Fermi National Accelerator Laboratory

FERMILAB-Pub-86/127-A
September 1986

COSMIC STRING INTERACTIONS

E. Copeland

and

N. Turok

Theoretical Astrophysics Group
Fermi National Accelerator Laboratory
Batavia, IL 60510

and

*Blackett Laboratory**
Imperial College
London SW7 2BZ

Abstract

The interaction between two intersecting cosmic strings is discussed and a simple model describing the process presented. It appears to be in reasonable agreement with the numerical results of Shellard for global strings and gives a new prediction for local strings. The model suggests that cosmic strings almost always reconnect the other way when they cross.

*Permanent address

Cosmic strings[1] provide an attractive theory for the origin of galaxies and clusters of galaxies.[2,3] Recently a lot of progress has been made in understanding the evolution of a network of strings and in comparing the predicted distribution of galaxies and clusters with observation.

Numerical simulation of evolving networks of cosmic strings indicate that the string evolves in a scaling solution, the density in strings remaining a fixed fraction of the total density.[4] Turok has used these simulations to calculate the two-point correlation function of loops, which agrees remarkably well with the observed correlation function of Abell's rich clusters of galaxies.[5] It has also been shown that strings produced at a symmetry breaking scale of $10^{16} GeV$ or so yield loops with the right masses to produce galaxies and clusters of galaxies.[6]

The above results rest crucially on the assumption that when two strings intersect one another they always reconnect the other way. This makes it possible for strings longer than the horizon to gradually chop themselves up into smaller loops. If strings simply pass through one another then as Albrecht and Turok showed[4] the density in strings rapidly comes to dominate, a cosmological disaster.

This issue is not only important in cosmology but may also be testable in the laboratory (although not of course at relativistic velocities). Flux vortices are observed in type II superconductors and the "flux-cutting" mechanism involving reconnection of adjacent strings has been invoked to account for the existence of a non-zero voltage across a superconductor subject to a magnetic field. It may also be possible to observe the effect more directly.[7]

So far the only information available on this question has been from numerical simulations by Shellard of the interactions of "global" strings.[8] Shellard found the important result that when such strings cross they do indeed almost always reconnect the other way.

In this letter we develop a simple analytical model to describe the process. First we show that an initially static configuration describing two straight crossed strings breaks apart into two reconnected strings. This gives us a time for reconnection t_R. Two moving strings cross one another in a time t_X which obviously decreases as the relative velocity increases. If t_X is less than t_R we expect the strings to pass through. Otherwise we expect them to reconnect. This gives us a predicted threshold velocity above which strings pass through and below which they reconnect. Over the angular range Shellard has been able to simulate our model agrees fairly well with his results. Our model also applies to gauge strings where it predicts a similar result. If further simulations confirm the analysis, this will remove one of the main uncertainties in the theory of cosmic strings.

First we discuss global strings. These are vortex solutions for the U(1) invariant Lagrangian[8]

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \frac{1}{2} h^2 (\phi^* \phi - f^2)^2 \tag{1}$$

where ϕ is a complex scalar and f its modulus in vacuo. ϕ obeys the equation

$$\partial_\mu \partial^\mu \phi + h^2 \phi (\phi^* \phi - f^2) = 0 \tag{2}$$

For a static straight string along the z axis we use the ansatz in cylindrical polar coordinates

$$\phi(r, \theta) = \rho(r) e^{in\theta} \tag{3}$$

where n is the integer winding number of the string. (2) yields

$$\rho'' + \rho'/r - \rho n^2/r^2 - h^2 \rho (\rho^2 - f^2) = 0 \tag{4}$$

At small $r\rho$ must vanish to keep ϕ single valued, and (4) yields $\phi \sim r^n$. For a single string we have at small r

$$\begin{aligned} \phi &= f \frac{r}{R} e^{i\theta} + 0(r^2) \\ &= f \frac{(x + iy)}{R} + 0(r^2) \end{aligned} \tag{5}$$

This defines R – it is the radius within which ϕ is far from its vacuum value. According to numerical calculations[8], $R \approx 1.72 (fh)^{-1}$.

Now consider two initially static straight strings crossed at right angles (Figure 1). The phase of ϕ is obtained by adding the phases of the two strings, the latter shown by dashed arrows. Thus near the origin we must have

$$\phi(0, \underline{x}) = \alpha \frac{(x + iy)(x - iz)}{R^2} f + 0(r^3), \tag{6}$$

with α a constant. (The linear term is absent because ϕ is zero along the y and z axes so $\frac{d\phi}{dy} = \frac{d\phi}{dz} = 0$. A term linear in x would have the wrong winding number. α is unknown at this stage, but for two strings crossing we would not expect α to be very much smaller than unity - this would imply that the crossed strings were much fatter than a single string. α is positive if ϕ is to match onto the configuration at infinity without additional zeros. We will assume this to be true.

How does such a configuration evolve? (2) yields

$$\ddot{\phi}(0,\underline{0}) = \underline{\nabla}^2\phi(0,\underline{0}) = 2\alpha R^{-2} f \tag{7}$$

so that for small times,

$$\phi(t,\underline{0}) \simeq \alpha R^{-2} f t^2 \tag{8}$$

Thus ϕ becomes real and positive at the origin. Figure 2 shows the phase of ϕ on two planes intersecting the origin. Both strings pass through abcd in the same direction – from (8) it follows by continuity that there are *two* zeros in abcd which move apart horizontally as t increases. By contrast the zero in the plane efgh disappears. Thus the two strings reconnect the other way and move apart.

If the two strings are initially at an angle $\theta \neq \frac{\pi}{2}$ we replace y in (6) by $y \sin\theta - z\cos\theta$ and instead of (8) we have

$$\begin{aligned} \phi(t,\underline{0}) &\simeq \alpha R^{-2} f t^2 (1-\cos\theta) \\ &\equiv f(\frac{t}{t_R})^2 \end{aligned} \tag{9}$$

where t_R gives the timescale for ϕ to approach its vacuum value and thus reconnection to be completed

$$t_R = \frac{R}{\sqrt{\alpha(1-\cos\theta)}} \tag{10}$$

It is interesting that the same value for t_R is obtained for $\theta = \frac{\pi}{2}$ if one considers the configuration to be described by two separate infinitely thin (Nambu) strings with right angle corners. One finds that the corner points move away from each other at a velocity $v = \frac{1}{\sqrt{2}}$ so the (Lorentz contracted) core radius $R/\sqrt{\alpha}\gamma$ is covered in $t_R = R/\sqrt{\alpha}\gamma v = R/\sqrt{\alpha}$ just as in (10).

We wish to compare (10) with the time it takes for two moving strings to cross one another. In the centre of mass frame where the strings have a velocity v perpendicular to their length, the radius R is Lorentz contracted, and the width of the string $W = 2R/\gamma$ with $\gamma = \frac{1}{\sqrt{1-v^2}}$. Thus the time for two strings to cross,

$$t_X = 2R/\gamma v \tag{11}$$

Comparing (10) and (11) we see that $t_R < t_X$ and so strings reconnect for all velocities v such that

$$\gamma v < 2\sqrt{\alpha(1-\cos\theta)} \tag{12}$$

or

$$v < \sqrt{\frac{4\alpha(1-\cos\theta)}{1+4\alpha(1-\cos\theta)}} \tag{13}$$

Note that it is the γ factor which, at very high velocities makes t_X smaller than t_R and thus prevents reconnection.

For a network of cosmic strings, the simplest assumption is that string intersections will occur isotropically, with the number of intersections between θ and $\theta + d\theta$ being proportional to the solid angle element $sin\theta d\theta = d(-cos\theta)$. Thus intersections should be uniformly distributed in $-cos\theta$.

In Figure 3 the threshold velocity v is plotted against $-cos\theta$ for $\alpha = .5$, 1 and 2. The result is not very sensitive to α for α of the order of unity or larger and indicates that for most collisions the strings will reconnect the other way.

Unfortunately with global strings there is a complication in relating our result to the simulations. There is a long range r^{-1} force between strings which is repulsive for parallel strings and attractive for antiparallel strings[8]. This means that the initial velocity of the strings is not equal to the velocity when they cross. The circles plotted in Figure 3 are Shellard's results when a correction to account for this effect was made.[8] However for angular separations much below $\frac{\pi}{2}$ the repulsive force between strings was too large to get them to cross at all. Thus it may not be possible to test the left half of Figure 3 with global strings. We do believe it applies for local strings however, as we discuss below. There is a further correction to Figure 3 for global strings due to the fact that the force between strings tends to twist the string as they approach and make them closer to antiparallel[8]. Thus for global strings the angular separation at crossing is larger than the initial angular separation. Since the curve is fairly flat anyway this is probably a small effect, but should also be taken into account.

Now we turn to the case of local strings – strings with gauge fields. The Lagrangian (1) is replaced by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(\partial_\mu + ieA_\mu)\phi|^2 - \frac{1}{2}h^2(|\phi|^2 - f^2)^2 \tag{14}$$

with equation of motion

$$(\partial_\mu + ieA_\mu)^2\phi = -h^2\phi(\phi^*\phi - f^2)$$

$$\partial^\nu F_{\mu\nu} = ie(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) + 2e^2 A_\mu\phi^*\phi \tag{15}$$

For a single string along the Z axis we use the ansatz in cylindrical polar coordinates

$$\phi = \rho(r)e^{i\theta} \qquad A_\theta = A(r) \tag{16}$$

with other components zero and find that for regularity $\rho \sim r$ and $A \sim r$ at small r. For our static configuration corresponding to two crossed strings we use the product of the ϕ field for two individual strings, as before, and the sum of the gauge fields. Because A vanishes near the origin and $\underline{\nabla} \cdot \underline{A} = 0$ we find exactly the same behaviour for $\phi(t, \underline{0})$ as in (9). Thus the evolution of the higgs field near the origin is unaffected by the presence of the gauge fields and the result (12) also applies. In this case, however, we expect the behaviour predicted in Figure 3 to be more accurately followed, since the forces between strings are short range[9].

In conclusion, we have presented an explanation for the numerical results on intersection of global strings and given a prediction for local strings. It would be useful to test the model by plotting diagrams of the phase of the higgs field as in Figure 1 and by determining the parameter α in equation (6). Hopefully this should also soon be possible with local strings[10].

If our model for cosmic string interactions is verified by detailed comparison to numerical simulations , it indicates that when strings cross they do almost always reconnect the other way. This will put the key assumption of earlier work[1–6] on cosmic strings on firmer ground.

Acknowledgements

We are grateful to A. Albrecht, M. Hindmarsh, T.W.B. Kibble and E.P. Shellard for helpful discussions. We also thank Fermilab and E. Kolb for hospitality during our visit and the SERC for financial support.

References

1. T.W.B. Kibble, *J. Phys.* **A9**, 1397 (1976). For a review, see A. Vilenkin, *Physics Reports* **121**, 263 (1985).

2. Ya.B. Z'eldovich, *M.N.R.A.S.* **192**, 663 (1980).

3. A. Vilenkin, *Phys. Rev. Lett.* **46**, 1169, 149(E) (1981).

4. A. Albrecht and N. Turok, *Phys. Rev. Lett.* **54**, 1868 (1985).

5. N. Turok, *Phys. Rev. Lett.* **55**, 1801 (1985).

6. N. Turok and R. Brandenberger, *Phys. Rev.* **D33**, 2175 (1986).

7. J.R. Clem, *Phys. Rev. Lett.* **B19**, 4486 (1979), *Phys. Rev.* **B26**, 2472 (1982).

8. E.P. Shellard, *D.A.M.T.P.* preprint, 1986.

9. H.B. Nielsen and P. Olesen, *Nucl. Phys.* **B61**, 45 (1973), L. Jacobs and C. Rebbi, *Phys. Rev.* **B19**, 4486 (1979).

10. R. Matzner, private communication.

Figure Captions

Figure 1: Configuration of fields corresponding to two initially static strings crossed at right angles, along the y and z axes. Dashed arrows show the phases of the field ϕ for each string individually on the minimum of the potential Vmin (inset). The phase of the configuration shown is obtained by *adding* these phases.

Figure 2: The phases of the resulting higgs field configuration in Figure 1 on the planes abcd and efgh intersecting the x axis. abcd has both strings passing through in the same direction, efgh has them passing through in the opposite direction.

Figure 3: The predicted threshold velocity v as a function of angle between the strings θ for three values of the parameter α described in the text. The circles show the approximate numerical results of Shellard from ref. 8. The deviation from horizontal in the numerical results is within the estimated errors[8].

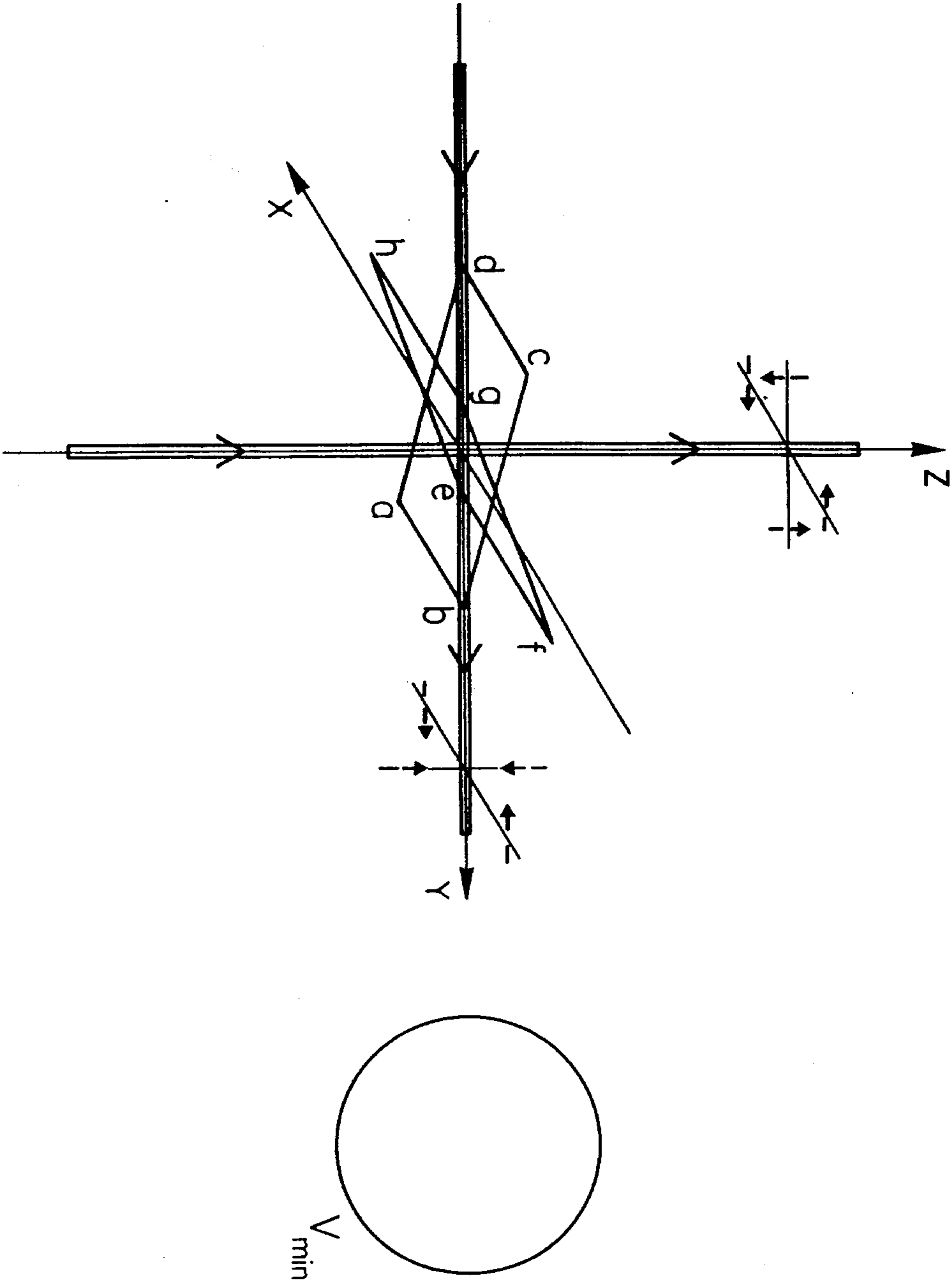

Figure 1

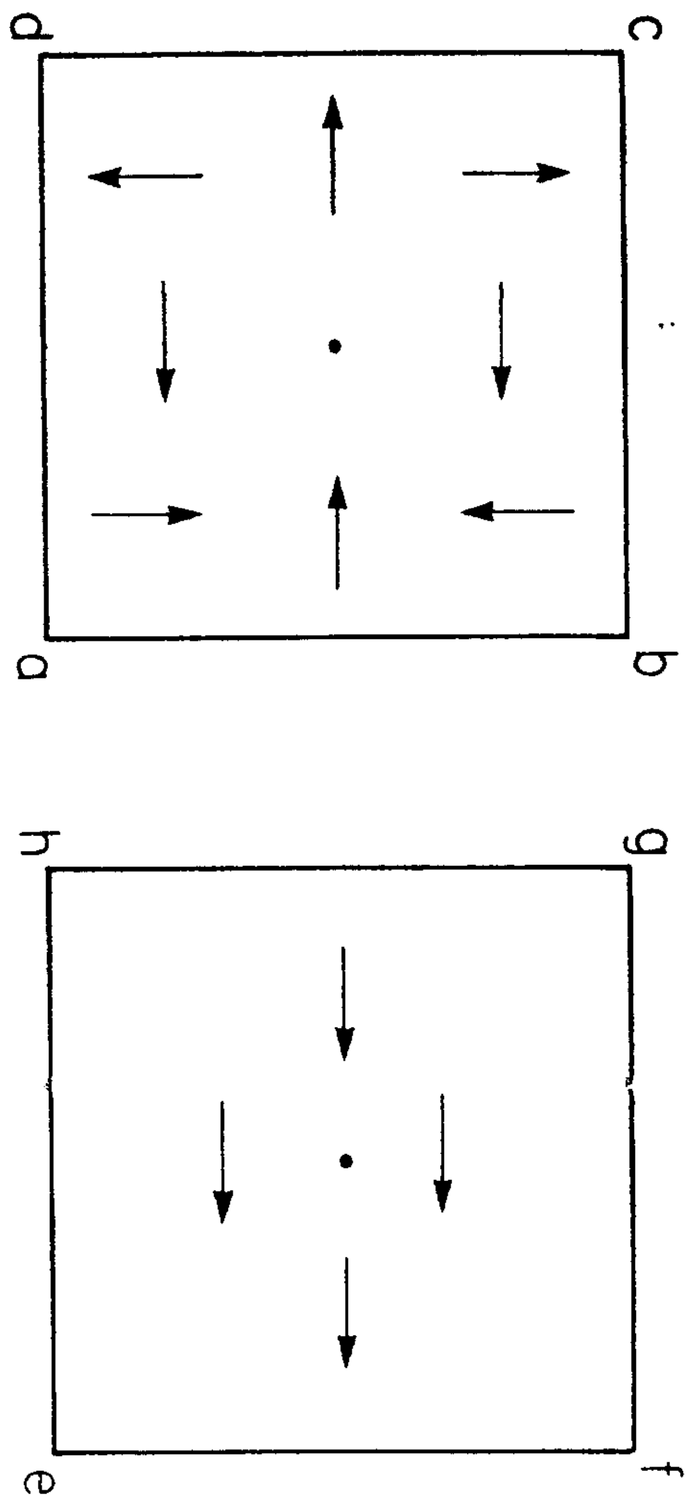

Figure 2

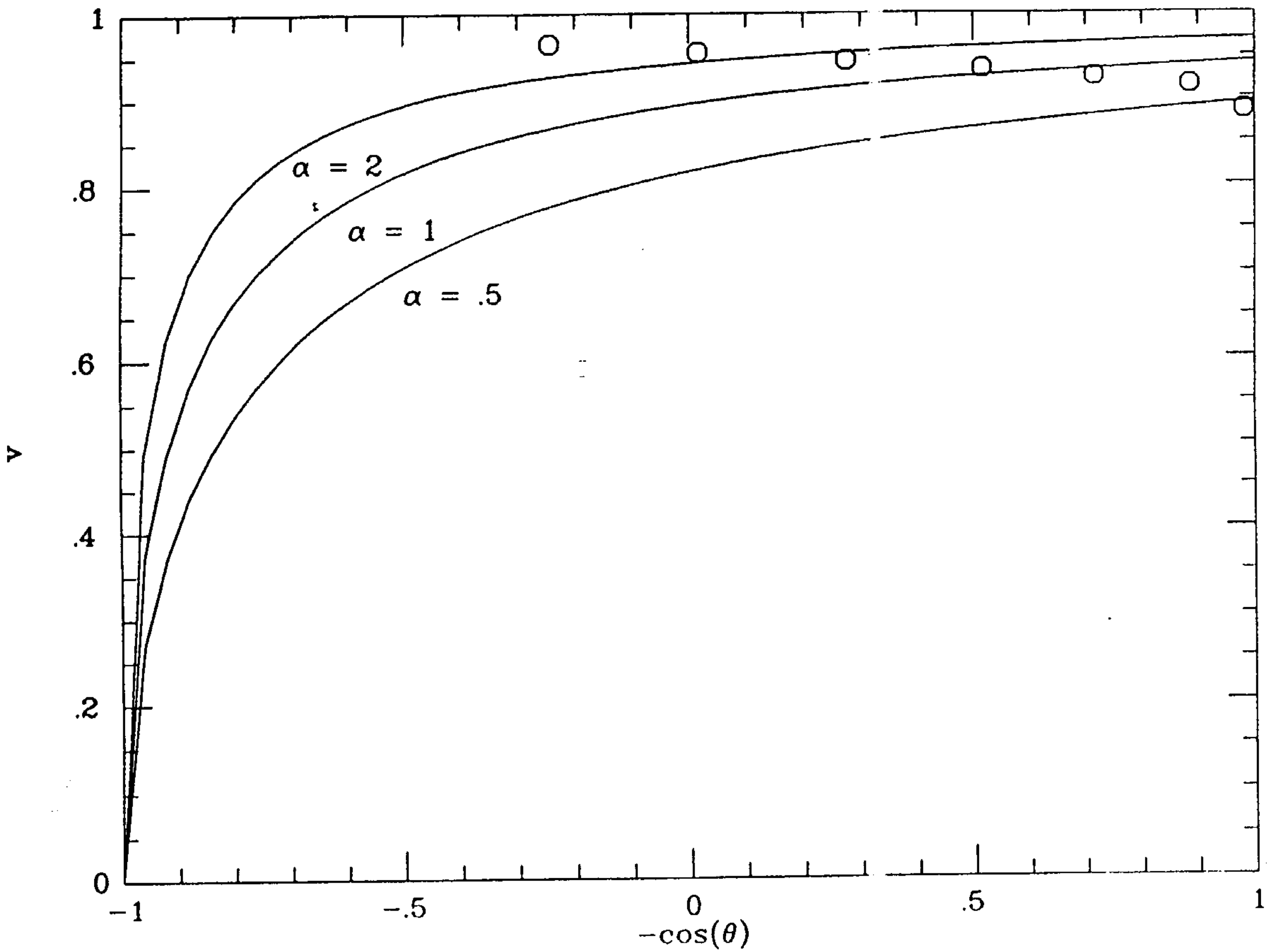

Figure 3