hereafter μ will be set to zero)

$$\begin{aligned} 2T^{-1}F = \sum_{r=-\infty}^{\infty} \sum_{m,n=0}^{\infty} D_{mn} \ln \big(r^2(2\pi T)^2 \quad &+ \quad m(m+2)a^{-2} \\ &+ \quad n(n+D-1)b^{-2}\big), \end{aligned} \tag{2.13}$$

where D_{mn} is a factor that counts the degeneracy

$$D_{mn} = (m+1)^2(2n+D-1)(n+D-2)!/(D-1)!n!. \tag{2.14}$$

The free energy given by Eq. 2.13 is, of course, infinite. To deal with the infinities, a regularization scheme will be found to extract the relevant finite part. For the purpose of regularization, each term in the sum can be expressed as an integral using the formula [8] [4]

$$\ln X = \frac{d}{ds}X^s \mid_{s=0} = \frac{d}{ds}\left(\frac{1}{\Gamma(-s)}\int_0^\infty dt\, t^{s-1}\exp(-tX)\right)_{s=0}. \tag{2.15}$$

The finite part of the free energy is given by

$$2T^{-1}F = \frac{d}{ds}\left[\frac{1}{\Gamma(-s)}\int_0^\infty dt\, t^{-s-1}\sigma_1(4\pi^2T^2t)\sigma_3(a^{-2}t)\sigma_D(b^{-2}t)\right]_{s=0}, \tag{2.16}$$

[4]This regularization is only valid for D =odd. The D =even case will be discussed below.

where the functions σ_i are given by

$$\sigma_i(x) = \sum_{n=0}^{\infty} \frac{(2n+i-1)(n+i-1)!}{(i-1)!n!} \exp[-n(n+i-1)x]. \tag{2.17}$$

The full expression for the free energy is quite difficult to evaluate, but the free energy is simple in several limits. In the "flat-space" limit the radius of S^3 is much larger than the radius of S^D ($a \gg b$) and $\sigma_3 \to (\sqrt{\pi}/4)a^3t^{-3/2}$. In the limit $a \gg b$ the free energy can be approximated by

$$F = \frac{\Omega_3 a^3}{b^4}\left(c_1 - c_2(bT)^4 - c_3(bT)^{D+4}\right), \tag{2.18}$$

where Ω_i is found from the volume of the i-sphere, $V_i = R^i\Omega_i$ with R the radius and $\Omega_i = (2\pi)^{(i+1)/2}/\Gamma[(i+1)/2]$. For S^3, the volume is $V_3 = R^3 2\pi^2$, and Ω_3 has the familiar form $\Omega_3 = 2\pi^2$. The term proportional to c_1 is the Casimir term (c_1 is c_N of Candelas and Weinberg [9]). The term proportional to $c_2 = \pi^2/90$ is the leading temperature-dependent term when $T \ll b^{-1}$. When $T \gg b^{-1}$, the term proportional to $c_3 = (2\zeta(D+4)/\pi^{3/2})\Gamma[(D+4)/2]/\Gamma[(D+1)/2]$ dominates. In the "low-temperature" limit the radius of the S^1 becomes large and $\sigma_1 \to (4\pi t T^2)^{-1/2}$. In the flat-space, zero-temperature limit only the term proportional to c_1 survives.

The internal energy is given in terms of the free energy, the temperature, and the entropy

$$S = -\left[\frac{\partial F}{\partial T}\right]_{a,b}, \tag{2.19}$$

by $U = F + TS$. The thermodynamic quantities ρ, p_3, and p_D are defined in terms of the internal energy:

$$\begin{aligned} \rho &= \frac{U}{\Omega_3\Omega_D a^3 b^D} \\ p_3 &= -\frac{a}{3\Omega_3\Omega_D a^3 b^D}\left[\frac{\partial U}{\partial a}\right]_{b,S} \\ p_D &= -\frac{b}{D\Omega_3\Omega_D a^3 b^D}\left[\frac{\partial U}{\partial b}\right]_{a,S}. \end{aligned} \tag{2.20}$$

The thermodynamic quantities in zero temperature, low temperature, and high temperature limits are given in Table 3. There are several obvious limits of Table 3. In the zero-temperature or in the low-temperature limits, dimensional reduction is possible. Upon integration over the internal dimensions the effective three-dimensional energy density and pressure is obtained by multiplication by $V_D = \Omega_D b^D$. After dimensional reduction the Casimir terms are proportional to $c_1 b^{-4}$.

	Casimir $T=0$	Low Temperature $0 \leq T \leq b^{-1}$	High Temperature $T \gg b^{-1}$	Monopole T=0
ρ	$c_1/\Omega_D b^{4+D}$	$(\pi^2/30)T^4/\Omega_D b^{4+D}$	$(D+3)c_3 T^{D+4}/\Omega_D$	$f_0^2/2b^{2D}$
p_3	$-c_1/\Omega_D b^{4+D}$	$(\pi^2/90)T^4/\Omega_D b^{4+D}$	$c_3 T^{D+4}/\Omega_D$	$-f_0^2/2b^{2D}$
p_D	$4c_1/D\Omega_D b^{4+D}$	0	$c_3 T^{D+4}/\Omega_D$	$f_0^2/2b^{2D}$
T^M_M	0	0	0	$(4-D)f_0^2/2b^{2D}$

Table 3: Contributions to thermodynamic quantities

The low-temperature limit after dimensional reduction is $\rho = 3p_3 \to (\pi^2/30)T^4$ and $p_D = 0$, which is the expected contribution for a spinless boson in 3+1 dimensions. In the high-temperature limit dimensional reduction does not make sense.

It is possible to perform a similar analysis for particles of higher spin. The technical details are more difficult, but the physics is quite similar.

•R_{MN} =RADIATION: [10] Consider the "high-temperature" ($T \geq b^{-1}$) "flat-space" ($a \gg b$) limit with $\Lambda = 0$. In this limit T_{MN} is isotropic in the sense that $p_3 = p_D \equiv p$ (see Table 3). The Einstein equations are

$$\begin{aligned} 3\frac{\ddot{a}}{a} + D\frac{\ddot{b}}{b} &= -8\pi\bar{G}\rho \\ \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= 8\pi\bar{G}p \\ \frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{D-1}{b^2} &= 8\pi\bar{G}p. \end{aligned} \tag{2.21}$$

In keeping with the flat space assumption the $2/a^2$ term has been dropped in R_{mn}. The equation of state is $\rho = Np$, where $N \equiv D+3$. The conservation law $T^{MP}{}_{;P} = 0$ implies

$$\rho\bar{\sigma}^{N+1} = \text{constant}, \tag{2.22}$$

where $\bar{\sigma} \propto (a^3 b^D)^{1/N}$ is the mean scale factor. Since $\rho \propto T^{N+1}$, there is a conserved quantity $S_N = (\bar{\sigma}T)^N$ that is constant. This is simply the total N-dimensional entropy.

The Einstein equations (or a subset of the Einstein equations and the $T^{MP}{}_{;P} = 0$ equation) can be integrated to give $a(t)$ and $b(t)$. A typical solution is shown in Fig. 2. Both scale factors emerge from a initial singularity. The scale factor for the internal space reaches a maximum and recollapses to a second singularity. As b approaches the second singularity a is driven to infinity. The parameter x/x_s in

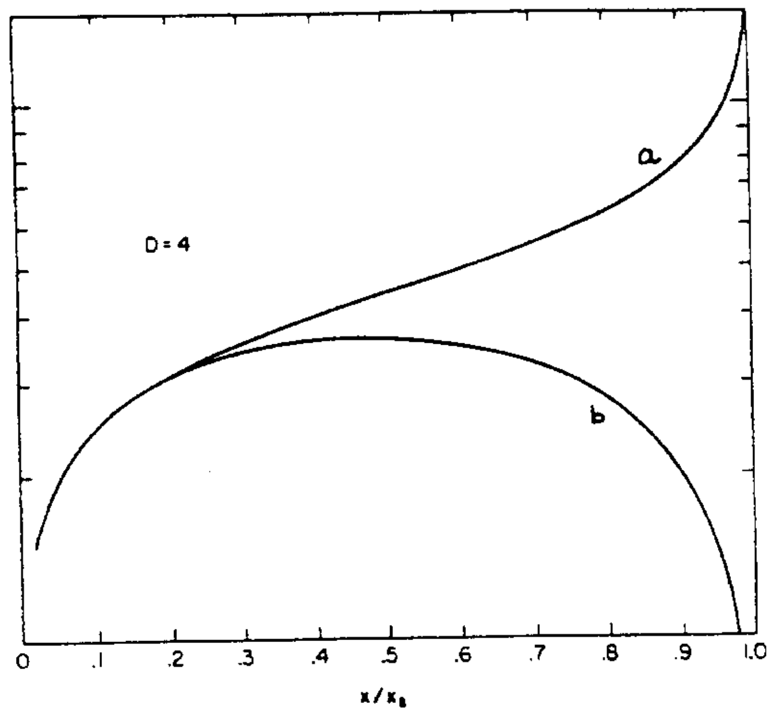

Figure 2: Evolution of the scale factors for R_{MN} =Radiation

Fig. 2 is a measure of the time in units of the time necessary to reach the second singularity.

The evolution of the temperature is shown in Fig. 3. The figure demonstrates the rather striking feature that as the second singularity is approached, the temperature increases. The expansion of a together with an increase of T seems unusual. However it is simply due to the conservation of entropy. In the region of growing T the mean volume of the Universe is actually *decreasing*, and the temperature must increase to keep S_N constant.

The assumption of the flat-space limit for S^3 can be easily justified. Imagine that the spatial geometry is $S^3 \times S^D$. If $a \geq b$ in the high-temperature region, once the maximum of b is reached, the S^3 will be inflated. The only requirement is that the curvature term, $1/a^2$, is small compared to the thermal term, $8\pi\bar{G}\rho$, at $b = b_{\text{MAX}}$.

In the approach to the second singularity the combination of expanding and contracting dimensions behaves like a Kasner model. A recurring feature in the analysis as presented in this review is that as the models become more baroque, there are limits in which the expansion can be approximated by only a part of the entire model. This is why consideration of the influence of individual terms contributing to T_{MN} is relevant.

In the period of increasing a and T, the entropy in the three expanding dimensions increases. Of course the *total* entropy is conserved, but in the approach to the second singularity entropy is squeezed out of the contracting dimensions into

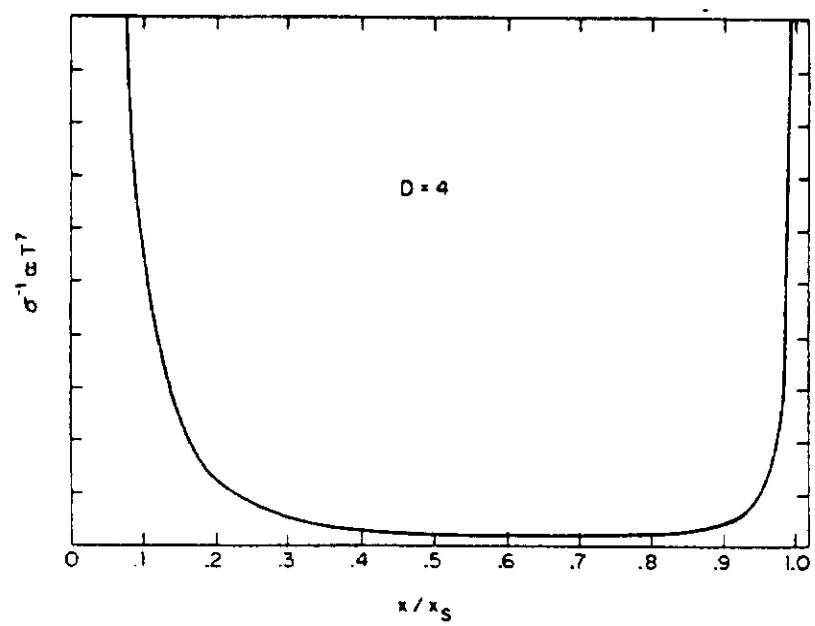

Figure 3: Evolution of the temperature for the solution of Fig.2

the expanding dimensions. The 3-entropy, S_3 will be defined as $S_3 = (d_{H3}T)^3$, where d_{H3} is the horizon distance in the 3-space

$$d_{H3} = a(t) \int_0^t dt'\, a^{-1}(t'). \tag{2.23}$$

In the approach to the second singularity, $d_{H3} \to \infty$ and $T \to \infty$, so $S_3 \to \infty$.

Before the second singularity is reached, two things must happen. First, there must be some mechanism to stabilize the internal dimensions. The other thing that must happen is that the high-temperature assumption will break down. The decrease of b outpaces the increase in T and eventually the assumption $T \geq b^{-1}$ will fail. When this occurs it is necessary to use the "low-temperature" form of the free energy and the only dynamical effect of the extra dimensions is the change in G. The increase in S_3 shuts off at this time. The conditions necessary to generate a significant amount of entropy in the three expanding dimensions have been studied. It is impossible to create an enormous amount of entropy without either very special initial conditions or extrapolating the solutions beyond the point where the high-temperature assumption breaks down.

• R_{MN} =Casimir + Λ [9]: The combination of Casimir forces plus a cosmological constant can lead to a classically stable ground state. With ρ, p_3, and p_D from Table 3, the Einstein equations in Eq. 2.7 becomes

$$3\frac{\ddot{a}}{a} + D\frac{\ddot{b}}{b} = -\frac{8\pi\bar{G}}{D+2}\left[\frac{(D+2)c_1}{\Omega_D}b^{-4-D} - \rho_\Lambda\right]$$

$$
\begin{aligned}
\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{2}{a^2} &= -\frac{8\pi\bar{G}}{D+2}\left[\frac{(D+2)c_1}{\Omega_D}b^{-4-D} - \rho_\Lambda\right] \\
\frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= \frac{8\pi\bar{G}}{D+2}\left[\frac{4(D+2)c_1}{D\Omega_D}b^{-4-D} + \rho_\Lambda\right] - \frac{D-1}{b^2}. \qquad (2.24)
\end{aligned}
$$

Note that the curvature of S^3 has been neglected ($1/a^2 \to 0$), and that the curvature term for S^D ($(D-1)/b^2$) has been moved to the right hand side of the $\mu\nu$ equation where it belongs.

The search for static solutions involves setting the left-hand side of the equations to zero. Setting the left-hand side to zero involves setting the time derivatives of *both* a and b equal to zero. The value of b for this static solution will be denoted as b_0. The first or the second equation determines b_0 in terms of ρ_Λ

$$
b_0^{-4-D} = \frac{\Omega_D}{(D+2)c_1}\rho_\Lambda. \qquad (2.25)
$$

Remembering that $\bar{G} = GV_D$ the $\ddot{b}$ equation can then be used to determine b_0 in terms of the Planck length

$$
b_0^2 = \frac{8\pi c_1(4+D)}{D(D-1)}l_{Pl}^2. \qquad (2.26)
$$

It is useful to rewrite the equations once again, this time in terms of b_0

$$
\begin{aligned}
3\frac{\ddot{a}}{a} + D\frac{\ddot{b}}{b} &= -(D-1)b_0^{-2}\left[\frac{D}{4+D}\left(\frac{b_0}{b}\right)^{4+D} - \frac{D}{4+D}\right] \\
\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{2}{a^2} &= -(D-1)b_0^{-2}\left[\frac{D}{4+D}\left(\frac{b_0}{b}\right)^{4+D} - \frac{D}{4+D}\right] \\
\frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= (D-1)b_0^{-2}\left[\frac{4}{4+D}\left(\frac{b_0}{b}\right)^{4+D} + \frac{D}{4+D}\right. \\
&\qquad \left. - \left(\frac{b_0}{b}\right)^2\right]. \qquad (2.27)
\end{aligned}
$$

Of course at $b = b_0$ the right-hand sides of the equations vanish.

In general there may be other interesting solutions to the system of equations. For instance in the limit where a and b *both* go to infinity, then the right-hand sides of all the equations approach a constant given by

$$
H^2 = \frac{D(D-1)}{4+D}b_0^{-2}. \qquad (2.28)
$$

In this limit the solution to the system is $a(t) = b(t) = \exp(\pm Ht/\sqrt{3})$. This solution describes exponentially growing scale factors for both S^3 and S^D.

The static minimum $b = b_0$ is stable against small perturbations, since $\delta b(t) = b(t) - b_0$ has no exponentially growing modes. However the existence of the exponentially growing solution for a and b implies that if b is ever large, it would grow without limit. This suggests that the static minimum is not stable against arbitrarily large dilatations. This point will be discussed in detail shortly.

In order to search for other solutions, and to study the semiclassical instability in compactification, the radius of the extra dimension will be expressed as a scalar field in a potential in four dimensions. The equation for $\ddot{b}$ looks like the equation of motion for a scalar field if the $\dot{b}^2$ term is neglected on the right hand side, and the left hand side is regarded as $\partial V(b)/\partial b$. The correct function of b to regard as the scalar field is determined by the kinetic part of the action. The kinetic part of the gravitational action is

$$S_k = -\frac{1}{16\pi\bar{G}} \int d^{4+D}x\sqrt{-g_{4+D}}R_k, \tag{2.29}$$

where R_k is the part of the Ricci scalar containing time derivatives of b:

$$R_k = -D\left[2\frac{\ddot{b}}{b} + (D-1)\left(\frac{\dot{b}}{b}\right)^2 + 6\frac{\dot{a}}{a}\frac{\dot{b}}{b}\right]. \tag{2.30}$$

Upon integration by parts and integration over the internal space the kinetic part of the action becomes

$$S_k = -D(D-1)\frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g_4}\left(\frac{b}{b_0}\right)^{D-2}\left(\frac{\dot{b}}{b_0}\right)^2. \tag{2.31}$$

If a scalar field ϕ is defined as

$$\phi(b) = \left[\frac{D-1}{2\pi D}\right]^{1/2}\left(\frac{b}{b_0}\right)^{D/2} m_{Pl} \tag{2.32}$$

it will have a canonical kinetic term. With this definition of ϕ the $\ddot{b}$ equation becomes

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\dot{\phi}^2}{\phi} = -\frac{dV}{d\phi}, \tag{2.33}$$

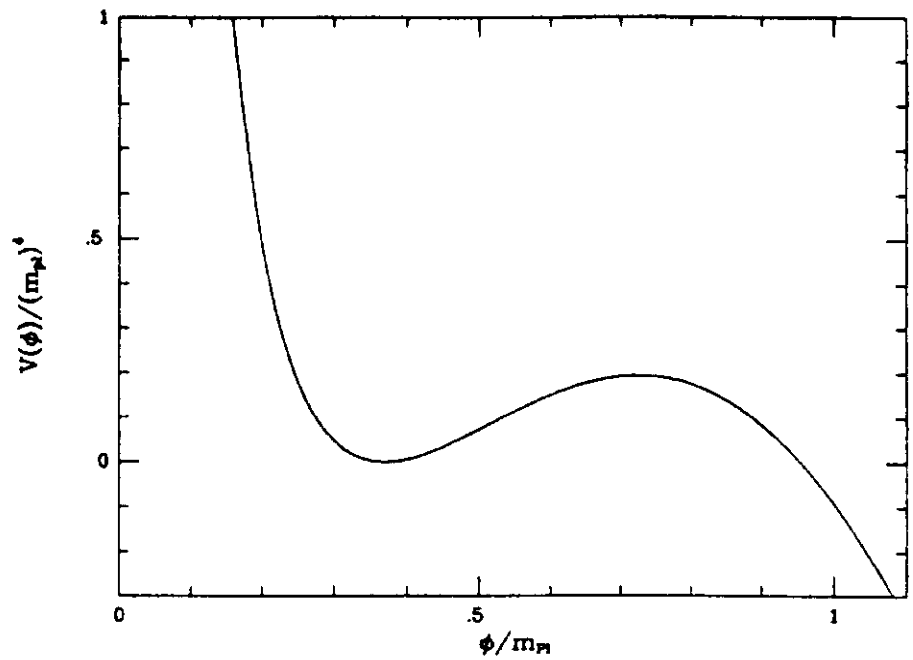

Figure 4: The potential for Casimir+Λ

where $dV/d\phi$ is the right hand side of Eq. 2.27 with the substitution of $\phi(b)$ for b. The potential is found by integrating $dV/d\phi$:

$$V(\phi) = \left(\frac{D(D-1)}{8\pi(D+4)}\right)^2 \frac{(D-1)}{c_1} m_{Pl}^4 \left\{ \left(\frac{\phi}{\phi_0}\right)^{-8/D} - \left(\frac{\phi}{\phi_0}\right)^2 \right.$$

$$\left. + \frac{D+4}{D-2}\left[\left(\frac{\phi}{\phi_0}\right)^{2(D-2)/D} - 1\right]\right\}, \tag{2.34}$$

where $\phi_0 = \phi(b_0)$ is the value of ϕ at the static minimum, $\phi_0 = [(D-1)/2\pi D]^{1/2} m_{Pl}$. There is an integration constant from integrating $dV/d\phi$ to find $V(\phi)$. The integration constant has been chosen to give $V(\phi_0) = 0$. A graph of $V(\phi)$ is given in Fig. 4 for $D = 7$ and $c_1 = 1$.

The figure illustrates several interesting features. The first feature is that the static minimum is perturbatively stable, but for ϕ greater than some value the potential is unstable. There is also a maximum to $V(\phi)$ that corresponds to $dV/d\phi = 0$ that corresponds to a solution with b static, but a expanding exponentially. A discussion of the semiclassical instability of the static solution will be discussed shortly.

•R_{MN} =Monopole+Λ [11]: The previous model used quantum effects from the Casimir effect to stabilize the extra dimensions against the cosmological constant. It is also possible to balance the effects of a classical field against the cosmological constant. Consider the Einstein-Maxwell theory in six space-time dimensions. The action for the model is given by

$$S = -\frac{1}{16\pi\bar{G}}\int d^6x\sqrt{-g_6}\left[R + \frac{1}{4}F_{MN}F^{MN} + 2\Lambda\right]. \tag{2.35}$$

The effect of the Maxwell field in the Einstein equations will through its contribution to the stress tensor

$$T_{MN} = F_{MQ}F_N^Q - \frac{1}{4}g_{MN}F_{PQ}F^{PQ}. \tag{2.36}$$

The ground state geometry will be assumed to be $R^1 \times S^3 \times S^2$, where as before $a \gg b$. The monopole ansatz has vanishing components of F_{MN} except for indices in the internal space:

$$F_{\mu\nu} = \sqrt{-g_2}\epsilon_{\mu\nu}f(t), \tag{2.37}$$

where $f(t)$ is a function of time and g_2 is the determinant of the S^2 metric. This ansatz, of course, satisfies the field equations for F_{MN}. The Bianchi identities can be used to express $f(t)$ in terms of the S^2 radius, $f(t) = f_0/b(t)$, where f_0 is a constant.

With the monopole ansatz for F_{MN} the non-vanishing components of the stress tensor are

$$T_{00} = \frac{1}{2}\frac{f_0^2}{b^4}; \quad T_{mn} = -\frac{1}{2}\frac{f_0^2}{b^4}g_{mn}; \quad T_{\mu\nu} = \frac{1}{2}\frac{f_0^2}{b^4}g_{\mu\nu}. \tag{2.38}$$

The contributions of the monopole configuration to ρ, p_3, and p_2 are given in Table 3. The Einstein equations with the cosmological constant plus monopole are

$$\begin{aligned} 3\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} &= -2\pi\bar{G}\left[\frac{f_0^2}{b^4} - \rho_\Lambda\right] \\ \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{2}{a^2} &= -2\pi\bar{G}\left[\frac{f_0^2}{b^4} - \rho_\Lambda\right] \\ \frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= 2\pi\bar{G}\left[3\frac{f_0^2}{b^4} + \rho_\Lambda\right] - \frac{1}{b^2}. \end{aligned} \tag{2.39}$$

The static solution in terms of f_0 is

$$\rho_\Lambda = \frac{f_0^2}{b_0^4} \qquad b_0^2 = 8\pi\bar{G}f_0^2. \tag{2.40}$$

To illustrate the potential it is again useful to express the Einstein equations in terms of b_0

$$
\begin{aligned}
3\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b} &= -\frac{1}{4b_0^2}\left[\left(\frac{b_0}{b}\right)^4-1\right] \\
\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}+2\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{2}{a^2} &= -\frac{1}{4b_0^2}\left[\left(\frac{b_0}{b}\right)^4-1\right] \\
\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+3\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= \frac{1}{4b_0^2}\left[3\left(\frac{b_0}{b}\right)^4+1-4\left(\frac{b_0}{b}\right)^2\right].
\end{aligned}
\tag{2.41}
$$

In addition to the static solution at $b = b_0$, there is a quasi-static solution at $b = \sqrt{3}b_0$ where b is static, but a increases exponentially $a = a_0\exp(Ht)$, where $H = \sqrt{2}/3b_0$. Finally, there is the solution as *both* a and $b \to \infty$ where both scale factors increase exponentially with rate $H = 1/2\sqrt{5}b_0$.

By the same methods as developed for the Casimir case, it is possible to define a scalar field and a potential for the scalar field. The potential is very similar to Fig. 4. This model is also unstable against large dilatations of the internal dimensions.

The monopole compactification was considered in $D = 2$ for simplicity. The extention to larger D will be considered in the section on inflation.

•$R_{MN} = R^2 + \Lambda$ [12]: The Casimir, monopole, and cosmological constant terms can arise in the Chapline-Manton action. Although terms such as R^2, $R_{MN}R^{MN}$, and $R_{MNPQ}R^{MNPQ}$ do not appear in the Chapline-Manton action, they are expected to be present in superstring theories, and probably all other extra-dimension theories as well. Consider the gravitational action for a theory with such terms given by

$$
\begin{aligned}
S = -\frac{1}{16\pi\bar{G}}\int d^{4+D}x\sqrt{-g_{4+D}}\,[R &+ 2\Lambda + a_1R^2 + a_2R_{MN}R^{MN} \\
&+ a_3R_{MNPQ}R^{MNPQ}(.
\end{aligned}
\tag{2.42}
$$

There is a $M^4 \times S^D$ solution if the following conditions are met:

$$
\begin{aligned}
0 &< D(D-1)a_1 + (D-1)a_2 + 2a_3 \\
0 &< (D-1)a_2 + 2a_3 \\
0 &< a_3 \\
\Lambda &= \frac{1}{4}\frac{D(D-1)}{D(D-1)a_1 + (D-1)a_2 + 2a_3}.
\end{aligned}
\tag{2.43}
$$

At the $M^4 \times S^D$ minimum, the value of b is

$$b_0^2 = 2D(D-1)a_1 + 2(D-1)a_2 + 4a_3. \tag{2.44}$$

The potential in this case is more difficult to analyze since there are higher derivative terms in the equations of motion. Nevertheless it has been shown that there is a solution corresponding to b ~constant and a increasing exponentially. Such a solution corresponds to a local maximum in the potential as in the Casimir or monopole cases. The difference in this case is that the location of this local maximum is a function of the a_i's, and for

$$a_3 = \frac{12/D - 3}{2 - 24/D(D-1)} a_2, \tag{2.45}$$

the local maximum will be at $b = \infty$. This means that the $M^4 \times S^D$ minimum is a true global minimum and is stable against large dilatations of the internal space. For the $D = 6$ case, the ghost-free action obtains for the case $a_3 = -a_2/4$, while Eq. 2.45 gives $a_3 = -5a_2/6$. The effect of the higher derivative terms in the equations of motion for $a(t)$ and $b(t)$ have been studied in both cases.

The possibility of using this model for inflation will be discussed below.

3. SEMICLASSICAL INSTABILITY OF COMPACTIFICATION [13]

In the Casimir $+\Lambda$ case, the monopole $+\Lambda$ case, and the $R^2 + \Lambda$ case where Eq. 2.45 is not satisfied, the static solution is not the true minimum of the theory. If the radius of the extra dimensions can be treated as a scalar field, it is possible to calculate the lifetime of the Universe against the decay of the false vacuum. The Casimir case will be used as an example.

Eq. 2.34 looks like the potential for a scalar field $\phi(x,t)$. The definition of ϕ in terms of b has been done to have the proper kinetic term for ϕ. With the four-dimensional gravitational degrees of freedom treated as a classical background, the problem of calculating the lifetime of the metastable state is identical to the decay of the false vacuum. For $D = 7$, $V(\phi)$ has a local minimum at $\phi_0 \simeq 0.37 m_{Pl}$, a local maximum at $\phi_m \simeq 0.725 m_{Pl}$, and a point degenerate with the local minimum at $\phi_T \simeq 0.96 m_{Pl}$ (see Fig. 4).

The potential can be approximated in the region $0 \leq \phi \leq \phi_T$ by (c_1 has been set to 1)

$$V(\bar{\phi}) \simeq 0.093\Lambda\bar{\phi}^2 - 0.159\Lambda\bar{\phi}^3/m_{Pl}, \tag{3.1}$$

where $\phi = \bar{\phi} + \phi_0$ has been shifted to place the metastable state at the origin. The potential has the form $V(\bar{\phi}) = M^2\bar{\phi}/2 - \delta\bar{\phi}^3/3$ for which the tunnel action has been calculated. The tunnel action is $S_E \simeq 205 M^2/\delta^2$ [14], which in terms of Λ and m_{Pl} is $S_E \simeq 165 m_{Pl}^2/\Lambda$.

The decay rate per unit four volume is

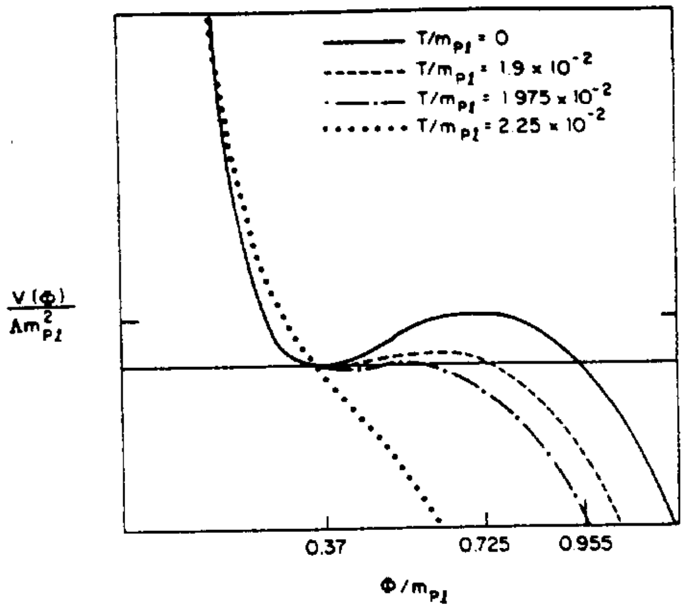

Figure 5: The temperature dependence of the Casimir potential

$$\Gamma \simeq m_{Pl}^4 \exp(-S_E), \tag{3.2}$$

where the pre-factor has been chosen as m_{Pl}^4 on dimensional grounds. In a matter-dominated Universe the probability for decay becomes of order unity in a time τ given by $\tau^4 \simeq 9\pi\Gamma/165 \simeq m_{Pl}^{-1} \exp(41 m_{Pl}^2/\Lambda)$. This is longer than the age of the Universe if $\Lambda \leq 0.3 m_{Pl}^2$.

In the Casimir case

$$\Lambda = \frac{D^2(D-1)^2(D+2)}{(D+4)^2 8\pi c_1} m_{Pl}^2 = 5.22 m_{Pl}^2/c_1 \quad (D=7). \tag{3.3}$$

In order to have the internal dimensions stay small for the age of the Universe requires $c_1 \geq 17.4$. For S^7 a single scalar field contributes $c_1 = 8.16 \times 10^{-4}$, so to satisfy the demand of longevity requires that there be more than 21,326 scalar fields. [5] Since the effective c_1's for higher-spin fields are larger, somewhat fewer are required.

There is also a finite-temperature instability present in the compactification. If the temperature-dependent terms in the free energy are included, the potential as a function of temperature has the form of Fig. 5. At high temperature the potential has no metastable state. The scalar field would not be trapped in the metastable phase if when $b \simeq b_0$ the temperature is large and temperature effects

[5] If there are N scalar fields, the effective c_1 is N times the c_1 for a single field.

are important. The temperature when $b = b_0$ depends upon the initial entropy. In a high-entropy initial condition the temperature will be large and compactification will not occur. The requirement that b should be trapped in the metastable state requires a low-entropy Universe, and the large entropy of the Universe must be created after compactification.

4. INFLATION AND EXTRA DIMENSIONS

The models of the previous section have illustrated the point that there are several mechanisms to force the internal space to be static and small. Although the mechanisms have different origins they all have in common the feature that there is a balance of forces at a particular value of $b \equiv b_0$. If $b \neq b_0$ there is an unbalanced stress in the vacuum. This unbalanced stress in the vacuum looks like a cosmological constant that can drive exponential expansion of all the dimensions, or just three dimensions. For instance in the monopole case discussed above, at $b = \sqrt{3}b_0$ there is a solution corresponding to static internal dimensions and exponentially expanding external dimensions. At $b = \sqrt{3}b_0$ the equation of motion for a is found from the (00) equation: $3\ddot{a}/a = 2/9b_0^2$, which has solution $a \propto exp(Ht)$, with $H^2 = 2/27b_0$

It is possible to imagine a scenario of new inflation where the exponential phase occurs for $b = \sqrt{3}b_0$, and is terminated when b settles to the local minimum at $b = b_0$. This is probably not a good example, because the potential is similar to the potential in Fig. 4, which is not the type of potential needed in new inflation. Even if for some unknown reason the Universe was ever in a configuration of $b = \sqrt{3}b_0$ and b static, quantum or thermal fluctuations would push b away from the unstable extremum. Even if it would roll in the correct direction toward the metastable minimum, the transition would be completed before sufficient inflation occurs.

A lesson learned from new inflation is that one should not be deterred by failure of simple models. For instance the $R^2 + \Lambda$ model is an existence proof that a model can be found. Recall that for a particular value of a_3/a_2 the potential does not turn over for large b and becomes flat. There can be a large amount of inflation as b evolves toward the ground state.

Inflation with the inflaton identified as the radius of the extra dimensions has some interesting features. In the evolution toward the ground state the radius of the extra dimensions grows, leading to an increase in the four-dimensional gravitational constant. The reheating is probably due to the change in the internal metric. For example, consider a minimally coupled scalar field χ with action

$$S \propto \int d^{D+4}x\, \chi \partial_M \left(\sqrt{-g_{4+D}}g^{MN}\partial_N \chi\right). \tag{4.1}$$

As b oscillates about the minimum of the potential there will be a non-zero value of $\dot{g}$ that results in an increase of χ. Although the details of the reheating remain to be worked out, the basic picture has been explored [12, 15].

All the models discussed above involve a $D+4$-dimensional cosmological constant that must be fine tuned to obtain the four-dimensional cosmological constant zero at b_0. All models (except the $R^2+\Lambda$ model with Eq. 2.45 satisfied) do not inflate and involve an unstable ground state. The introduction of the cosmological constant in the higher dimensional theory is not attractive. The fine tuning certainly must be incorrect. The unstable ground state cannot be ruled out, but seems undesirable. It would be nice if the existence of extra dimensions would lead to inflation.

Surely any realistic model should work without fine tuning of Λ. One might expect a realistic model to work for any effective value of Λ, and any change in Λ would simply lead to a change in b_0. In other words, if the vacuum energy would change, the only physical result would be a slight readjustment of b_0. This would be very attractive, since any cosmological constant produced as a result of SSB could be completely absorbed by a small change in b_0 and it would be unnecessary to fine tune Λ at high energies to account for phase transitions at low energies. Without extra dimensions there is nothing to do with the vacuum energy produced in phase transitions. Extra dimensions may provide a rug under which to sweep unwanted vacuum energy. After all, some vacuum energy is needed to keep the extra dimensions static.

The prospect of inflation from extra dimensions has not been realized in a realistic model, but there are no realistic models for compactification. In the Chapline-Manton theory there are two massless scalar fields, the dilaton and the radius of the internal dimensions. Perhaps one, or both, of these fields are the dilaton. Both fields have the promising feature that at the classical level they have flat potentials. The possibility of a unique field configuration that will lead to inflation is interesting.

The instability for large b in the Casimir and monopole models can be removed by considering combinations of the models.

• R_{MN} =ALL OF THE ABOVE [16]: Before combining the contributions it is useful to extend the analysis to products of spheres. Assume a ground state geometry of the form $R\times S^3\times\sum_{i=1}^{\alpha} S_i^d$, with metric g_{MN} =diag$(1, -a^2(t)\tilde{g}_{ij}(x),\ -b_1^2(t)\tilde{g}_{\mu\nu}(y), \ldots, -b_\alpha^2(t)\tilde{g}_{\rho\sigma}(y))$. The D extra dimensions are split into α d_i-spheres ($\sum d_i = D$). The stress tensor will be extended in a similar way by the definition of additional p_{di}. In the monopole and the Casimir cases, the large-b instability was caused by the presence of a cosmological constant, which was unbalanced as $b\to\infty$. For a stable ground state a cosmological constant is probably impossible. The Einstein equations without a cosmological constant are

$$
\begin{aligned}
3\frac{\bar{a}}{a}+\sum_{i=1}^{\alpha} d_i\frac{\bar{b}_i}{b_i} &= -\frac{8\pi\bar{G}}{D+2}(\rho - T_M^M(\\
\frac{\bar{a}}{a}+2\frac{\dot{b}^2}{b^2}+\frac{\dot{a}^2}{a^2}\sum_{i=1}^{\alpha}\frac{\dot{b}_i}{b_i}+\frac{2}{a^2} &= \frac{8\pi\bar{G}}{D+2}(p_3 - T_M^M(
\end{aligned}
$$

$$\frac{\ddot{b}_i}{b_i} + (d_i - 1)\frac{\dot{b}_i^2}{b_i^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}^i}{b^i} + d_i\frac{\dot{b}_i}{b_i}\sum_{j\neq i}\frac{\dot{b}_j}{b_j} + \frac{d_i - 1}{b_i^2} = \frac{8\pi\tilde{G}}{D+2}\left(p_{di} - T^M_M\right). \quad (4.2)$$

with the last equation for each internal sphere and $T^M_M = \rho - 3p_3 - \sum_{i=1}^{\alpha} d_i p_{di}$.

For forces to balance at a unique value of $b = b_0$ it is necessary to have contributions to T_{MN} that have different dependences on b. For this reason a combination of Casimir and monopole forces will be considered.

The generalization of the $D = 2$ monopole ansatz will be used. An antisymmetric tensor field of rank $d_i - 1$ has a field strength $F_{M,N,\ldots Q}$ of rank d_i and has a natural Freund-Rubin ansatz on the d_i-sphere. The stress tensor in terms of the field strength is

$$T_{MN} = F_{MP\ldots Q}F_N^{\;P\ldots Q} - \frac{1}{2d_i}g_{MN}F_{SP\ldots Q}F^{SP\ldots Q}. \quad (4.3)$$

With this assumption the monopole configuration leads to

$$\begin{aligned} \rho = -p_3 &= \sum_{i=1}^{\alpha}\frac{1}{2}\left(\frac{f_{0i}}{b_i^{di}}\right)^2 \\ p_{di} &= \frac{1}{2}\left[\left(\frac{f_{0i}}{b_i^{di}}\right)^2 - \sum_{j\neq i}\left(\frac{f_{0j}}{b_j^{dj}}\right)^2\right]. \end{aligned} \quad (4.4)$$

The generalization of the Casimir forces for products of spheres is also straightforward. The first generalization is a single sphere in even dimensions. For even dimensions there is an additional contribution to the free energy proportional to $\ln(2\pi\mu^2 b^2)$, where μ is a parameter that sets the scale of the path integral. This parameter can be set by imposing certain conditions on the effective potential. The second generalization is to products of spheres. The free energy becomes (ignoring the ln term)

$$F = \Omega_3 a^3 \sum_{i=1}^{\alpha}\frac{c_{i1}}{b_i^4}, \quad (4.5)$$

which leads to the thermodynamic quantities

$$\begin{aligned} \rho &= -p_3 = \left(\prod_{i=1}^{\alpha}\Omega_i b_i^{di}\right)^{-1}\sum_{i=1}^{\alpha}\frac{c_{1i}}{b_i^4} \\ p_{di} &= \frac{4}{d_i}\left(\prod_{i=1}^{\alpha}\Omega_i b_i^{di}\right)^{-1}\frac{c_{1i}}{b_i^4} \end{aligned} \quad (4.6)$$

The first example of combining Casimir and monopole forces is a single internal D-sphere. Ignoring here and below the possible logarithmic dependence of the Casimir force for even dimensions, the Einstein equations are

$$\begin{aligned} 3\frac{\ddot{a}}{a} + D\frac{\ddot{b}}{b} &= -\frac{8\pi\bar{G}}{D+2}\left[\frac{(D+2)c_1}{\Omega_D}b^{-4-D} + (D-1)f_0^2 b^{-2D}\right] \\ \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{2}{a^2} &= -\frac{8\pi\bar{G}}{D+2}\left[\frac{(D+2)c_1}{\Omega_D}b^{-4-D} + (D-1)f_0^2 b^{-2D}\right] \\ \frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= \frac{8\pi\bar{G}}{D+2}\left[\frac{4(D+2)c_1}{D\Omega_D}b^{-4-D} + 3f_0^2 b^{-2D}\right] \\ &\quad -\frac{D-1}{b^2}. \end{aligned} \tag{4.7}$$

From the first two equations it is obvious that either c_1 or f_0^2 must be negative in order to have $\ddot{a}$ and $\ddot{b}$ vanish at b_0. The combination of the first two equations and the last equation gives

$$b_0^2 = \frac{D(D-1)^2}{8\pi(D+2)(D-4)c_1}l_{Pl}^2; \qquad b_0^{D-6}l_{Pl}^2 = -\frac{\Omega_D 8\pi(D-4)}{D(D-1)}f_0^2. \tag{4.8}$$

For $D < 4$, f_0^2 must be positive and c_1 must be negative. Although c_1 is positive for scalar fields on spheres, the sign of the Casimir force is notoriously slippery, and for other spins or other geometries it could easily be negative. For $D > 4$, c_1 must be positive and f_0^2 must be *negative*. Therefore this simple model is only viable for $D < 4$.

There are other problems with the model. If the potential is constructed along the lines of the previous section it is found that the static extremum is a local *maximum* of the potential. The potential is shown in Fig. 6. The point $\phi/\phi_0 = 1$ is the point where a *and* b are static. The potential becomes flat for large b, but there is a small b instability. This potential is sicker than Casimir+Λ or monopole+Λ. The same problem occurs for a product of D-spheres for the internal space.

The presence of fermion condensates in the Chapline-Manton action can cure the problem. Assume that $\mathrm{Tr}\bar{\chi}\Gamma_{MNP}\chi$ and $\bar{\lambda}\Gamma_{MNP}\lambda$ also have the Freund-Rubin form on a product of three S^3's. [6] The radius of one of the S^3's will be assumed to be much larger than the other two radii which will be assumed to be equal. If all other background fields are set to zero, a classically stable ground state with potential given by Fig. 7 is obtained. The new ingredient present in this model is that the presence of the fermion condensates change the right hand side of the Einstein equations. For the monopole+Casimir example on a single S^D, the coefficients of the monopole terms in the (00) and ($\mu\nu$) equations were fixed to be

[6] The dilaton is assumed to be a constant in space-time, $\sigma = \sigma_0$. The dilaton field equation gives $(H_{MNP})^2 = (3/2)\exp(\sigma_0/2)H_{MNP}(\mathrm{Tr}\bar{\chi}\Gamma^{MNP}\chi)$.

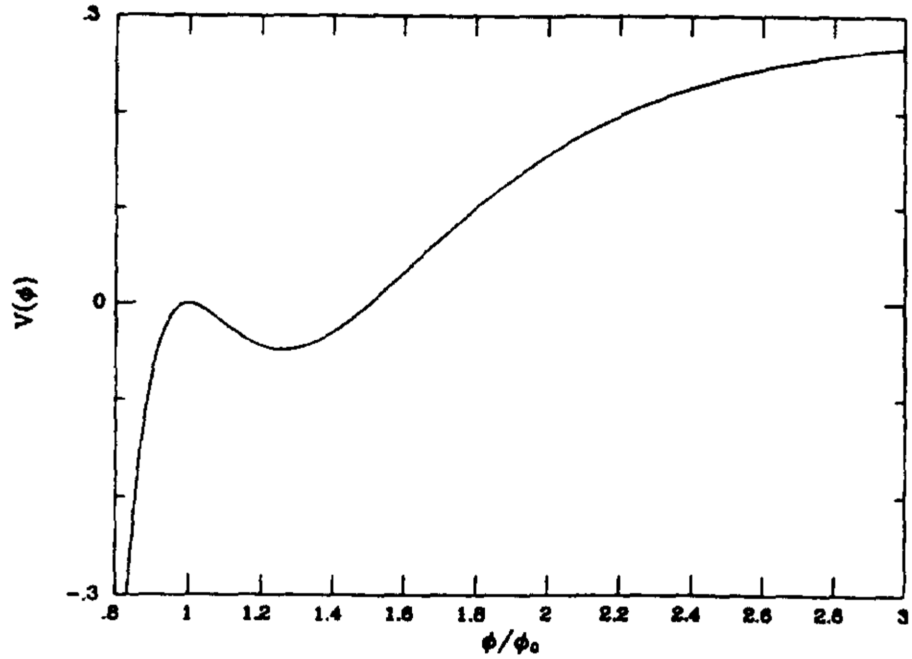

Figure 6: The potential for the Casimir + monopole case

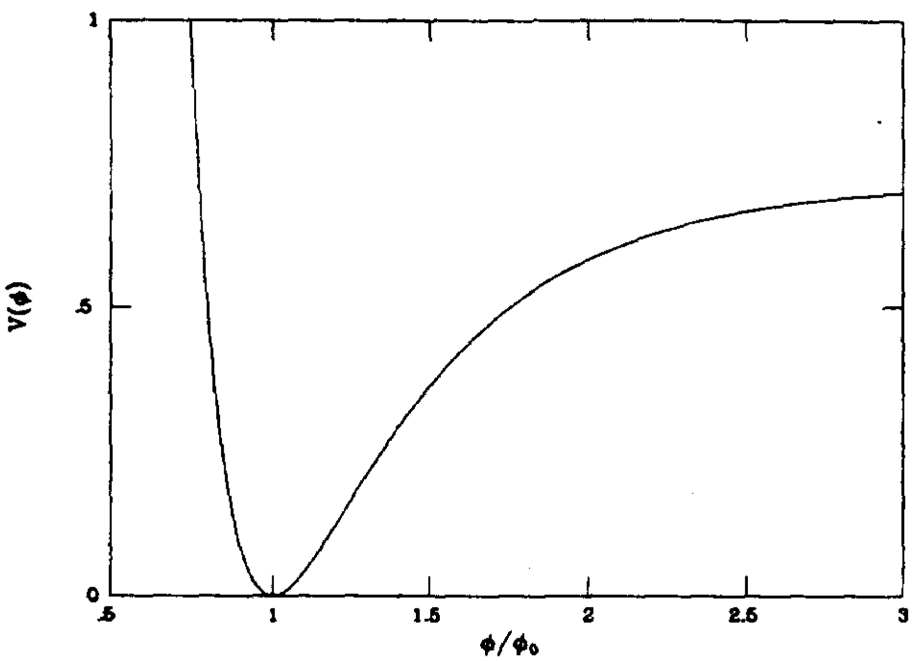

Figure 7: A possible potential for the Chapline-Manton action

THEORY	a	b
Open	$9/2$	$\pi\sqrt{8}(\alpha')^{1/2}$
Closed	10	$\pi\sqrt{8}(\alpha')^{1/2}$
Heterotic	10	$\pi(2+\sqrt{2})(\alpha')^{1/2}$

Table 4: Density of states for superstring theories: $\rho(m) \propto m^{-a}\exp(bm)$ as $m \to \infty$

in the ratio $(D-1)/3$ (see Eq. 4.7). With the addition of fermion condensates this is no longer true. A stable ground state can be found (at least in the limit that the radii of the two internal S^3's are not too different).

It should be noted that the potential in Fig. 7 is *not* the potential for inflation. The effective four-dimensional cosmological constant vanishes as b becomes large. This is simply because as $b \to \infty$ there are no stresses in the vacuum to drive inflation. This is rather different than the usual case that the further a scalar field is displaced from the origin, the larger the cosmological constant.

One of the lessons from new inflation is that there is a lot to be learned by models that fail. All of the models for stable extra dimensions and inflation from extra dimensions either fail or have some very undesirable features. Hopefully the lessons learned from these failures will point the way to a more attractive model.

5. LIMITING TEMPERATURE IN SUPERSTRING MODELS [17]

The thermodynamic properties of string theories have been studied for many years. All string models have a density of states $\rho(m)$=number of states with mass between m and $m+\delta m$ that increase exponentially with mass for large mass. In the large mass limit

$$\rho(m) = cm^{-a}\exp(bm). \tag{5.1}$$

The constant c will be uninteresting. The constants a and b depend upon the theory. Some examples are given in Table 4. In Table 4 α' is the "Regge slope" of the string theory. For superstrings α' is expected to be of order m_{Pl}^{-2}.

The traditional way to discuss the thermodynamics of superstrings is to start with the canonical ensemble. The partition function for the canonical ensemble is

$$\ln Z = \frac{V}{(2\pi)^9}\int dm\,\rho(m)\int d^9k\,\ln\left[\frac{1+\exp\left[-(k^2+m^2)^{1/2}/T\right]}{1-\exp\left[-(k^2+m^2)^{1/2}/T\right]}\right]$$

$$\simeq \; V \sum_{n=0}^{\infty} \left[\frac{1}{2n+1}\right]^5 \int_\eta dm\, m^{-a} \exp(bm) m^5 K_5[(2n+1)m/T], \qquad (5.2)$$

where V is the (9-dimensional) spatial volume, η is the mass below which the exponential form of ρ is a bad approximation, and K_n is a modified Bessel function of the second kind. Using the limiting form $K_n(x) \to x^{-1/2}\exp(-x)$ the partition function may be expressed in terms of the incomplete gamma function

$$\ln Z \simeq \left(\frac{TT_0}{T_0 - T}\right)^{-a+11/2} \Gamma\left[-a + \frac{11}{2}, \eta\left(\frac{T_0 - T}{TT_0}\right)\right], \qquad (5.3)$$

where $T_0 = b^{-1}$.

The partition function diverges for $T \geq T_0$. The pressure (p), average energy ($\langle E \rangle$), and specific heat (C_V) are given in terms of $\ln Z$ by

$$p = T\frac{\partial \ln Z}{\partial V}; \quad \langle E \rangle = T^2 \frac{\partial \ln Z}{\partial T}; \quad C_V = \frac{d\langle E \rangle}{dt}. \qquad (5.4)$$

For $a \leq 13/2$, all diverge as $T \to \infty$. For $a > 13/2$, p and $\langle E \rangle$ approach a constant as $T \to T_0$. For $a > 15/2$, C_V also approaches a constant. If the thermodynamic quantities approach a constant as $T \to T_0$, T_0 is *not* a limiting temperature. Therefore the open string has a limiting temperature, but the closed or heterotic string does not. What is happening in this case is that the energy fluctuations are becoming so large that the thermodynamic description based upon the canonical ensemble breaks down. In this case it is more appropriate to use the microcanonical ensemble. When the microcanonical ensemble is used it is found that the most likely configuration is that one string carries almost all the energy and the remaining strings have very little energy. The specific heat in this case is *negative*.

The negative specific heat is quite interesting. A system of strings cannot come into thermal equilibrium with a heat bath. The negative specific heat also obtains for black holes. A possible connection between black holes and superstrings has been the subject of recent speculation.

6. GUT SYMMETRY BREAKING IN EXTRA DIMENSIONS

It has been shown that the phase transitions associated with spontaneous symmetry breaking have a multitude of interesting physical and cosmological effects. In theories with extra dimensions there is a new type of mechanism for symmetry breaking that does not depend upon the Higgs mechanism. The new mechanism depends upon a topological non-trivial nature of the internal space and will be referred to as topological symmetry breaking (TSB) [18].

In the absence of external sources the vacuum configuration for gauge fields is $F^a_{MN} = 0$. If the fields are defined on a topologically trivial manifold, the vanishing of F implies that $A^a_M = 0$ also. However if the manifold is not simply connected, then the vanishing of F in the vacuum does *not* imply that $A^a_M = 0$. $A^a_M \neq 0$ implies that the gauge symmetry is broken.

To determine the details of symmetry breaking the relevant quantity is the Wilson line $\vec{\mathcal{U}}$ related to the path-ordered exponential

$$\vec{\mathcal{U}} = P \exp \left(\oint_\Gamma \vec{A}_\mu dx^\mu \right) \tag{6.1}$$

where Γ represents some path in the manifold. If there are non-contractible paths in the manifold, then $\vec{\mathcal{U}} \neq 1$ and the original symmetry $\mathcal{G}$ is broken to some subgroup $\mathcal{H}$ that commutes with $\vec{\mathcal{U}}$. The Wilson lines replace adjoint Higgs fields.

This mechanism has very many interesting properties. Of interest here are the properties relevant for cosmology. The first question of interest is whether the symmetry will be restored at high temperature. Does $\vec{\mathcal{U}}$ go to unity if the system is put in a heat bath? Assuming there is a cosmological phase transition with this mechanism are topological defects (monopoles, cosmic strings, domain walls) produced in the transition? What is the dynamics of the evolution of the system to the ground state? If the system is away from the ground state at high temperature, can inflation occur in the evolution to the ground state?

Finally, in general there may be several possible ground states associated with different $\mathcal{H}$'s (including $\mathcal{H} = \mathcal{G}$). At the classical level at zero temperature they all have the same energy, namely zero. At finite temperature the state with the most massless degrees of freedom will have the lowest free energy. This will correspond to the unbroken state. As the temperature decreases a strong coupling phase will occur and massive bound states will form and the number of massless degrees of freedom in the unbroken state will fall below the number in one of the broken state. Will there follow a cascading of symmetry and does it have any physical effect. These questions are unanswered at present and are under investigation.

The Higgs mechanism and SSB has proved to be an interesting part of early Universe cosmology. It is likely that TSB will also.

7. REMNANTS

The final aspect of extra dimensions and cosmology that will be considered here is the survival of a stable massive particle somehow connected with extra dimensions. Before discussing specific particles it is useful to recall some facts about the survival of massive particles. The expansion of the Universe generally stops the annihilation of massive particles (mass M) at a temperature T_f given by

$$x_f \equiv M/T_f \sim \ln \left(m_{Pl} M \sigma_0 \right) , \tag{7.1}$$

where σ_0 is related to the annihilation cross section σ_A by

$$\langle |v|\sigma_A\rangle = \sigma_0 \left(\frac{M}{T}\right)^{-n} . \tag{7.2}$$

It is useful to compare the density of particles under consideration (denoted as ψ) to the entropy density. After annihilation freeze out and if entropy is conserved this ratio will be constant in the expansion. After annihilation ceases, the ratio of ψ's to entropy is given by

$$Y_\psi \sim \frac{x_f^{n+1}}{m_{Pl} M \sigma_0}. \tag{7.3}$$

In general $\sigma_0 \propto M^{-a}$. Since the effective annihilation cross section decreases with mass, the more massive a particle, the more likely it is to survive annihilation. For masses close to the Planck mass and $\sigma_0 \simeq M^{-2}$, annihilation is not effective and a particle would survive with $Y_\psi \sim 1$, i.e., about as abundant as photons. This would be a great embarrassment, since it would result in a contribution to Ω from the massive particles of about 10^{26} or so. Creation of entropy, as in inflation, could greatly reduce this number. If inflation occurs and the universe is reheated to a temperature of $T_f \ll M$, the ratio of ψ to entropy would not be determined by freeze out, but would be determined by $\exp(-M/T_R H)$. It is likely that this number is too small to be interesting today, but it is possible to imagine that M is just small enough to result in an interesting value of Y_ψ.

Here "interesting" means a value large enough to one day be detectable, but small enough not to be already ruled out. The most general limit on the abundance of massive stable particles comes from the overall mass density of the Universe. For a particle of mass M, the limit $\Omega h^2 \leq 1$ implies $Y_\psi \leq 5 \times 10^{-27}(m_{Pl}/M)$, or $n_\psi \leq 1.4 \times 10^{-23}(m_{Pl}/M)\ \mathrm{cm}^{-3}$. The most useful limit is in terms of the flux of ψ's, $F_\psi \leq 10^{-16}(m_{Pl}/M)\ \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$. It is likely that very massive particles would be trapped in the galaxy and contribute to the mass density of the galaxy. In this case the limit is more restrictive. The relevant limit as a function of M is shown in Fig. 8. It is denoted "ρ_G."

Now consider candidates for ψ.

• *PYRGONS* [19]: In Kaluza-Klein theories there is an infinite tower of four dimensional particles corresponding to the non-zero modes of the harmonic expansions in mass eigenstates of the higher-dimensional fields. These non-zero modes are called Pyrgons.

In the five-dimensional theory the mass spectrum of the pyrgons is a series of spin-2 particles with mass $m_k = kR^{-1}$, where k is an integer and R is the radius of the internal space (in the five-dimensional theory the internal space is a circle). In the five-dimensional theory the $k = 1$ pyrgons are stable. This is because the charge operator is proportional to the mass operator. The zero modes are neutral and the $k = i$ mode has charge $e_k = i$. The kth pyrgon can decay to k number of $k = 1$ pyrgons, but the $k = 1$ pyrgons cannot decay to zero modes.

In more complicated Kaluza-Klein theories the mass spectrum is more complicated, but the general features remain, namely that there are zero modes and

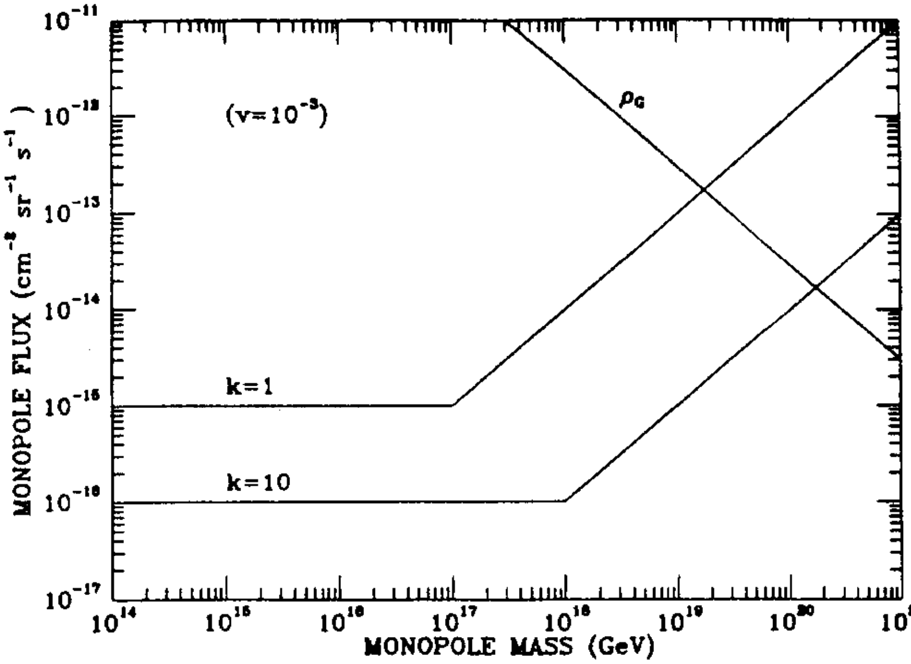

Figure 8: Flux limits as a function of mass

massive modes with mass proportional to the inverse of radii in the extra dimensions. The question of stability of the pyrgons is a more complicated one. In general there may be selection rules that prevent some massive modes from decaying. Such a selection rule is present in $N = 8$ supergravity models with an S^7 as the internal space. In general, the only reason one might imagine the pyrgons to be stable is if the pyrgon has a quantum number that is not represented by zero modes, which will be assumed to include only the observed particles. One possibility is if the pyrgon breaks the relationship of electric charge and triality. If the pyrgon is color neutral with fractional electric charge, or is fractionally charged but a color singlet it could not decay to the known particles (so long as SU_3 of color is unbroken). The second possibility is that the pyrgon has a quantum number that is not shared with any new particle.

In superstring theories the gauge symmetries arise from a different source, but there still might be excitations of the extra dimensions that are stable. There might also be excited string states that are stable. In the heterotic superstring there are 8,064 zero modes, 18,883,584 $k = 1$ modes, 6,209,272,160 $k = 2$ modes, ... (remember the increase is exponential!). Some of these massive modes might be stable.

• *MONOPOLES*: Just as GUT monopoles correspond to topological defects in the orientation of the vacuum expectation value of a Higgs field, there are magnetic monopoles in Kaluza-Klein theories that correspond to topological defects in compactification [20]. The Kaluza-Klein monopoles satisfy the Dirac quantization condition $ge = 1/2$ and have masses given by $m_M \sim m_{Pl}/e \sim 10^{20}$GeV. The cosmological production of Kaluza-Klein monopoles is uncertain because there is nothing that corresponds to a Kibble mechanism. It is unclear what the high-temperature behavior of the SSB will be [15]. In this case the SSB corresponds to the process of compactification, i.e., the symmetry breaking $\mathrm{Diff}^{D+4} \rightarrow \mathrm{Diff}^4 \times I$ where Diff^n is the diffeomorphism group in n dimensions and I is the isometry group of the internal space. Since the symmetry breaking that gives rise to the Kaluza-Klein monopoles is topological in nature, the restoration of the symmetry cannot be studied by classical methods.

In theories with TSB, there are additional topologically stable excitations. There are magnetic monopoles and particles with fractional electric charge [22]. The striking feature of these particles is that the minimum magnetic charge is some integer times the Dirac quantum, $g_{\mathrm{MIN}} = k g_{\mathrm{DIRAC}}$. The minimum electric charge is also determined by the integer k, $e_{\mathrm{MIN}} = e/k$. The expected cosmological abundance of these particles has not been estimated. The present flux of the magnetic monopoles is limited by the Parker bound, which is the maximum number of monopoles that can be present without "shorting out" the galactic B-fields. The Parker limit as a function of mass and magnetic charge is shown in Fig. 8 [23]. Of course, it is always possible to avoid the Parker limit if the monopoles are abundant enough that coherent oscillations of the monopoles are the source of the galactic B-field [24].

There are perhaps other possibilities for massive stable particles. The searches for massive stable particles in cosmic rays should be pushed. The detection of any particle with mass comparable to the Planck mass would have enormous implications for both particle physics and cosmology.

ACKNOWLEDGEMENTS

This work was supported by NASA and the Department of Energy. I would like to thank Richard Holman and Frank Accetta for a careful reading of the manuscript. The section on inflation has been greatly influenced by Michael Turner's writings on the subject.

REFERENCES

1. M. J. Duff, B. E. W. Nilsson, and C. N. Pope, *Phys. Rep.* **130**, 1 (1986).

2. J. H. Schwarz, *Superstrings* (World Scientific, Singapore, 1985).

3. E. W. Kolb, M. J. Perry, T. P. Walker, *Phys. Rev. D* **33**, 869 (1986).

4. G. Chapline and N. Manton, *Phys. Lett.* **120B**, 105 (1983).

5. A. Chodos and S. Detweiler, *Phys. Rev. D* **21**, 2176 (1980).

6. J. Barrow, *Phys. Rep.* **85**, 1 (1982).

7. A. Hosoya, L. G. Jensen, J. A. Stein-Schabes, Fermilab preprint.

8. S. Randjbar-Daemi, A. Salam, and J. Strathdee, *Phys. Lett.* **135B**, 388 (1984).

9. P. Candelas and S. Weinberg, *Nucl. Phys.* **B237**, 397 (1984).

10. E. Alvarez and M. Belen-Gavela, *Phys. Rev. Lett.* **51**, 931 (1983); E. W. Kolb, D. Lindley, and D. Seckel, *Phys. Rev. D* **30**, 1205 (1984).; R. B. Abbott, S. Barr, and S. Ellis *Phys. Rev. D* **30**, 720 (1984).

11. Y. Okada, *Phys. Lett.* **150B**, 103 (1985).

12. Q. Shafi and C. Wetterich, *Phys. Lett.* **129B**, 387 (1983).

13. J. Frieman and E. W. Kolb *Phys. Rev. Lett.* **55**, 1435 (1985).

14. A. Linde, *Nucl. Phys.* **B216**, 421 (1983).

15. M. Yoshimura, *Phys. Rev. D* **30**, 344 (1984).

16. F. Accetta, M. Gleiser, R. Holman, and E. W. Kolb, *Nucl. Phys.* B276, 501 (1986).

17. M. Bowick and S. Wijewardhana, *Phys. Rev. Lett.* **54**, 2485 (1985).

18. Y. Hosotani, *Phys. Lett.* 129B, 193 (1983).

19. E. W. Kolb and R. Slansky, *Phys. Lett.* **135B**, 378 (1984).

20. R. Sorkin, *Phys. Rev. Lett.* **51**, 87 (1983); D. Gross and M. J. Perry, *Nucl. Phys.* **B226**, 29 (1983).

21. J. A. Harvey, E. W. Kolb, and M. J. Perry *Phys. Lett.* **149B**, 465 (1984).

22. X.-G. Wen and E. Witten, *Nucl. Phys.* **B261**, 651 (1985).

23. H. M. Hodges, E. W. Kolb, and M. S. Turner, Fermilab preprint.

24. I. Wasserman, S. Shapiro, and R. Farouki, *Comments on Astrophysics* **X**, 257 (1985).